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CONTENTS.

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[Continued on page (III) of Cover.

THE APPLICATION OF PROPELLER FANS TO THE COOLING OF ELECTRICAL MACHINES.

By RALPH POOLE, Associate Member.

(*Paper first received* 20*th August, and in final form* 18*th December*, 1934; *read be ore* THE INSTITUTION 14*th February*, 1935.)

SUMMARY.

In a recent paper* the author dealt with the design of propeller fans, and in the present paper he discusses the advantages of propeller fans as applied to the cooling of electrical machines.

Illustrations are given of various types of electrical machines fitted with propeller fans, and the author points out the flexibility of design as compared with that of the centrifugal fan.

Special tests show the advantages of a fan designed according to modern aerodynamics over the simple " paddle wheel " type of fan. Reference is made to fans designed to give maximum slip-stream rotation. The author points out that the windage, efficiency of a machine with fan is far more important than that of the fan alone, and the most efficient fan does not necessarily give the most efficient machine. Great care must be taken in computing from friction and windage tests the actual power taken by the fan. The fan can be designed to rotate the air more efficiently than can the machine itself, so that the increased fan power may be more than balanced by the reduced machine windage.

The author stresses the need for good inlet conditions for the satisfactory operation of propeller fans. Tests made upon a turbo-alternator illustrate the effect of guide vanes in the air intake to propeller fans, and the author points out that both circumferential and radial guide vanes are essential.

Discussing the problem of external fans for cooling large turbo-alternators, the author shows that two fans in series on a variable load permit greater efficiencies, and therefore greater flexibility, than two fans in parallel.

Tests on two propeller fans in series bear out the author's statements.

High efficiencies are claimed for a special scheme of series propeller fans which simplifies the dampers in a turbo-alternator ventilation scheme.

INTRODUCTION.

The developments in the science of aerodynamics have done much towards solving the problem of cooling electrical machines.

When the quantity of air passing through a machine is small the actual windage loss has relatively little effect upon the overall efficiency of the machine. The increasing tendency to rely upon forced convection for cooling has, however, made the efficiency of the ventilating fan of greater importance.

It is not surprising, therefore, that the vast amount of knowledge available on the design of aeroplane propellers has been utilized in the design of highly efficient propeller or axial-flow fans.

In this paper the author discusses the application of propeller fans to the cooling of electrical machines, and in order to make certain of the points clear the following notes on aerofoil theory are included.

* "Theory and Design of Propeller Fans," Institution of Civil Engineers, Selected Engineering Papers, 1935, No. 4988.

AEROFOILS.

Whilst all bodies are subject to a resisting force or drag when passing through a fluid, it is found that certain shapes known as aerofoils experience a force at right angles to the direction of motion, many times greater than the drag forces.

This force, normal to the direction of motion, is known as the lift. The magnitude of the lift and drag depends upon the dimensions of the aerofoil, its velocity relative to the fluid, the attitude relative to the fluid, and finally the density of the fluid.

The projection of the aerofoil section upon the line drawn as a common tangent to the cambered under-

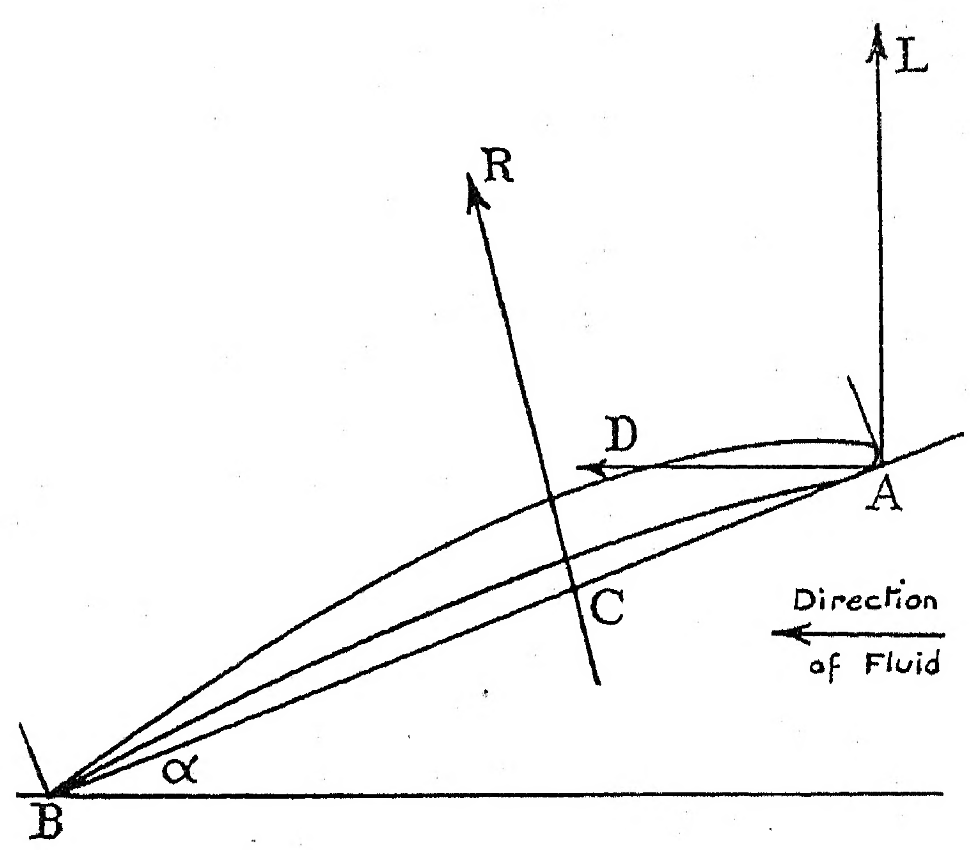

FIG. 1.

surface is known as the chord (see Fig. 1). When the underside is convex the chord is that line joining the centres of curvature of the nose and tail. The angle of incidence can now be defined as that angle which the chord line makes with the direction of motion of the aerofoil relative to the fluid.

For an aerofoil having a total projected surface S travelling (at a given angle of incidence) with a velocity v relative to a fluid of density ρ, the lift force L may be expressed as

$$L = K_L S \rho v^2$$

and the drag force D as

$$D = K_D \rho S v^2$$

K_L and K_D being constants.

Thus $K_L = L/S\rho v^2$ and is known as the " lift coefficient."

Also $K_D = D/S\rho v^2$ and is known as the " drag coefficient."

For an aerofoil of rectangular plan, S is equal to the product of the length or span and the chord.

The value span/chord is known as the "aspect ratio."

The Lift Force.

It is well known that when a rotating cylinder is moved laterally through a fluid it is subject to a lift force at right angles to the direction of motion, and this lift force ceases with the rotation. The phenomena is analogous to that occurring when a wire carrying a current is placed in a magnetic field, the distortion of the main field by the presence of the circular field being similar to the distortion of the main fluid stream by the rotating surface of the cylinder. Actually it has been found that there is a boundary layer of fluid which is stationary relative to the cylinder; the lift force may therefore be said to be due to the circulation of the fluid relative to the main stream.

It is by assuming the presence of such a circulation of fluid around an aerofoil that the lift force can be explained. Thus in Fig. 2 the aerofoil has a velocity v relative to the fluid and there is a clockwise circulation around the section of v_1. Hence the relative velocity beneath the aerofoil is reduced to $(v - v_1)$, with a consequent increase in pressure.* On the upper surface

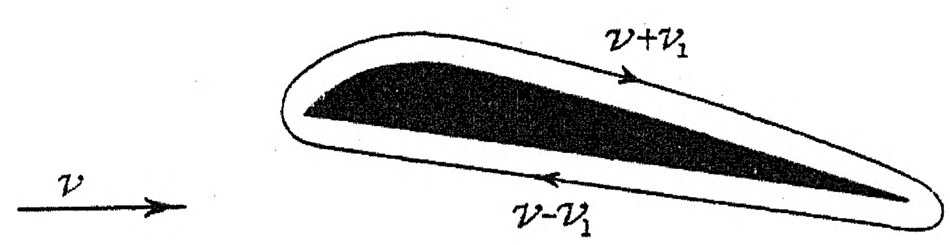

FIG. 2.

the relative velocity increases to $(v + v_1)$ causing a reduction in pressure. The aerofoil therefore experiences an upward lift force.

Actually the circulation is periodic, its frequency decreasing as the angle of incidence increases. The frequency is also directly dependent upon the velocity.

It is found that with all aerofoils there is a sudden reduction in the mean circulation beyond a certain angle of incidence, accompanied by a rapid increase in drag. When this state is reached the aerofoil is said to have stalled.

The Drag Force.

The drag force upon an aerofoil is made up of two distinct parts, one being independent of the angle of incidence and hence of the lift coefficient.

This first part is known as the "profile drag" and this itself may be subdivided into:—

(1) Skin friction, and

(2) Turbulent wake, or "form drag," as it is often called.

Prandtl has shown that the air in immediate contact with the surface of the aerofoil is stationary, whilst an infinitely small distance away the air is in motion; thus whilst the main body of the fluid may act as though it were not viscous, the thin boundary layer is in shear and offers a resistance to motion depending upon the viscosity of the fluid. This resistance is generally known as "skin friction."

The lift force was explained by the existence of a difference between the relative velocities on the upper and lower sides of the aerofoil. It will be understood that when the two streams unite at the tail of the aerofoil there will be tendency for the upper stream to roll over the lower, owing to its higher velocity and its different direction. This causes a weak vortex sheet to spring from the tail, the loss of energy due to this being known as the "form drag."

The second constituent of the drag is somewhat more complicated and is closely associated with tip leakage. Considering an aerofoil of finite length, it is evident that the pressure will be zero at the ends and maximum (negative or positive) in the centre. On the underside there is an increase in pressure, whilst on the upper

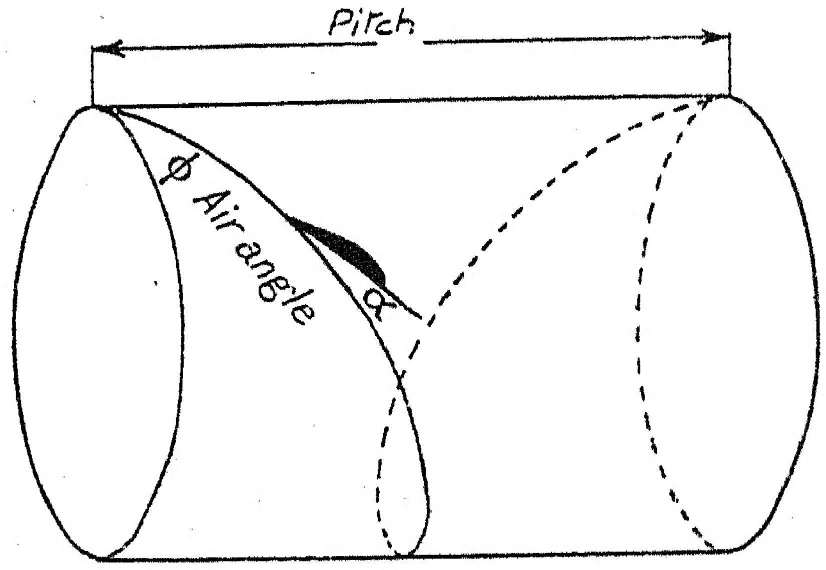

FIG. 3.

Air pitch = (air velocity)/r.p.m.

Geometric pitch = $2\pi r \tan(\phi + \alpha)$.

surface there is a decrease, hence the air tends to flow outwards from the centre on the pressure side and inwards from the ends on the suction side.

At the tip, air will flow across the end from the pressure side to the suction side. An extremely complicated vortex system is set up, causing the flow, in the normal case, to be deflected downwards. This is in effect equivalent to changing the direction of the fluid relative to the aerofoils, the effective angle of incidence being reduced. Hence the lift force becomes tilted backwards and has a component parallel to the direction of motion. This component is known as the "induced drag."

For an aerofoil of infinite length the induced drag is zero, hence for a given angle of incidence a change from finite to infinite aspect ratio involves not only an increase in lift coefficient but a decrease in drag.*

THE SIMPLE BLADE-ELEMENT THEORY OF PROPELLER DESIGN.

The nature of the forces to which an aerofoil is subjected when placed at some angle of incidence relative to a fluid moving at constant velocity has been described.

Drzewiecki suggested that a propeller or screw blade

* Bernoulli's theorem states that if there are no losses the total energy at any point is the same. Thus $P + \frac{1}{2}\rho v^2 =$ constant, P being the static pressure. Thus an increase in velocity causes a decrease in static pressure, and vice versa.

* For a more detailed treatment of the subject the reader is referred to Glauert's "Aerofoil and Airscrew Theory."

might be treated as if it were an aerofoil moving relative to the fluid, with the slight difference that its path was a helix instead of a straight line.

Hence, given the velocity of advance and the angular velocity, the resultant velocity can be found, and from the known characteristics of the aerofoil as measured in a wind tunnel it should be possible to calculate the thrust, knowing, of course, the angle which the aerofoil makes with its direction of motion. Fig. 3 shows the path of the blade relative to the fluid, the distance moved forward per complete revolution being known as the "air pitch." The geometric pitch is that distance moved forward per revolution when the blade chord actually falls along the line of motion, and the difference between air pitch and

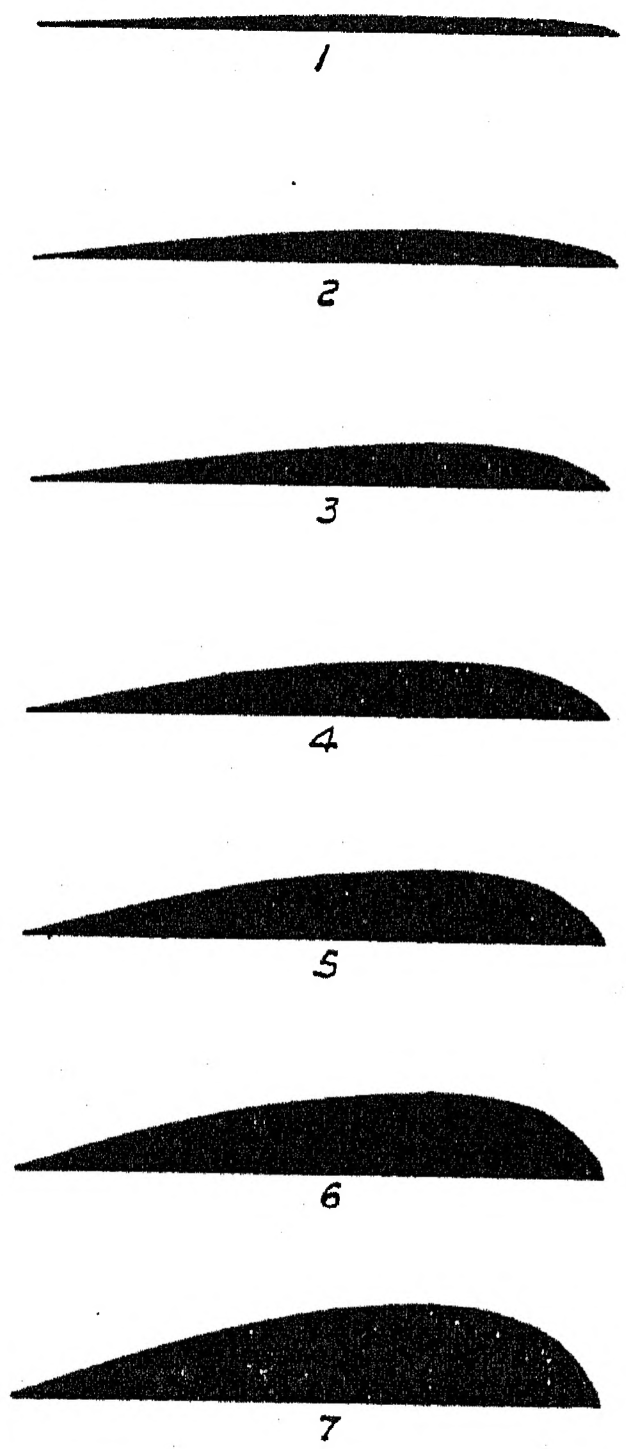

FIG. 4.—(Reproduced by kind permission of the Comptroller, H.M. Stationery Office.)

geometric pitch depends of course upon the angle of attack.

The later developments of the theory as applied to fan design are treated in the author's paper.

AEROFOIL TESTS.

As long ago as 1912 the Advisory Committee for Aeronautics published the results of an interesting set of tests upon a series of aerofoils of varying thickness. The aspect ratio was 6 and the author has therefore corrected the results for an infinite aspect ratio. Seven different sections were tested; these are reproduced to scale in Fig. 4. It will be noticed that No. 1 differs little from a flat plate.

Referring to Fig. 5, which gives the variation of absolute lift coefficient with angle of attack, we notice that the maximum lift of No. 1 (the thinnest section) is practically the same as that of No. 7 (the thickest section), but occurs at a much larger angle of attack. There is, however, a very sharp drop in the lift curve of the latter

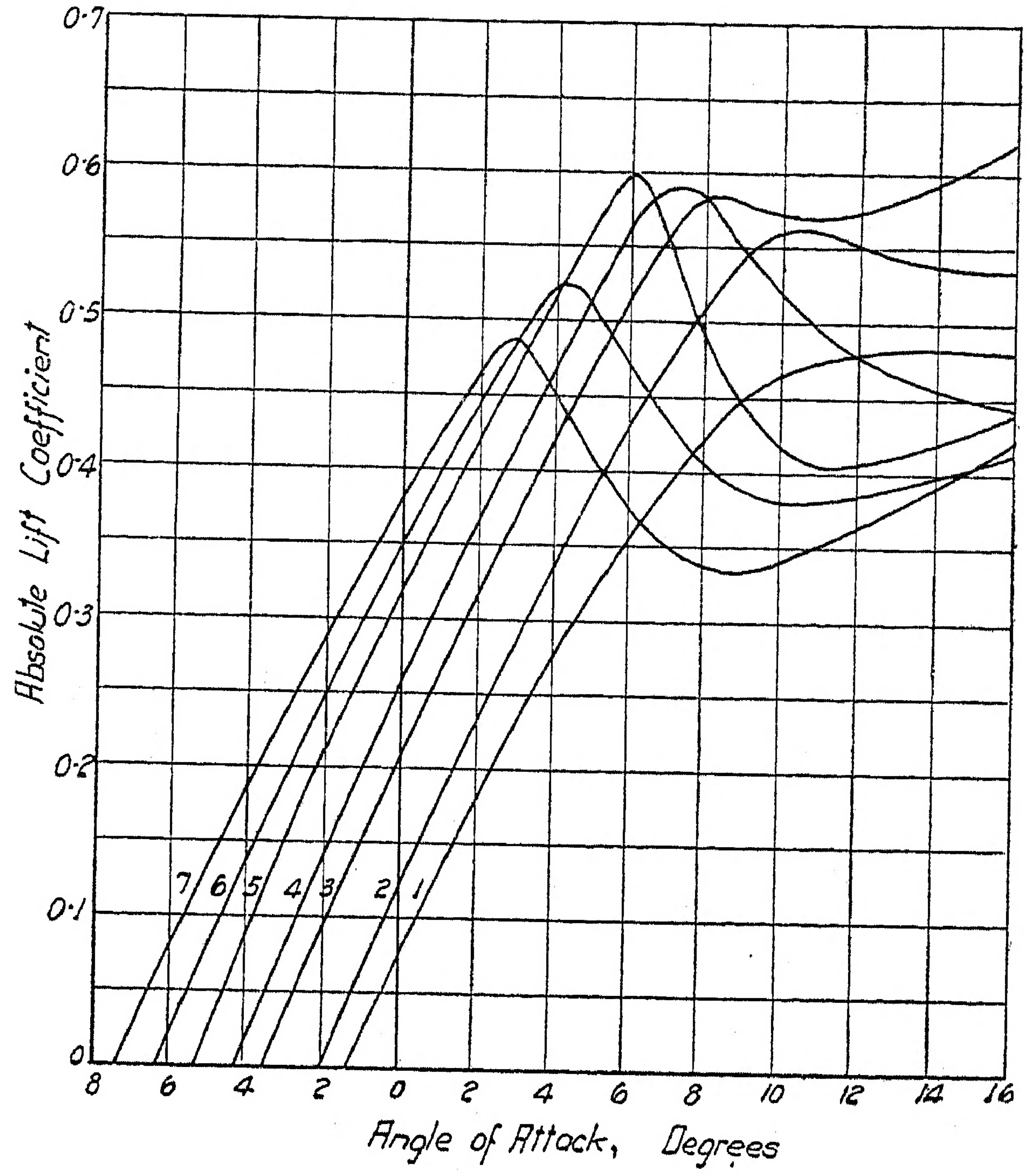

FIG. 5

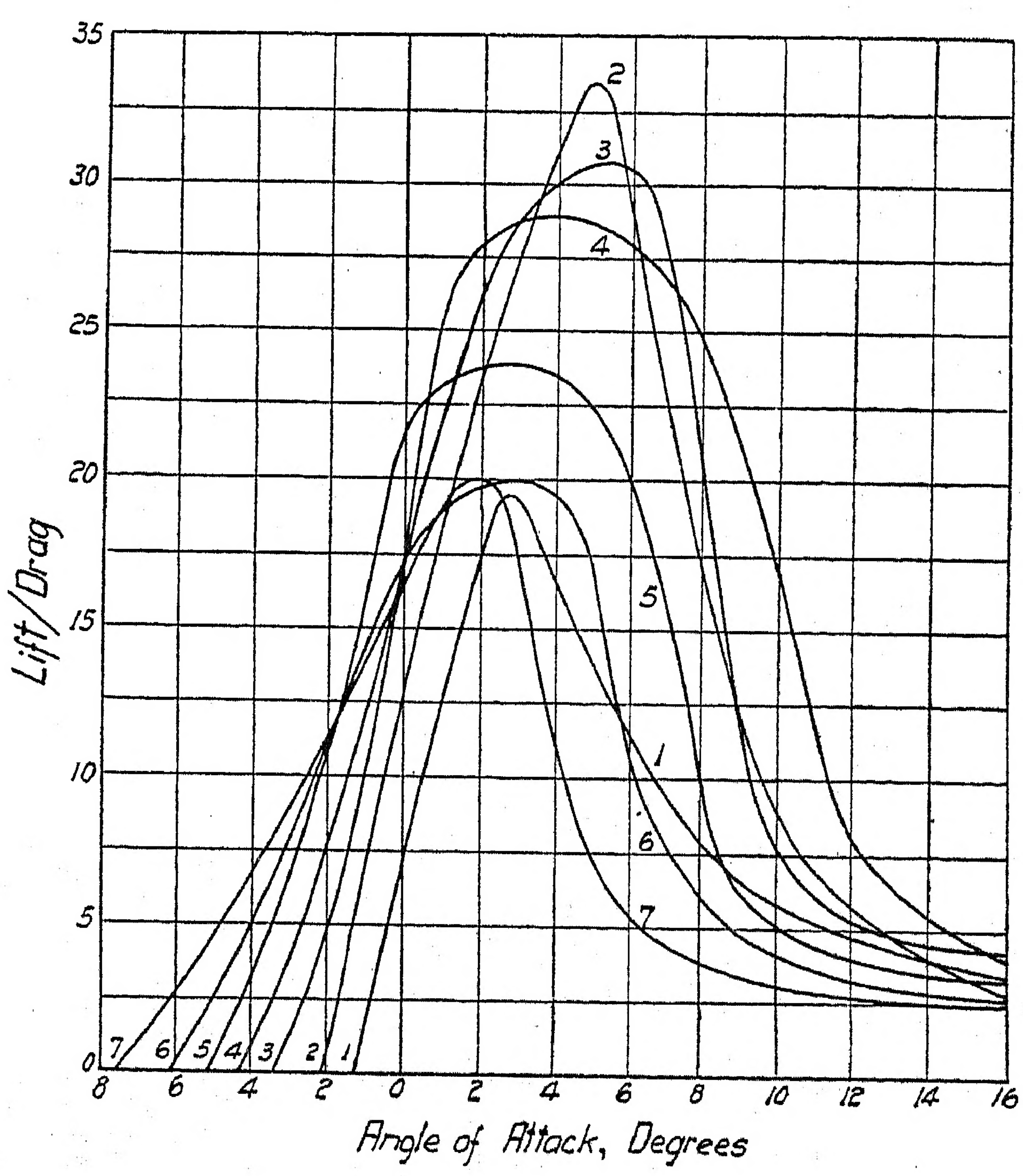

FIG. 6.

which does not occur in the case of No. 1. Since we require a section which has a flat lift curve combined with a high lift, No. 3 appears to be most suitable.

However, before making a final selection we must refer to the lift/drag curve, Fig. 6, to obtain some idea of the efficiency of the section.

Here No. 2 appears to be most efficient but has a very sharp peak and would therefore be unstable. The most stable is No. 4 with a maximum value of lift/drag of about 29. It should be noticed that the thin section has a lift/drag of only 19 and its characteristic is very sharp. For the design of fans the choice would therefore fall between Nos. 3 and 4, and from a mechanical strength point of view the final choice would rest with No. 4 with a lift coefficient of 0·47 at the maximum lift/drag of 29.

pitch is low, since a small error in blade angle gives a relatively large error in output and efficiency, the difficulty being increased if the low-pitch fan is required for a relatively high pressure, calling for a large number of blades. The presence of a large number of blades, or a high solidity, increases the rotation in the slip-stream, and therefore introduces a further loss of energy, which can only be recovered by the use of guide vanes or by rotating two series fans in opposite directions.

The propeller fan is most adaptable to the cooling of d.c. machines, since it can be fitted at the end remote from the commutator and made to blow through the machine and across the commutator, keeping out all

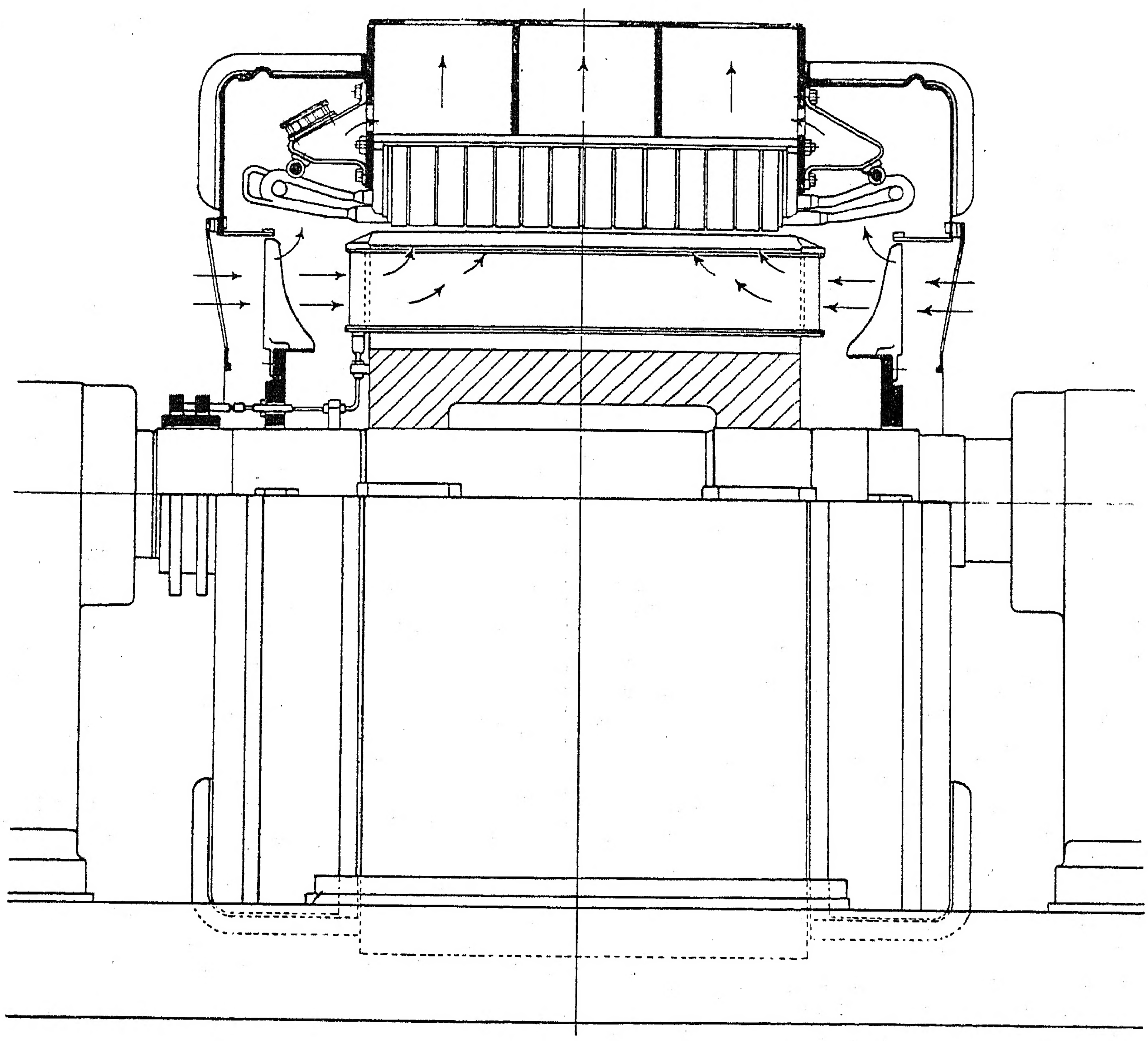

FIG. 7.—Sectional arrangement of a salient-pole synchronous motor.

FANS FOR COOLING ELECTRICAL MACHINES.

In the case of fans used for cooling electrical machines, there are limitations set by the general construction of the machine itself, and it is often found that the dimensions of the fan are decided by factors other than the fan output. This is a serious drawback in the design of a centrifugal fan, which is not very flexible in application, but it is found that propeller fans can be arranged to suit most cases, even when the centrifugal fan fails.

On the point of efficiency the propeller has a decided advantage, since if the air speed is not too high there is no great difficulty in obtaining an efficiency of 80 per cent on the static head only, whilst on the total head it is possible to obtain even higher values. The most difficult fan of the propeller class to design is that in which the

brush dust. Furthermore, this arrangement does not interfere with the accessibility of the brushgear.

In order to maintain accessibility a centrifugal fan would have to be fitted at the rear end of the machine. Its diameter would therefore have to be reduced sufficiently to allow the air to turn axially into the machine. Such a scheme gives poor armature ventilation, in addition to lengthening the machine. Salient-pole motors and alternators are ideal cases for the application of propeller fans, and Fig. 7 shows the sectional arrangement of such a machine. The rotor has propellers taking in air at each end and discharging into the spaces between the poles and then out through the vent spaces in the stator.

Fig. 8 shows propellers applied to the cooling of an induction motor having combined axial and radial

ventilation with double inlet as in the case of the salient-pole machine. In this case the air outlet is at the bottom of the machine, between the yoke feet.

Pipe ventilation becomes simple when propeller fans are adopted, as there is no longer any need for a volute or whirling chamber. Furthermore, reversibility is no longer a problem since propeller fans can be designed to operate efficiently in both directions, but it will be realized that the air flow will be reversed. This may prove a disadvantage in certain cases of pipe ventilation.

Propeller fans have been successfully applied to turbo-alternators, and Fig. 9 shows a sectional arrangement of a small high-speed machine. One end of the machine is shown with a plain shrouding around the fan, but the other has been drawn to illustrate the type of guides which are recommended when it is desired to take full advantage of the possible gain in efficiency. (The use of guide vanes is discussed more fully on page 300.)

There is no doubt that the propeller fan could be used effectively to augment the ventilation of large induction motors, which at present are almost invariably self-cooled by means of radial vents in the rotor.

Centrifugal fans are sometimes provided on the ends of the rotor beneath the stator winding, but they do little towards cooling the stator core and in many cases reduce the amount of air which would normally pass through the rotor core.

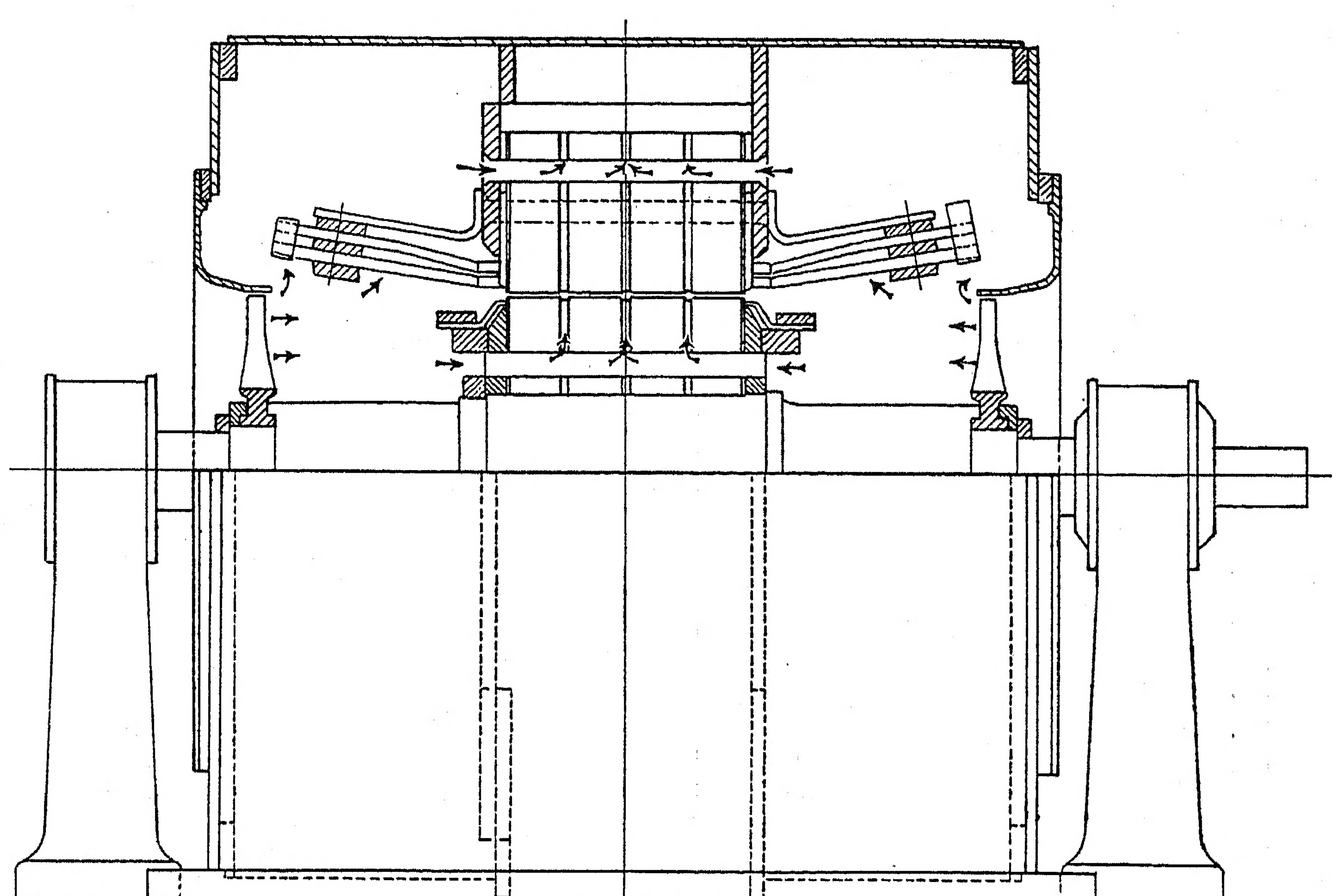

FIG. 8.—Section of a high-speed propeller-ventilated induction motor.

Comparison Between Aerofoil-Bladed Fan and Flat-Bladed Fan.

Perhaps the earliest axial-flow fans used for cooling electrical machines were of the paddle-wheel type illustrated in Fig. 10 (see Plate 1, facing page 300), no attempt being made to provide a fairing round the tips.

It has been found that the maximum volume of air was delivered when the blades were set at 45°, although this did not give the maximum mechanical efficiency.

Special tests have been carried out to compare the output of the paddle-wheel type fans with that of their more scientific successor. The rotor of the actual machine tested fitted with aerofoil type fans, is shown in Fig. 11 (see Plate 1).

Afterwards these fans were replaced by the type shown in Fig. 10 and the tests repeated.

In order to facilitate the measurement of the quantity of air passing into the rotor and out through the stator vents, a wooden duct 4 ft. long and 3 ft. square was fitted to the top of the chimney-type stator. The outlet area was divided into 9 equal sections and air measurements were taken over each section with a rotating vane anemometer.

The velocity of the air passing through the endbell spaces was measured by means of a special Pitot tube. This tube is a development of the static tube described by C. J. Fechheimer* and has been in successful use by the author's firm for some years. An article by L. S. Marks,† describing work on a similar instrument, was published recently.

The advantage of the new tube is that the static pressure and velocity pressure are measured at points very close together. It is therefore very suitable for measuring air flow in electrical machines.

The results of the tests are summarized in the Table.

Comparing the results of the tests with fairing fitted around the fans, the aerofoil fan shows an increase in air quantity of 30 per cent compared with the flat-plate fan, and there is a reduction in friction and windage of 7·5 per cent.

Assuming that the static pressure varies as (volume)^2

* *Mechanical Engineering*, 1927, vol. 49, p. 871.
† *Journal of the Franklin Institute*, 1934, vol. 217, p. 201.

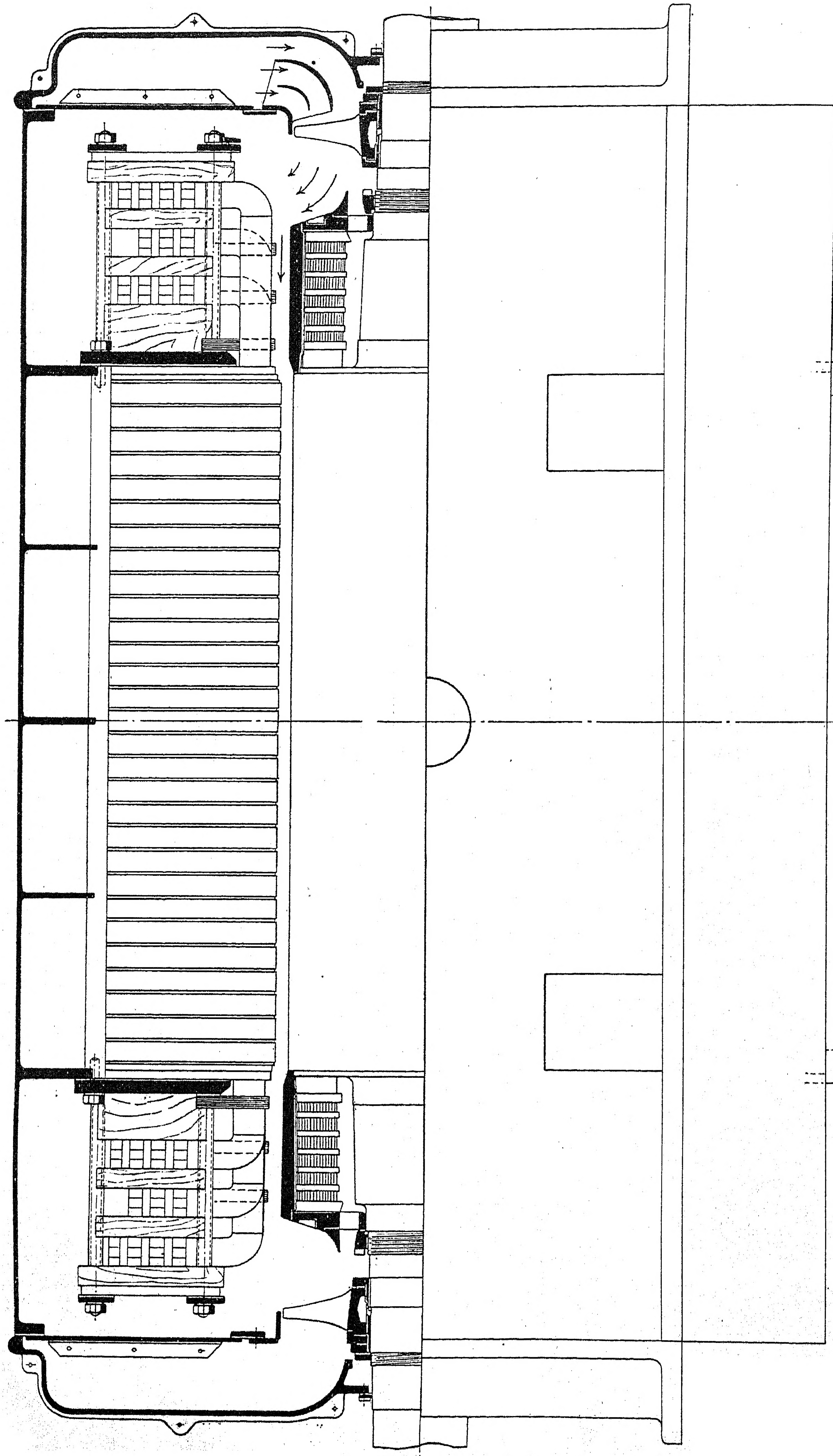

FIG. 9.—Section of propeller-ventilated turbo-alternator: 6 000 kW, 7 500 kVA, 6 600 V, 3-phase, 60 cycle, 3 600 r.p.m.

TABLE.

Propeller-Fan Tests on Salient-Pole Alternator.

Condition: Cylindrical fairing fitted around fan.

	6-bladed flat-plate fan. Constant angle of 45°	12-bladed aerofoil section. Fan angle at root 31°
Air quantity through chimney	4 940 cub. ft. per min.	5 570 cub. ft. per min.
Air quantity each endbell	1 670 cub. ft. per min.	2 570 cub. ft. per min.
Total air quantity	8 280 cub. ft. per min.	10 710 cub. ft. per min.
Friction and windage	12·85 kW	11·9 kW
Friction and Windage without Fans 9·6 kW.		
Condition: Cylindrical fairing removed.		
Air quantity through chimney	4 670 cub. ft. per min.	4 720 cub. ft. per min.
Air quantity each endbell	1 745 cub. ft. per min.	1 050 cub. ft. per min.
Total air quantity	8 160 cub. ft. per min.	6 820 cub. ft. per min.
Friction and windage	12·9 kW	13·12 kW

The bearing friction of the machine would be of the order of 1 to 1·5 kW.

(this is not true since the rotor acts as a fan in series with the main fans for the stator circuit), an assumption which is somewhat optimistic, and subtracting the measured friction and windage without fans,* the mechanical output of the aerofoil fans compared with that of the flat-plate fans is equal to

$$1\cdot3^3 \times (12\cdot85 - 9\cdot6)/(11\cdot9 - 9\cdot6) \text{ times} = 3\cdot1 \text{ times},$$

that is, an increase of 210 per cent.

The effect of the fairing is more marked in the case of the aerofoil fan than in that of the flat-plate fan. The former shows a decrease in air quantity of 36 per cent with an increase in friction and windage of 1·22 kW, or about 10 per cent. Without the fairing the flat-bladed fans are superior, the quantity being reduced by only 1·5 per cent with no appreciable increase in friction and windage. It is of great interest to note that without the fairing the quantity passing through the endbells increased at the expense of the rotor, as would be expected, but in the case of the aerofoil fan there was a reduction in quantity through both endbells and chimney.

The flat blades would be working well beyond the stall point and therefore a change in conditions would produce little effect upon output. The solidity of the aerofoil fan was rather high and the blade angle comparatively small, hence there would be a great increase in drag due to the removal of the fairing, in addition to an increase in the return flow at the tips, the fan working in what is known as the " vortex ring state."

THE EFFECT OF INLET CONDITIONS UPON THE PERFORMANCE OF A PROPELLER FAN.

The air inlet to the internal fans of a turbo-alternator is very restricted and whilst the uneven distribution of air is permissible in the case of a centrifugal fan (although there is of course a loss of pressure) it is disastrous to the satisfactory operation of a propeller fan. This is due to the fact that such distribution causes the angle of attack to vary over a wide range along the blades, since the latter depends upon the air velocity, which in the design stage is assumed to be uniform over the fan disc.

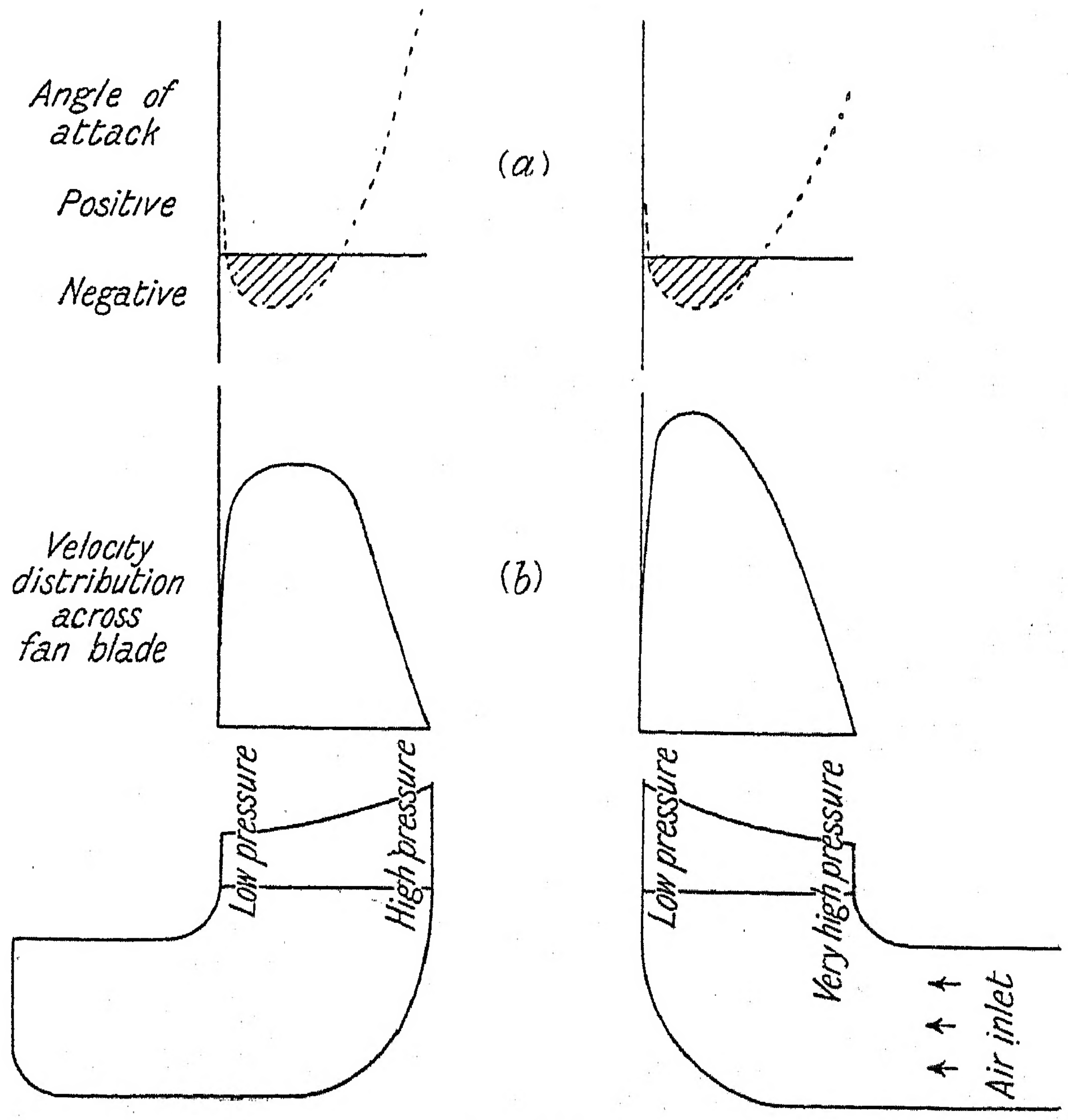

FIG. 12.

Fig. 12 represents the inlet for two blades, one immediately on the inlet side and the other diametrically opposite.

It is well known that at a right-angle bend the air velocity is highest at the side most remote from the inlet, so that the velocity distribution for the lower blade would resemble that shown in Fig. 12(*b*). The angle of

* This method is not strictly correct but actually favours the poorer fan.

attack is of course smallest where the velocity is highest and might therefore vary across the blade as shown in Fig. 12(*a*), the pressure being low at the root and high at

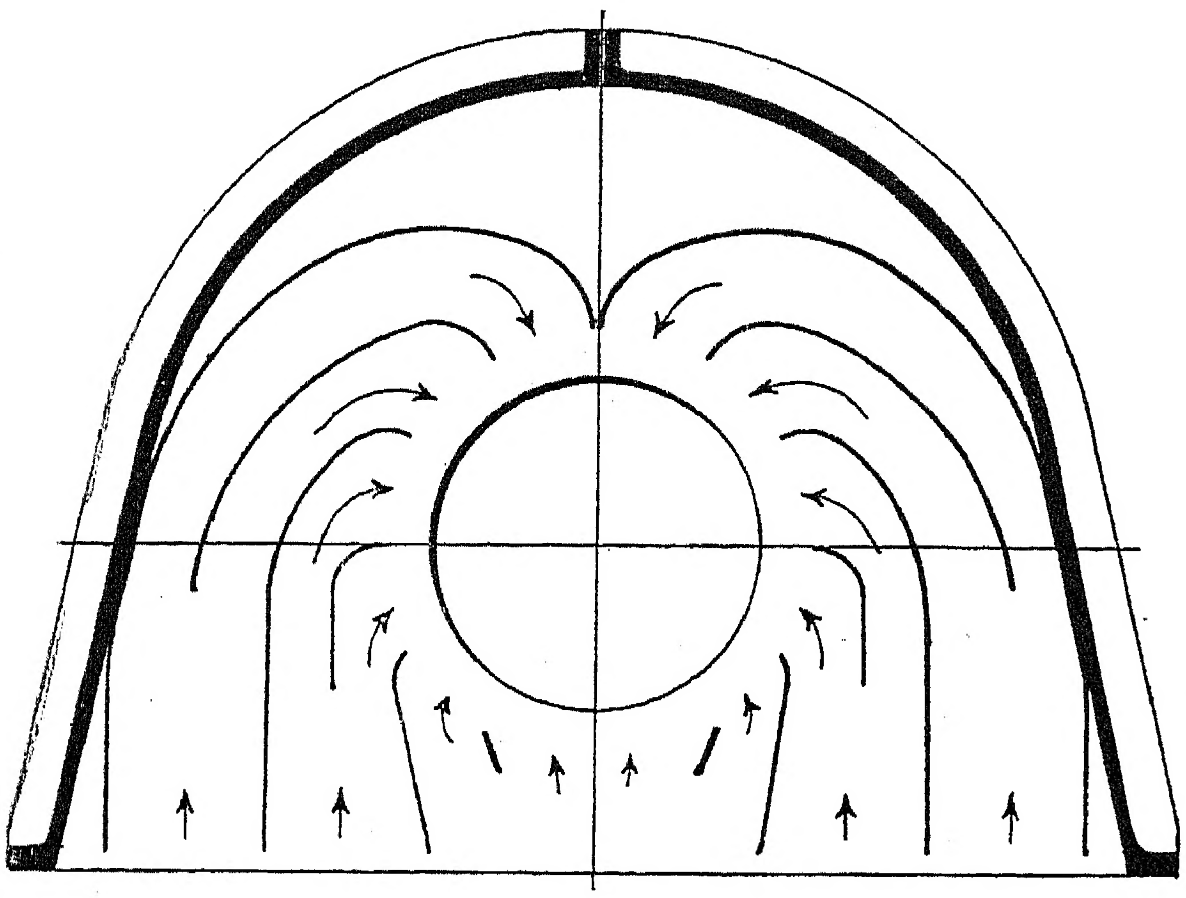

FIG. 13.

the tip. On the other hand the upper blade is subject to a high velocity at the tip and a comparatively low velocity at the root, the pressure therefore being low at

The Use of Guide Vanes in cases of Restricted Inlet.

In order to find the effect of using guide vanes in the inlet, experiments were carried out upon the small turbo-alternator illustrated in Fig. 9.

It will be noticed that the left hand of the drawing shows a plain baffle ring around the tips of the blades, whilst the right hand illustrates guide vanes at the inlet to the fan. In addition radial guide vanes were fitted to the intake or silencer (see Fig. 13) in order to distribute the air uniformly around the fan.

Fig. 14 (see Plate 2) is an end view of the alternator with half of the intake removed to show the arrangement of the guide vanes and fan.

In order to measure air quantities a special trunking was erected upon the top of the frame, the inspection covers being removed to provide a convenient outlet for the air. The trunking was about 4 ft. long, and external resistance was introduced by means of layers of cloth placed beneath a sheet of expanded metal. The air velocities were measured by means of a rotating-vane anemometer, the outlet of the duct being divided into a large number of sections, separate readings being taken over each. Static pressures were measured in the endbell of the machine.

A complete series of tests was taken and the results

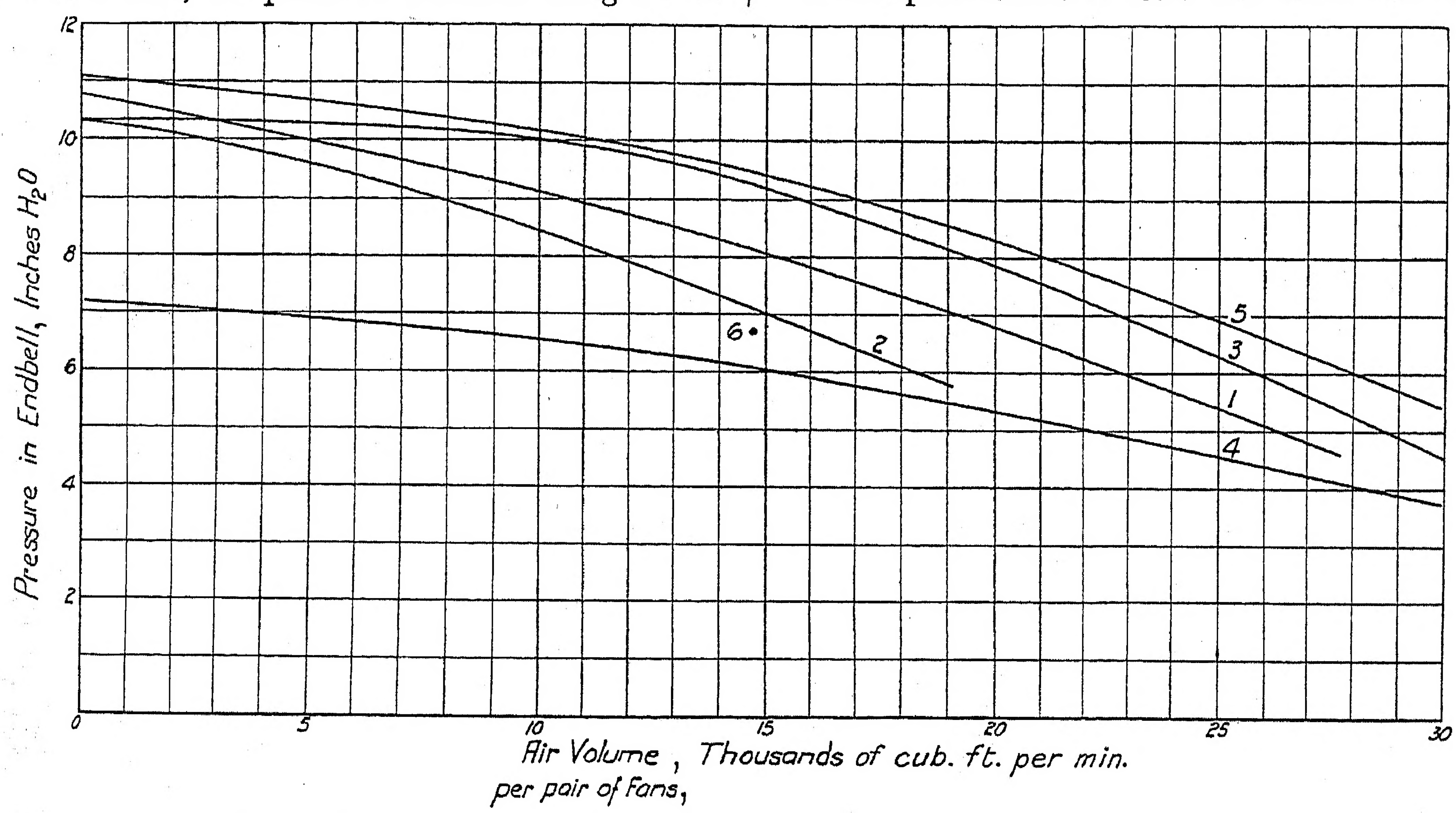

FIG. 15.

Test 1. Temporary baffle ring only.
Test 2. Outlet guides only.
Test 3. Silencers and all guide vanes.
Test 4. As Test 3, but inlet guides removed.
Test 5. As Test 3, but outlet guides removed.
Test 6. Silencers only.

the tip and high at the root. Careful tests confirm this and show that there is a serious amount of return flow under these conditions, with a consequent reduction in output and efficiency.

In order to prevent this unsatisfactory state of affairs it is essential that there should not only be radial guide vanes to split the inlet or feed into a number of sections, but there must also be circumferential guide vanes, preferably of aerofoil section, to reduce the loss of energy due to the change from radial to axial motion, at the same time dividing the inlet to the fan into a number of separate annular sections.

are shown in Fig. 15, the output being the total for the two fans.

Test No. 1 was carried out with only the temporary baffle-rings around the fans, the intakes or silencers being removed. It is interesting to note that this condition gave results considerably worse than those obtained when the silencers were fitted together with the guide vanes (see Curves 3 and 5).

The removal of the inlet guides, however, caused a serious reduction in output. A comparison of curves 2, 5, and 3, appears to show that outlet guides tend to reduce the output. It has already been shown that it is

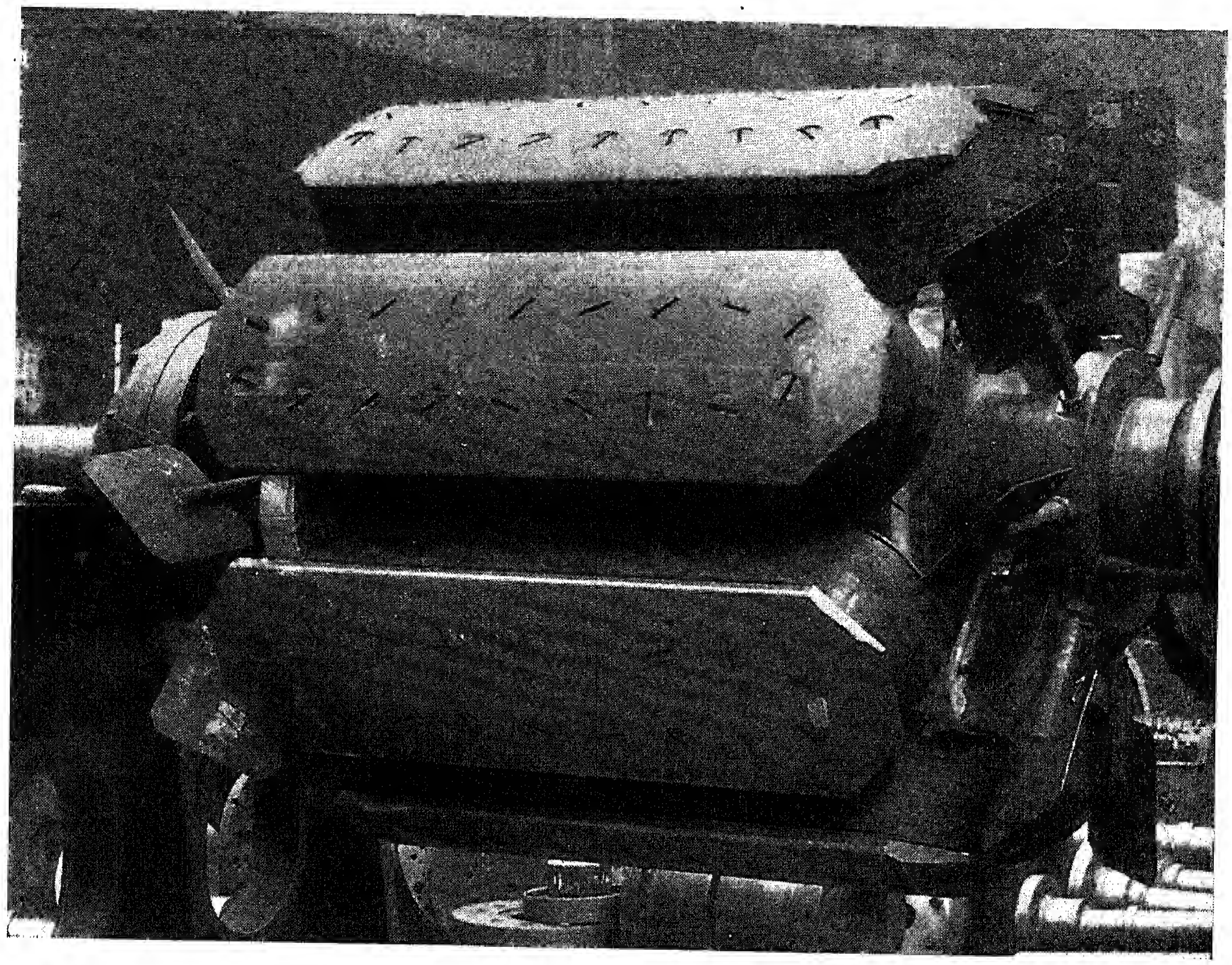

FIG. 10.—Paddle-wheel fans mounted on rotor of a synchronous motor.

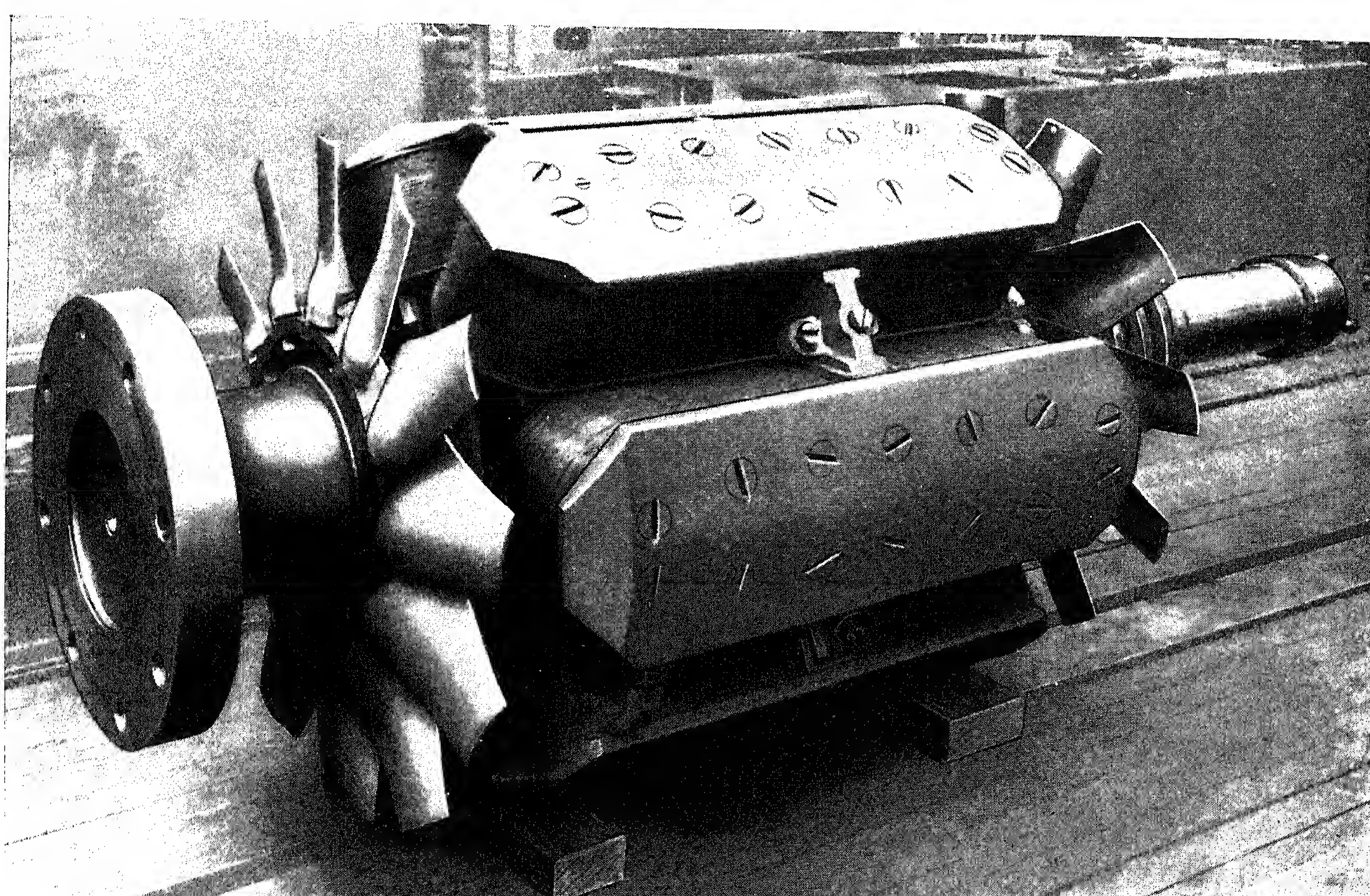

FIG. 11.—Arrangement of aerofoil-type fans on a 3 125-kVA, 3 400-volt, 3-phase, 50-cycle, 1 000-r.p.m. geared alternator.

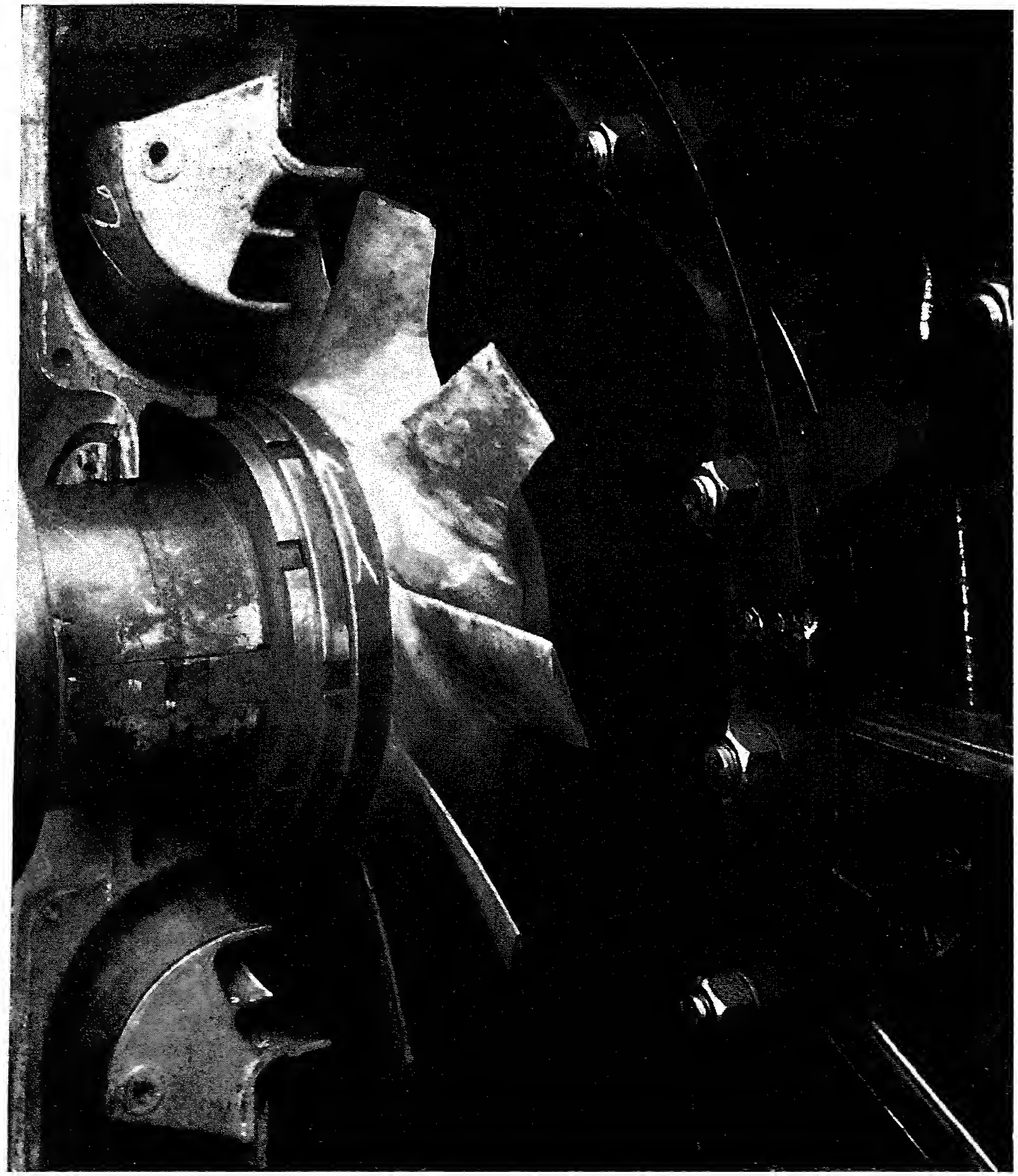

FIG. 14.—Close-up of turbo-alternator with part of air intake removed to show arrangement of propeller fan and guide vanes.

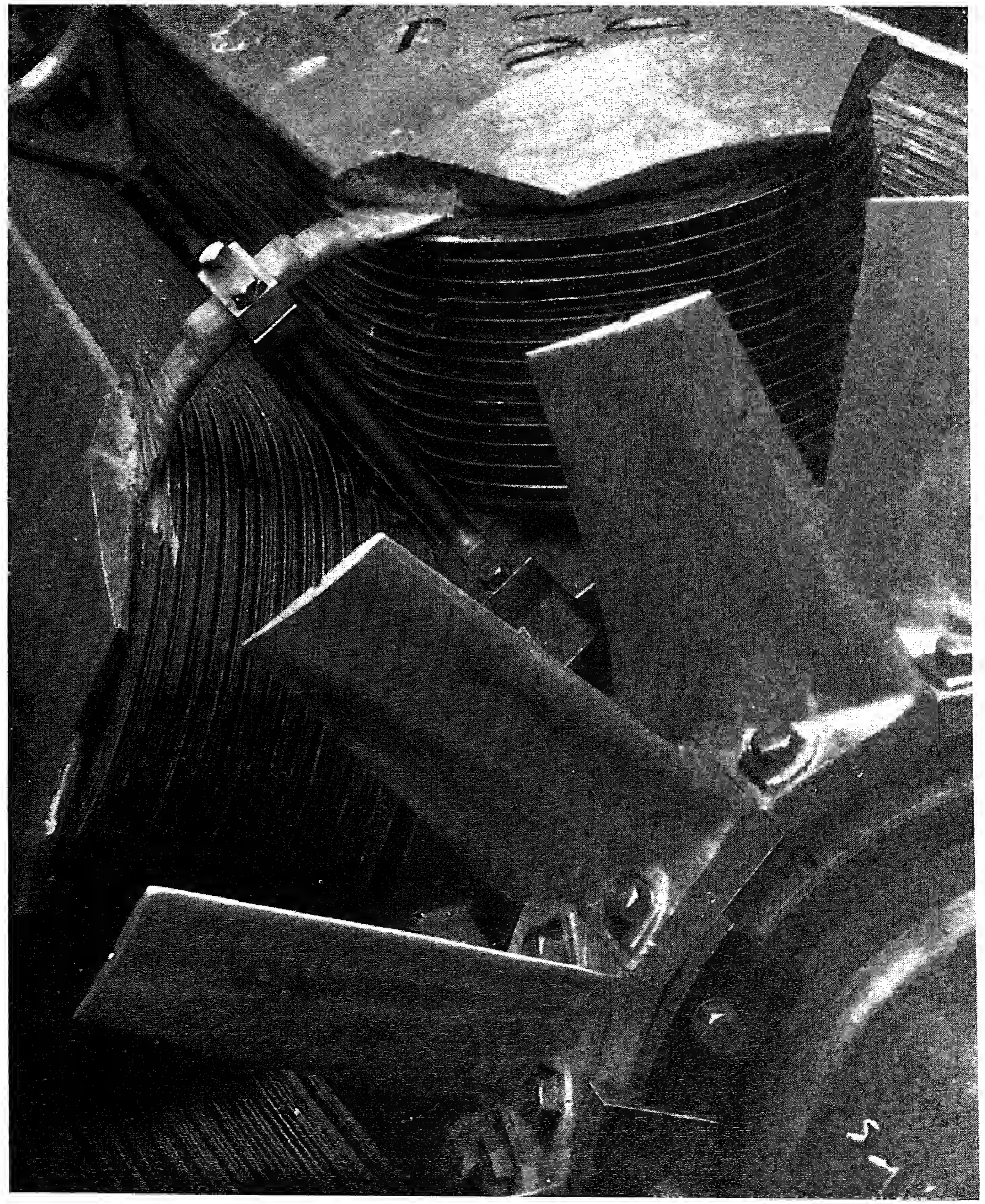

FIG. 20.—Close-up showing propeller fans mounted on rotor of synchronous condenser.

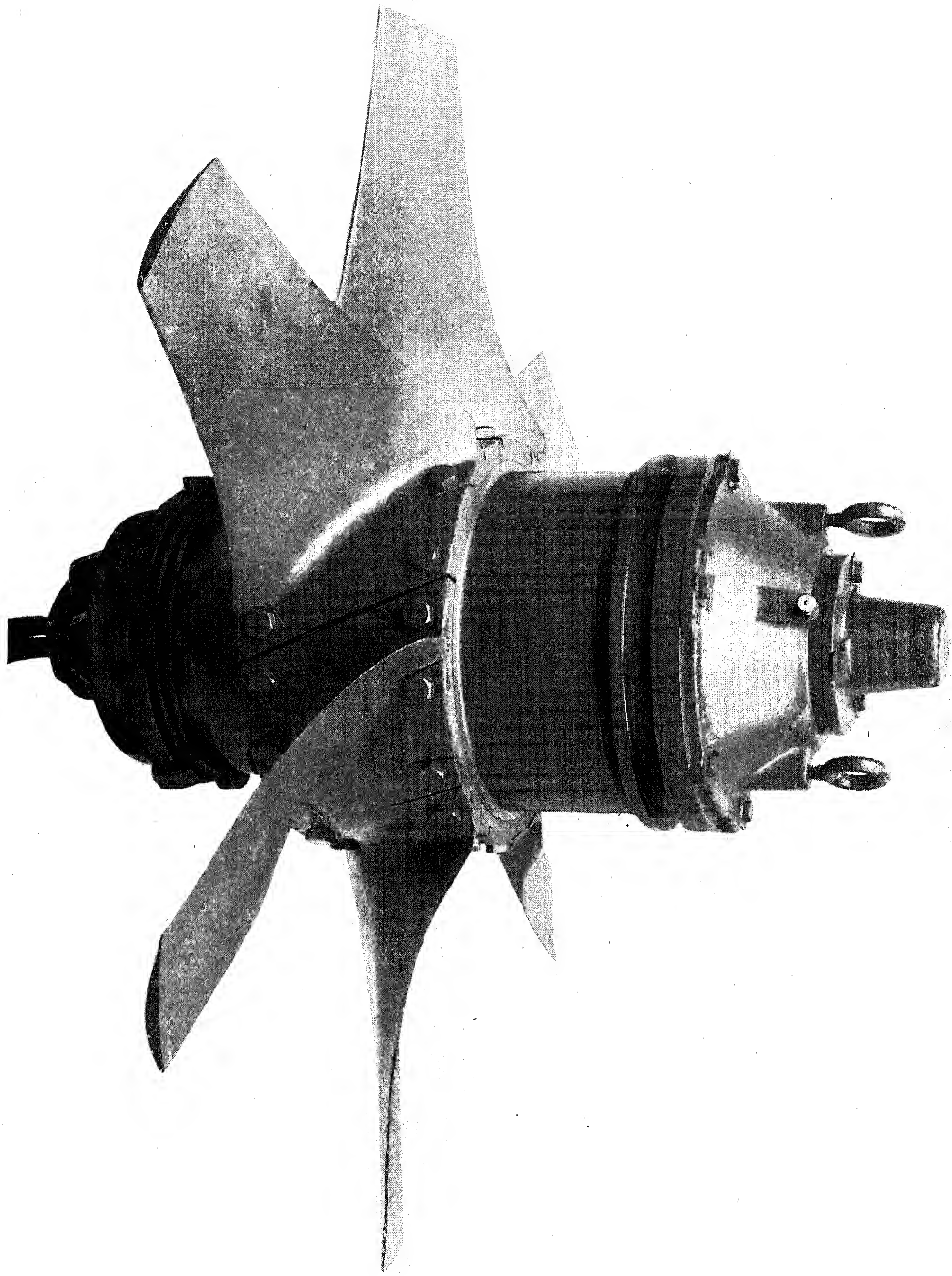

FIG 21.—External propeller fans for cooling multipole induction motors for Hungarian State Railways. Diameter over tips 48 in. approx. Blades mounted on rotor of internal-stator type of motor.

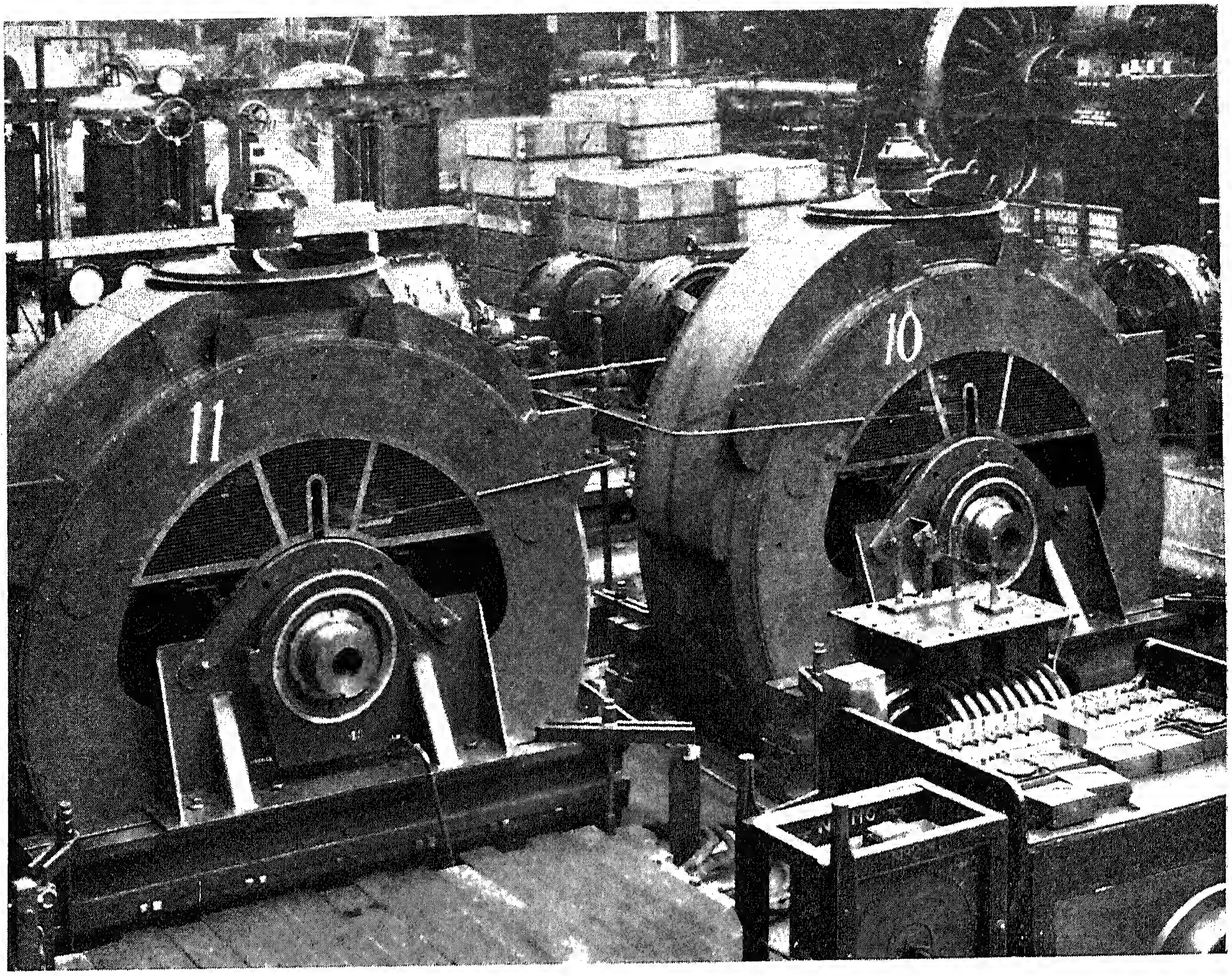

FIG. 22.—View showing fans mounted on top of induction motors.

an advantage to rotate the air before forcing it through rotating ducts, and the drop in output may be due to the increased entry loss in the rotor vents and air-gap.

The final test, No. 6, gives a single point on the pressure-volume curve for the condition shown on the left hand of the section drawing Fig. 9, that is, with intake and plain baffle-ring only. The fitting of the inlet guides (see Curve 5) therefore increases the pressure from 6·65 in. to 8·5 in. of water, i.e. nearly 28 per cent greater output.

Loss of Energy in forcing Air through Rotating Ducts.

In many cases the temperature of the rotor limits the output of a machine, and special attention has therefore been paid to the problem of forcing air through rotor vent ducts.

Tests have proved beyond doubt that the pressure required increases with the speed of rotation. As pointed out by the author,* the loss is proportional to $(v_A^2 + v_R^2)$, where v_A is the axial air velocity and v_R the rotational velocity of the duct.

This increase in energy loss is merely due to the angle of approach being changed, causing in turn a decrease in the effective cross-section of duct.

The energy of rotation of the air is separate, and is supplied as torque through the shaft and via the walls of the duct.

Now consider a propeller blowing air into a rotating duct. The air leaving the fan will be rotating in the same direction as the duct. The energy loss at entry will now be proportional to $v_A^2 + (v_R - v'_R)^2$, where v'_R is the rotational velocity of the air leaving the fan.

Moreover, the rotational velocity to be imparted by the rotor duct will become $v_R - v'_R$.

There is therefore a reduction in the entry loss, or the static pressure required to force the air into the duct, and also a reduction in the torque required to rotate the air.

Maximum-Rotation Fans.

In order to take full advantage of this saving in energy the author has introduced a special design of propeller fan, the aim being to give the maximum slip-stream rotation. This does not result in the most efficient fan according to the normal method of calculating efficiency, but gives a higher machine efficiency when such fans are used for rotor cooling.

The increased power taken by the fan is more than balanced by the reduction in windage of the rotor itself; thus the use of the maximum-rotation fan enables more air to be forced through the rotor for a given power.

It has been shown theoretically that the solidity of a fan designed for maximum slip-stream rotation must be of the order of $2\sqrt{2}$ for maximum efficiency. Such a figure requires an immense number of blades, since solidity equals $Bc/(2\pi r)$, B being the total number of blades, and c the actual blade width at radius r. It will be understood, therefore, that in such fans the blades overlap and the blade angle is large.

In practice the solidity is fixed by the permissible cost of the fan.

* "Design of Fans for Cooling Electrical Machines" (published by the Association of Engineering and Shipbuilding Draughtsmen, 1932–33).

Friction and Windage Tests.

It is not generally realized that input tests with fan and with fan removed do not give the true power taken by the fan. The energy taken up in rotating the air passing through the rotor is supplied as torque, apart from that required to drive the fan. Thus when the fan is removed there is a reduction in the windage of the rotor. On the other hand an increase in the amount of air delivered by the fan also results in an increase in the power taken to rotate the air passing through the rotor. The difference in the friction and windage readings for two fan outputs will therefore not give the true change in the power taken by the fan.

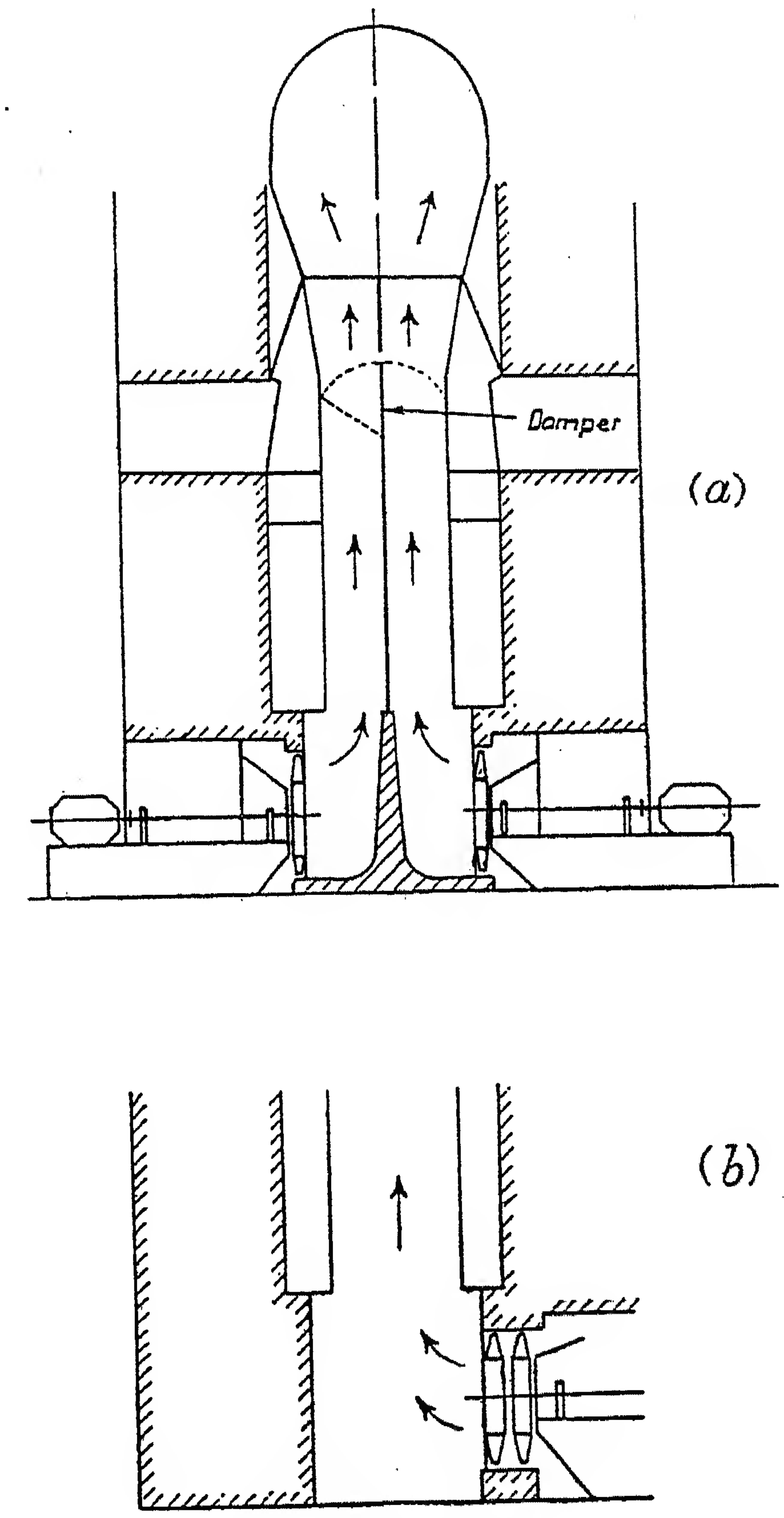

FIG. 16.

Friction and windage tests will of course give reliable results when the amount of air passing through the rotor is very small.

EXTERNAL FANS.

Series versus Parallel Fan Ventilating Systems.

It is interesting to examine the respective merits of series and parallel ventilating systems as applied to conditions in which it is desired to reduce the air supply on occasion and yet maintain a reasonable efficiency. The arguments set forward are not intended to apply to conditions where variable-speed drives are suitable.

A particular case may be mentioned as an illustration,

viz. the ventilation of a large turbo-generator by means of external fans.

In the past it has been customary to use two centrifugal fans, the system being so arranged that each fan feeds one half of the machine at full output. At smaller loads, by means of a damper either fan can be made to supply the whole of the machine, the other fan being closed down. A similar arrangement using propeller fans is shown in Fig. 16(a).

It will be realized that there is a definite reason for using two driving motors instead of one of variable speed. It is not necessary to keep spares for the full output, and in the case of breakdown it is possible to run for some considerable time with only one fan whilst the faulty motor is replaced.

Consider the case of a turbo-alternator requiring a total air quantity of 30 000 cub. ft. per min. at 10·0 in. water gauge from two fans. In Fig. 17 curves are drawn showing the variation of pressure with volume for the two fans together. Drawing the resistance curve for the machine, assuming $P = kV^2$, this must cut the P-V curve at a point corresponding to the output 30 000 cub. ft. per min. at 10·0 in. water gauge. An assumed efficiency curve has been added, showing 75 per cent at the design output for the two fans in parallel.

From the data given, the P-V curve and efficiency curve for a single fan can be drawn, the pressure of 10·0 in. water gauge and efficiency 75 per cent occurring at a volume of 15 000 cub. ft. per min.

The point at which the resistance curve cuts the P-V curve for the single fan indicates the quantity of air when only one fan supplies both halves of the machine.

The corresponding efficiency is low—of the order of 42 per cent.

If, however, it is decided to use two screw fans in series, so designed that when the second is stationary it does not interfere with the efficient working of the first, there is no need for special dampers to change over the circuit, the fans feeding direct into the whole of the machine [see Fig. 16(b)].

Furthermore it can be arranged that the operating point of a single screw of the series arrangement occurs at the point of maximum efficiency as shown on the curves, thus giving an efficiency over 1·5 times that of the single fan of the parallel system.

So much for the gain in efficiency due to operating conditions.

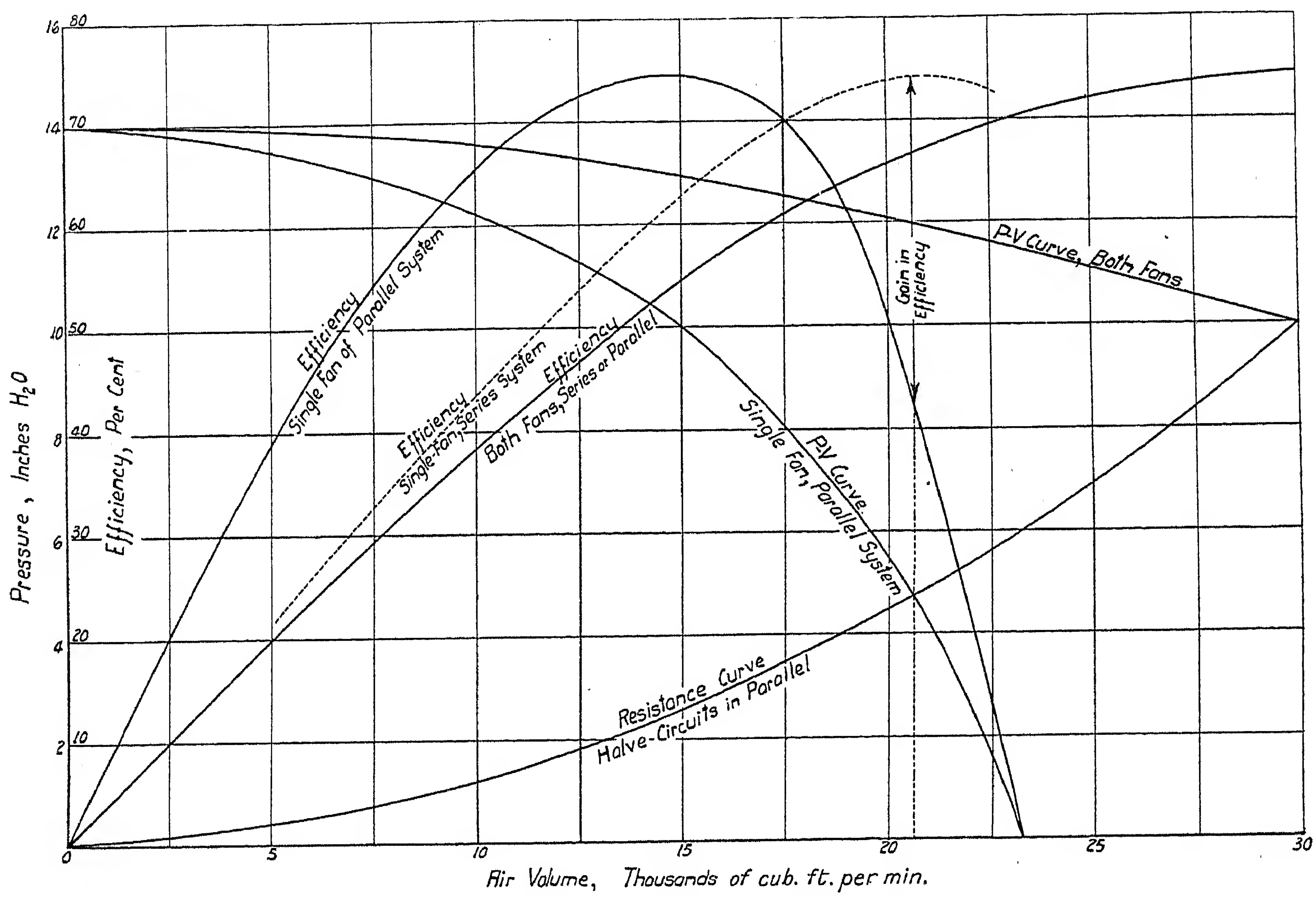

FIG. 17.—Series and parallel fan characteristics.

Series Fans Rotating in Opposite Directions.

There is a further argument in favour of the series system; by rotating the fans in opposite direction the efficiency of the two fans may be much higher than that of a single fan, since the energy of rotation at the outlet of the first can be converted into static pressure. Furthermore, the second fan can be so designed that when stationary it will act as guide vanes for the first and the efficiency will therefore be higher than that of a single fan.

Tests have been carried out with two identically designed fans (but of opposite rotation) to illustrate the

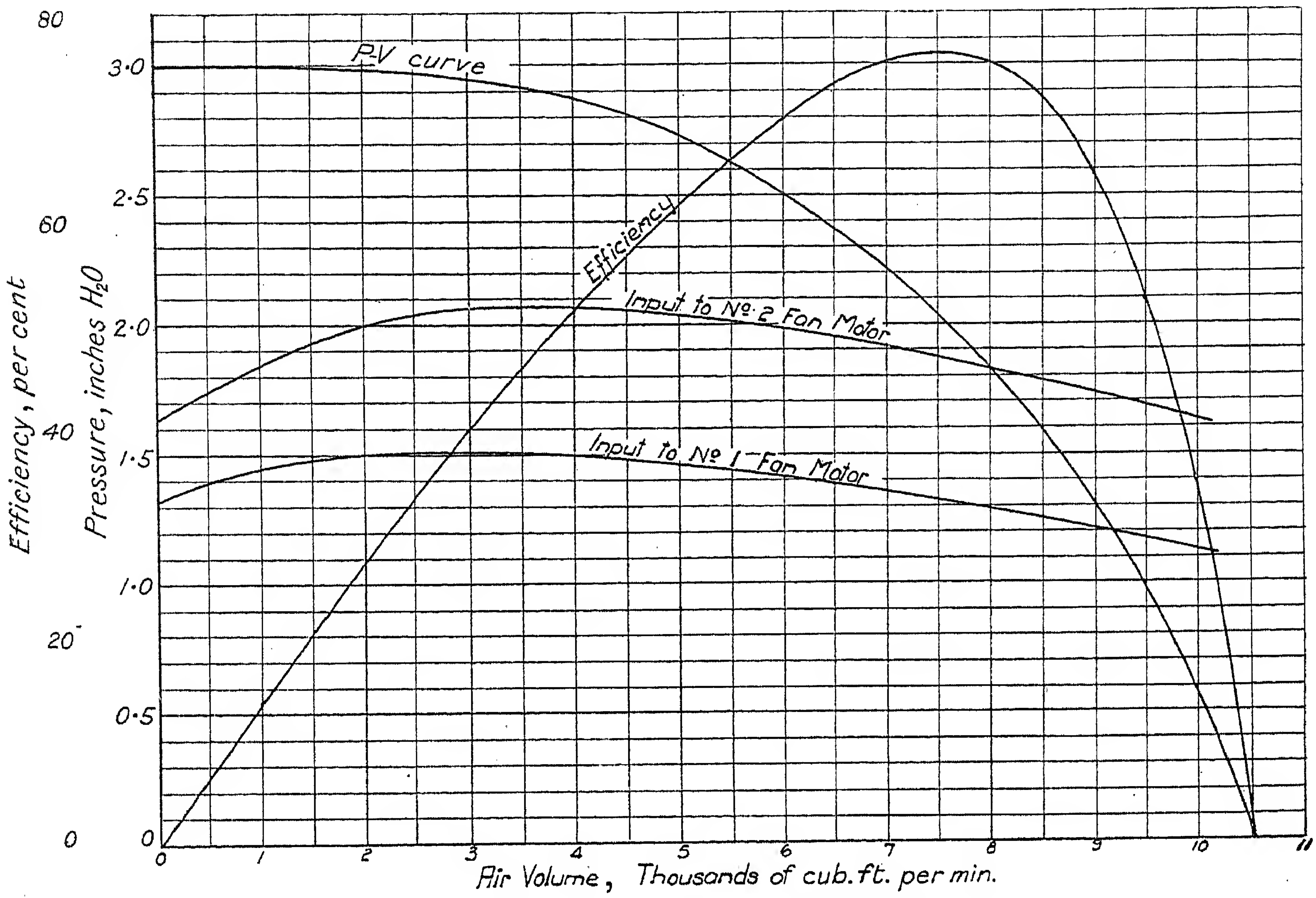

FIG. 18.—Propellers in series, rotating in opposite directions; both fans driven.

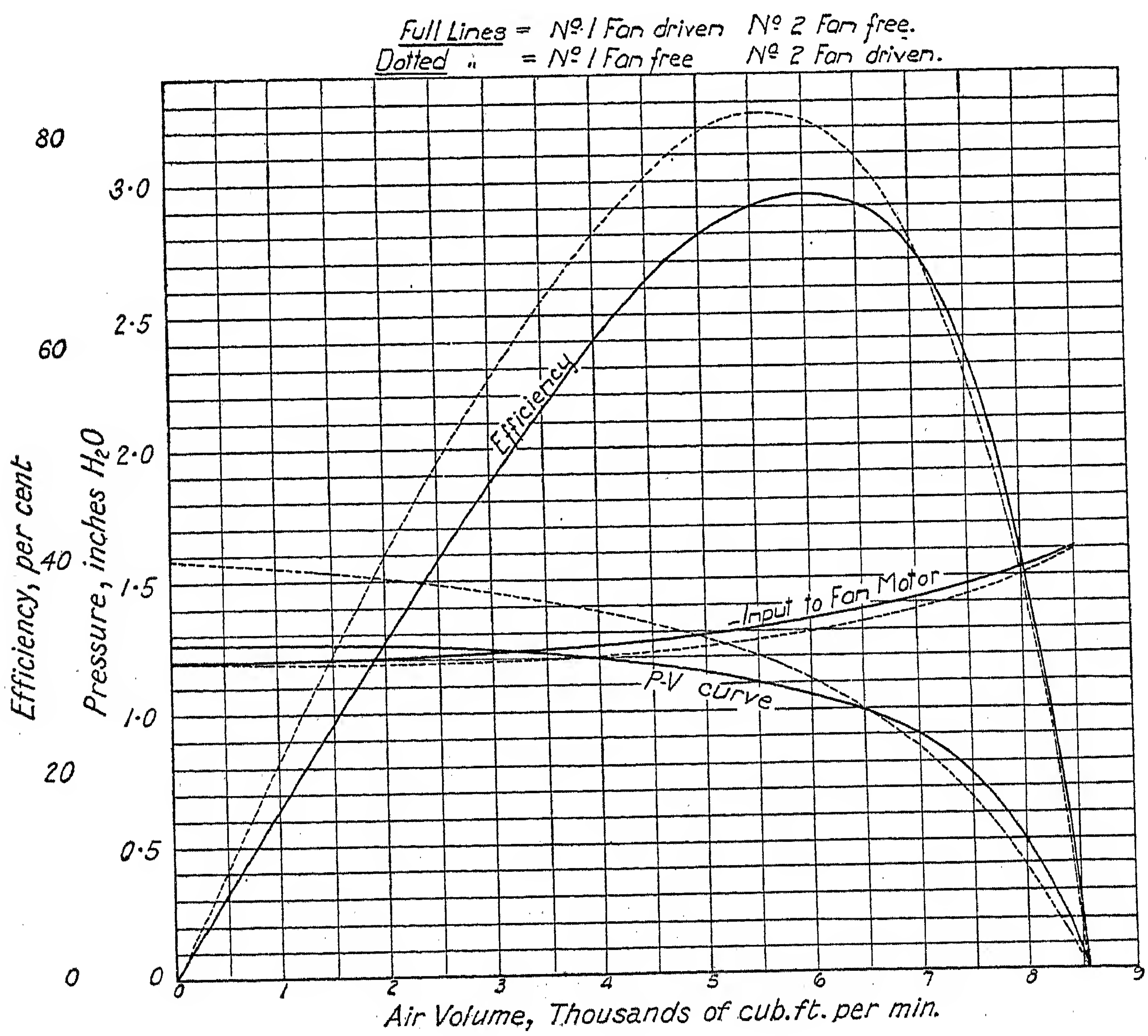

FIG. 19.

Full lines: No. 1 fan driven; No. 2 fan free.
Dotted lines: No. 1 fan free; No. 2 fan driven.

small movement of the point of maximum efficiency and also the fact that there is not much interference between the fans when one is stationary (see Figs. 18 and 19). Unfortunately, it was found that there was a certain difference in manufacture between the two screws which made the output of one slightly less than that of the other; this to some extent reduced the value of the test.

The main point, however, is very well proved, and it will be noticed that the increased velocity head when the two fans were running was more than covered by the recovered energy of rotation (see Fig. 19). Theoretically, for similar fans, as the rotational velocity of the second fan relative to the fluid is $(\Omega+2\omega)/\Omega$ times that of the first (Ω being the angular velocity of the fans, and 2ω the angular velocity of the slipstream leaving the first fan), the torque required to rotate the second fan will be higher. This is confirmed by the power curves in Fig. 19, No. 2 fan requiring approximately 30 per cent more power than No. 1.

Special Case of Series Propeller Fans.

The author has shown* that it is possible to design propeller fans so that when the second stage is closed down it will act as guide vanes to the first stage.

Thus not only is the efficiency of the double stage higher than that of a single fan, but with one closed down the efficiency is higher than that of a fan alone. It is obvious that such a system is very adaptable to the cooling of large turbo-alternators, giving high efficiency together with a simplified damper system. Furthermore, it is only necessary to design each propeller of a series system for half the pressure that would be required in the corresponding parallel scheme. This results in a reduction in the leakage across the fan, with a corresponding increase in efficiency. The cost may also be reduced, since the peripheral speed need not be so high, as in general the outer diameter of a propeller fan is fixed by the pressure rather than by the air quantity required.

Internal and External Propeller Fans.

Two interesting examples of the application of propeller fans to electrical machines are illustrated in Figs. 20, 21, and 22 (see Plates 2, 3, and 4). Fig. 20 shows a fan built up of separate blades bolted to a steel hub which has machined flats. The complete fans are pressed on to the shaft of a large 1 500-r.p.m. constant-speed machine, a synchronous condenser. In this case double inlet ventilation is used.

On the other hand, Fig. 21 illustrates a special external fan having blades bolted to the circular rotor body of a central-stator-type, 1 480-r.p.m. induction motor. The complete fan unit is mounted on the top of a low-speed multipole induction motor (see Fig. 22). This scheme has the distinct advantage that it permits good ventilation independent of the speed of the main motor.

Conclusion.

Experience shows that propeller fans can be designed to suit almost all machines; the deciding feature is ultimately cost. For high pressures it is essential that the radial tip clearance shall be small in the case of a propeller fan, but this requirement increases the cost considerably. The final decision therefore rests between cost and efficiency and is a matter for the customer rather than the manufacturer. It has been found that in certain cases the propeller fan is quieter than the centrifugal fan, but in general the former requires more careful design to prevent noise. Special attention is being paid to this problem and much useful information has been obtained from recent experimental work, the results of which may shortly be published. The author drew attention to the most important features in a recent discussion.*

Although the propeller fan is most suitable for cooling transformers, no reference has been made to this problem in the present paper. It is possible that in the near future the question of transformer cooling will be dealt with as a separate subject, i.e. since the type of fan, high-pressure or low-pressure, may determine the arrangement of cooling tubes. While forced ventilation may become economical, this is a matter for further investigation by transformer designers.

In conclusion the author wishes to acknowledge the kind encouragement received from his chief, Mr. G. A. Juhlin, Member, and to thank Mr. J. S. Peck, Member, Director of Metropolitan-Vickers Electrical Co., Ltd., for kind permission to publish the paper.

Discussion before The Institution, 14th February, 1935.

Mr. D. B. Hoseason: The author describes a number of successful applications of propeller-type fans, and it may be of interest to give details of one unsuccessful attempt to use this type of fan; the more so, because the construction of the particular motor appeared to be specially favourable. Fig. A shows a half-section of a 110-h.p. 960-r.p.m. squirrel-cage motor, with axial ducts and the air flow in the active portions of the motor parallel to the shaft. Fig. B shows the same stator fitted with a new design of end bracket and a propeller-type fan. Five different designs of propeller-type fans were tried out, with the results shown in Table A. In each case, we were endeavouring to obtain approximately 10 per cent more air than was given with the standard design of centrifugal fan. It will be observed from the table that all our efforts in this respect were unsuccessful.

Since energy consumption is one of the prime reasons for employing propeller fans, at the conclusion of the tests two further centrifugal fans were made up of the same outside diameter as the best of the propeller fans. One of these centrifugal fans had radial blades and the other had blades which sloped back in accordance with the principles indicated in a recent paper.† Taking the energy absorbed by the best propeller-type fan, i.e. that built in March, 1934, as 100 per cent, the radial-blade centrifugal fan gave 42 per cent more air with 86 per cent more energy consumption. The sloping-blade fan

* See Institution of Civil Engineers paper referred to on page 293, and also Patent 399619 (R. Poole and A.E.I., Ltd.).

* *Journal I.E.E.*, 1934, vol. 75, p. 431.

† D. B. Hoseason: *Journal I.E.E.*, 1931, vol. 69, p. 121.

gave 18 per cent more air with 47 per cent more energy. It can be deduced from these figures that a smaller-diameter centrifugal fan would give the particular air quantity provided by the propeller-type fan and would

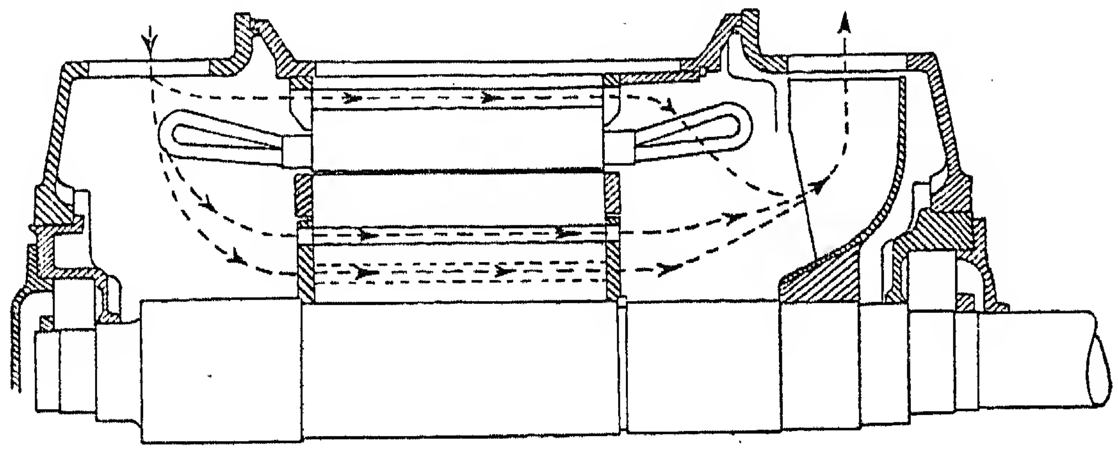

FIG. A.—110-h.p., 960-r.p.m. squirrel-cage motor with centrifugal fan.

absorb substantially less energy. Needless to say, the cost of the smaller-diameter centrifugal fan is much lower than that of the aluminium propeller-type fan.

In referring to his tests on flat-bladed propeller fans and those of aerofoil section, the author states (page 299) " without the fairing the quantity passing through the endbells increased at the expense of the rotor, as would be expected, but in the case of the aerofoil fan there was a reduction in quantity through both endbells and chimney." Has the author any explanation of why the two cases should be different?

It appears to me that the propeller fan has two rather important deficiencies. Fig. 2 emphasizes the feature of aerofoil sections, viz. that the air speed over the back of the aerofoil is greater than that over the underside. If, however, we draw several aerofoils, to represent the conditions existing in a propeller-type fan, we find that, considering a stream of air passing through any pair of aerofoils, the velocity across the air stream changes from high (H) to low (L) as in Fig. C. If we wish to obtain a high water-gauge from a propeller-type fan, then the difference between the rates of flow in the blading on either side must be considerable. I suspect that this factor has a very important bearing on the difficulty which we experience in making high-pressure propeller-type fans, particularly for low speeds of rotation. The second objectionable feature of aerofoil fans

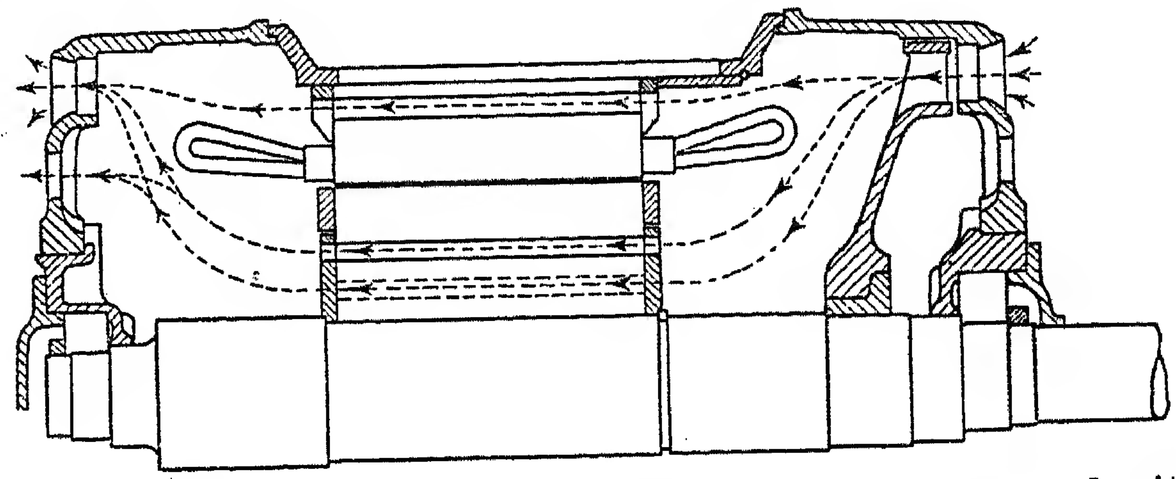

FIG. B.—110-h.p. 960-r.p.m. squirrel-cage motor fitted with propeller-type fan.

is the very small angles of the blading, as shown in Figs. 6 and 11. Wherever there are such small angles, the conditions are liable to be critical.

Considering the problem of cooling a body by passing air over it, the temperature-rise is given by the equation* $\theta_t = \theta_g + \theta_s + \theta_m$, where θ_g is the temperature-drop from the hot spot to the surface over which the air flows, θ_s is the difference between the temperatures of

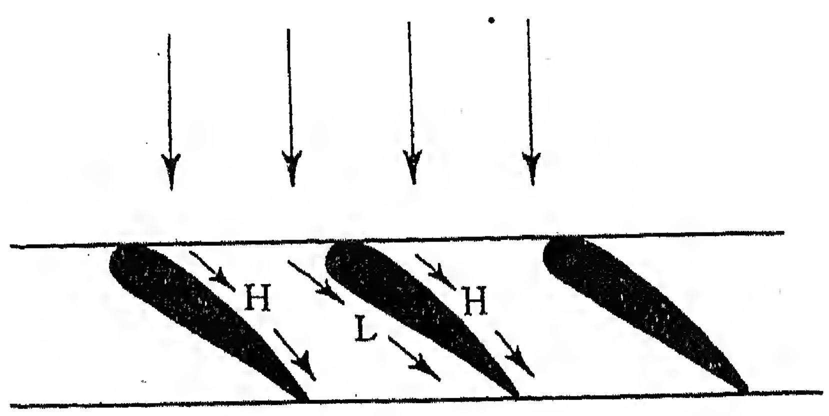

FIG. C.—Diagram showing high- and low-velocity air flow through a pair of propeller-type fan blades.

the surface and the air stream, and θ_m is the temperature-rise of the air itself above atmosphere. Now in the majority of electrical machines θ_s—the middle term—

* *Journal I E.E.*, 1931, vol. 69, p. 121.

TABLE A.

Date	Type of fan	Form of blade	Outside Diameter	Suction or Blowing	Air quantity: Actual	Air quantity: Per cent
			in.		cub. ft. per min.	
Aug., 1929	Centrifugal	Radial	18·8	Suction	960	100
Dec., 1929	Propeller	Flat blades at 35°	18·8	Suction Blowing	575 278	60 29
April, 1932	Propeller	Aerofoil with outlet guide vanes. Cast aluminium	19·0	Suction	680	71
Sept., 1932	Propeller	Aerofoil, no guides. Cast aluminium	19·0	Blowing	720	75
March, 1933	Propeller	Aerofoil, no guides. Cast aluminium	22·5	Blowing	768	80
March, 1934	Propeller	Aerofoil, no guides. Cast aluminium	20·75	Blowing	730	76

predominates. If we omit turbo-alternators, where the depth of core is very considerable, θ_s is of the order of 50 or 60 per cent of θ_t. In other words, if the internal temperature-rise in a large generator is 50 deg. C., then approximately 25 deg. C. is being spent in transferring the heat from the surface to the air stream. The transfer of heat from a surface to an air stream is a function of the velocity or turbulence of the air rather than of its volume, and air velocity means water gauge. It seems to me, then, that the type of fan which we want for electrical machines is the high-water-gauge fan rather than the high-volume fan. Our experience has all tended to show that for medium and low speeds the propeller fan is essentially a high-air-volume, low-water-gauge device, while for high water-gauge figures the centrifugal fan is supreme. I therefore fear that, for a machine in which air is passed at high velocity through axial ducts, the propeller-type fan is not really suitable; therein lies the explanation of our success with the centrifugal fan and our failure with the propeller type on the motor shown in Fig. A.

Mr. W. L. Cowley: The problems discussed by the author are rather different from those with which we meet in aeronautical questions. In the first place, his problem is linked up with cooling. In aerodynamical work we must cool our hot surfaces with air which has as little eddy motion as possible; otherwise the machine experiences considerable losses through aerodynamic resistance. In the case of electrical machinery, however, it appears to me that a large amount of eddying may be beneficial, and it may explain to some extent why Mr. Hoseason found such considerable differences between the types of fans he uses. If a fan throws out eddying air through passages which themselves produce eddying, then it may be that cooling will be improved. I suggest that this problem should be analysed in detail from the heat side, as well as from the point of view of the quantity of air discharged by the fan.

Dealing with the question of the air flow through the ducts, it is evident that one fan may be suitable for one machine but not for another, merely because the passage-ways through the machines are different in the two cases. How these passages should be designed is a very difficult question. In aeronautical work we design our passages to give as small a resistance as possible. Electrical engineers, on the other hand, might welcome eddying of the air in order that it might scrub the surfaces and carry away the heat. In the wind tunnel at the National Physical Laboratory great care is taken to get the air to turn smoothly round corners. We have to use guide vanes, and without these the efficiency of our wind tunnel would drop considerably. The machines shown in the paper have no provision of that nature, and I suppose it would be extremely difficult for the designer to shape his passages so as to give good air flow as we get it in the aeronautical world. Nevertheless, I think considerable attention should be focused on that part of the subject. No doubt in the future, when the general design of a machine is being considered, primary importance will be attached to these passages. It may be that, by suitably arranging the general lay-out around the passages, one may be able to get increased power, as the output of an electrical machine depends very much upon the cooling of the conductors. It would be very interesting if the author could state what would be gained in present machines by an increase, say, of n per cent, in the cooling. One would visualize that if, for example, the cooling were increased 4 times, we could carry twice the current in the conductors and double the power of the machine.

The water-gauge question mentioned by Mr. Hoseason is intimately bound up with the aerodynamical resistance of the ducts. The effectiveness of a fan would be governed by the shape of the passage-ways.

It is very interesting to find that the science of electrical engineering is joining hands with what is apparently such a diverse subject as aerodynamics. Prof. Prandtl developed the aerofoil theory and brought about considerable advancement in aeronautics from a theory which is really almost an electrical theory. I think it would be very useful if a joint meeting of the Institution and the Royal Aeronautical Society were held to discuss such problems as those raised in the present paper.

Dr. N. A. V. Piercy: It would have been useful if the author had given a detailed statement of the process adopted in the design of a propeller in a particular case. Thermodynamical considerations apart, the airscrew must win, and if experiments seem to contradict this it appears necessary to suppose that the airscrew is not quite suitably designed in those cases.

The blade element theory referred to in the paper is, of course, incapable of giving a design to specification unless joined with the momentum theory of Rankine and Froude or with the vortex theory, either of which resets the blade angles so as to realize the desired angles of incidence. I assume that the author takes this matter into account. The present application is more complicated than the original aeronautical problem—although not calling for such a precise solution—and it would be interesting to know clearly how the author met his special difficulties. Generally, a knowledge of two things appears to be required at the outset in designing under the present conditions, (1) the velocity field at the airscrew disc, which depends upon the radius, and (2) the *distribution* of the energy output required, which determines the form of the blade loading. Owing to the complicated nature of the problem one would then expect only an approximation, and from this point of view the method adopted in some cases of bolting the blades to machined flats appears optimistic; spigot attachment would allow of a final adjustment of the blade angle as a whole.

The author has every reason on his side for his advocacy of guide vanes. He might, however, have gone farther with them, and employed them, for instance, in the ventilating of the large turbo-generator by external fans. I find it difficult to believe that the fan itself is the best means of imparting strong rotation to the stream; such rotation will be produced partly by profile drag. Under the conditions of working in many electrical cases light stresses permit the use of thin aerofoil sections having, at the Reynolds numbers concerned, a much smaller profile drag than that of the 1912 N.P.L. aerofoils, which were small and were tested at a low speed. Possibly a profile drag only one-third as great as that of the aerofoil No. 4 referred to in the paper may be

assumed. A great increase in rotation appears to spell the production of a large amount of unnecessary vorticity. Some application of aerofoil guide vanes, such as a fixed windmill, might be sought as an alternative.

Two other points occur to me. First, the pressures used by the author are sufficiently high to make important the question of casual leakage, apart from that past the propeller tips. The second point is that some use might possibly be made of the regenerative outlet cone which has been found effective in wind tunnels. For instance, if the fans shown mounted on the top of the induction motors of Fig. 22 were drawing the air upward, their replacement by exhaust funnels with larger-diameter fans at the outlet would improve efficiency by easing the kinetic energy out of the stream.

In regard to the thermodynamics of the problem, aeronautical engineers often manage to transfer a lot of heat to a fluid which is apparently in generally smooth motion. It has been shown that the depth of air heated is of the same order in such cases as that affected by viscosity. Nevertheless the boundary layers are turbulent, so that the transport of heat is by molar and not molecular action, and the flow is not laminar in the precise sense of the word.

Prof. F. G. Baily: While the paper shows clearly the advantage of an aerofoil shape of blade in producing a smooth flow of air along a well-designed path, it deals only with the volume passed per second, whereas the real figure required is the amount of heat removed, i.e. volume × rise in temperature. It is useless to increase the volume of air by improvement of air passages if the freer passage allows the air to get through without intimate contact with the surfaces to be cooled. The problem resembles more the design of a steam boiler than the ventilation of a room, and if forced draught is available the boiler tubes are put directly in the way of the air flow. Air at a moderate speed will slide past smooth flat surfaces with little heat transference, though at high speed eddy-currents form and there is more effective mixing and consequent cooling action. What is more required, however, is forcible impinging against the surface, and preferably at a slant, to allow the heated air to get away after contact. There must of course be sufficient air to carry the heat and still be well below the machine temperature on emergence, and with the gentle flow that comes from induced draught the obstruction must not be overdone; but with direct fan pressure adequate air can be forced through, and, as the power absorbed depends on $P \times V$, the smaller volume compensates to a considerable extent for the larger pressure.

While an easy entrance to the fan is a direct advantage, the use of guide vanes to lead the air straight into the revolving ducts shifts the best cooling action from the ducts to the guide vanes, where it is of little use. The violent change of direction as the air enters the ducts (without guide vanes) is immediately effective in cooling, and the air continues turbulent through the duct. Broadly, the advantage of the aerofoil blade seems to depend on an undisturbed flow, whereas good cooling requires just the reverse. While it would be rash to disbelieve the conclusions, the figures in the paper are scarcely convincing for the real result required. A few thermometers put into the measured outflow would have answered the question completely, if sufficient time had been allowed for attaining full-load steady conditions. This means at least several hours for each test; but no novelty in design is accepted without such a trial.

Mr. R. Borlase Matthews: At the beginning of the War the efficiency of the air-screw was in the neighbourhood of $22\frac{1}{2}$ per cent; but at the end of the War, thanks to the team work of mathematicians and to air-tunnel work, its efficiency was about $83\frac{1}{2}$ per cent. These figures give an indication of the possibilities of increasing the efficiency of fans when applied to electrical work.

A few years after the termination of the War, as a result of my experience in air-screw design I took out a patent for propellers for cooling turbo-alternators and big electrical machinery; but seeing that compared with the total power required in operating a generator that required for the fan is comparatively small, to economize by 10–80 per cent in an already small amount was not considered worth while. The same thing applied as regards noise. I should like to point out that by scientific design one can reduce the noise produced by the propeller and the air current.

In criticism of the paper, it seems to me that the top edges of the ends of the blades should be slightly more rounded in form; this modification would probably reduce waste eddy currents. Eddy currents may be of advantage in the cooling of electrical machinery, but in any case we should learn how to control them.

The ordinary portable electric table fan could be made very much more efficient by applying the science of aerodynamics to the design of the blades. Another application of this science is to the design of electrically operated propeller fans used for the distribution of heat artificially and also for ventilation purposes. Still another development, which has not really started in this country but which is used a very great deal on the Continent, where this aerodynamic design is again required, is that of air propeller fans for the transport of hay, straw, and other similar farm crops. By this method the crops are passed pneumatically from the delivery wagons direct into the lofts or barns of the farm. There are two problems, one where the crop passes through the fan blades themselves—and it requires careful design of these blades to ensure that the crop is not chopped up and spoilt—and the other where the crop is conveyed by means of a fan-operated ejector. The latter method is less efficient, but it avoids damage to the crops.

I should like to support Mr. Cowley's suggestion that this subject should be jointly discussed by the Institution and the Royal Aeronautical Society. It seems to me, however, to be more a matter for a select joint committee. It is not a question merely of applying aerodynamical principles to the design of fans; we want to apply those same principles to any rotating part. We shall have to see to it that right through our designs of electrical machinery the churning of air in unnecessary and eddying forms is entirely eliminated.

Prof. W. M. Thornton: I should like to call the author's attention to a paper* published in 1909 which

* *Electrician*, 1909, vol. 63, p. 706.

refers to some work done in my laboratory, where we ran a machine in a vacuum. We came to the following conclusion: "The final result is that very nearly one-half of the heat expended in exciting the field coils is dissipated by the total action of convection, while the remaining half is removed by the combined action of radiation and conduction. And, further, that one-half of that due to convection, that is one-quarter of the total heat dissipated, is accounted for by the fanning effect of the rotating armature." These or similar figures are the data on which I presume the author is going to work in designing his fans.

Mr. R. Poole (*in reply*): In reply to Mr. Hoseason, I agree that the propeller fan is unsuitable for cooling small low-speed machines of the type illustrated in Fig. A. The principal cause of failure is the relatively small quantity of air required in proportion to the static pressure.

The efficiency of a blade is largely dependent upon the aspect ratio and the pitch or angle. For a fan of given outside diameter, at a fixed speed, a reduction in air quantity is obtained by increasing the inside diameter, i.e. shortening the blades, or alternatively by reducing the blade angle. In general these measures reduce the efficiency of the fan. The aspect ratio of the blade can of course be maintained by using a greater number of narrower blades, but there is great difficulty in cheaply manufacturing small blades of good shape.

In the comparative tests of the flat-bladed fan and the fan with aerofoil blades, the former had 6 blades set at an angle of 45°. The solidity of the fan, i.e. the ratio of total blade width to circumference, was small, and the blade was therefore working beyond the stalling point. There was thus set up a considerable centrifugal action and the removal of the shroud permitted the air to escape radially, consequently increasing the quantity passing through the endbells. The aerofoil fan, however, was operating much below the stalling point and the removal of the shroud permitted a greater contraction of the air stream passing through the fan, with a consequent reduction in quantity through both endbells and stator.

I do not agree with Mr. Hoseason that the difference in velocity between the upper and lower side of a propeller-fan blade is one of its deficiencies. A similar condition occurs in a correctly designed centrifugal fan, which derives much of its pressure from the circulation around the blades, just as in the case of the propeller fan.

Apparently Mr. Hoseason is unaware of the fact that the aerofoil theory is applicable to the design of centrifugal fans, even if the blades are made of flat sheet metal. His remarks regarding small "angles of attack," if true, would be applicable to both types of fans.

An examination of the pressure/volume curve of a propeller fan does not support the suggestion of critical conditions. It is a fallacy to assume that the angle of attack in a propeller fan varies much with air quantity. Fig. D shows the velocity diagram of a blade element having an axial air velocity V, a peripheral velocity $r\Omega$, and a resultant velocity W. The component $a'r\Omega$ is the mean interference rotation of the air with relation to the blade element.

Reducing the air quantity to one-half, i.e. making the axial velocity $0 \cdot 5\ V$, has the effect of increasing the rotation of the slip stream. The value of $a'r\Omega$ is therefore increased and the angle of attack is not very much changed (see Fig. E). It has been explained* that the rotational interference $a'r\Omega$ is made up of two parts, one due to lift (i.e. contributing to the static pressure), the other due to drag. The reduction in axial air velocity has the effect of increasing the amount of rotation due to drag. The torque remains almost constant, since the increased rotation is set up in a reduced quantity of air.

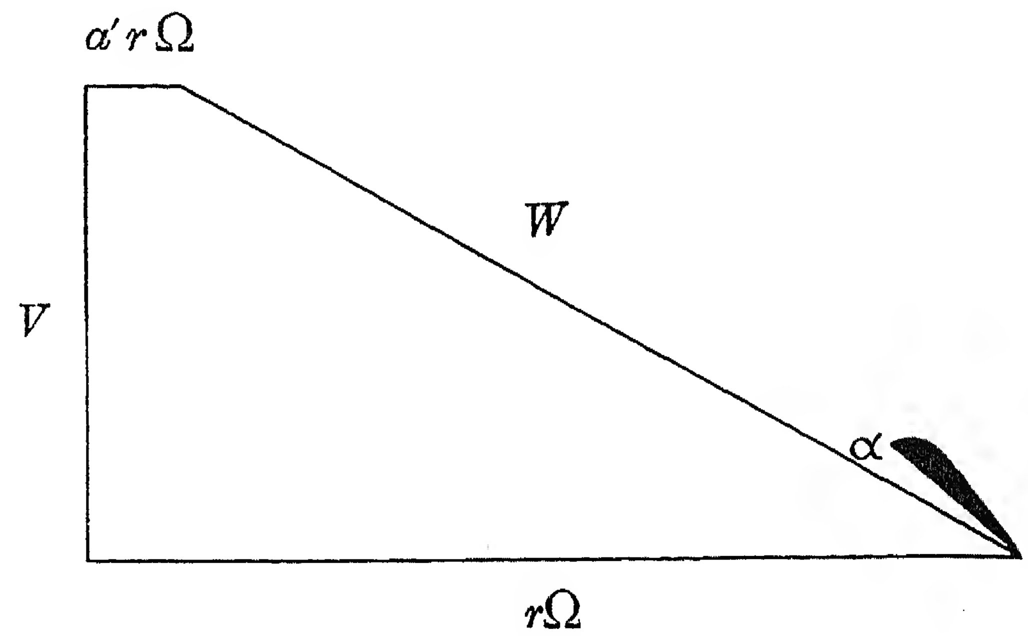

Fig. D.

In reply to Mr. Hoseason's statement that a high-water-gauge fan is preferable to a high-volume fan, this remark is surely only applicable to small machines.

In well-cooled machines the output for a given length increases almost as the square of the diameter. The losses, and consequently the quantity of air required for cooling, may also be considered to increase as the square of the diameter. The 110-h.p. machine instanced by Mr. Hoseason may be compared with larger machines of the same speed, as follows:—

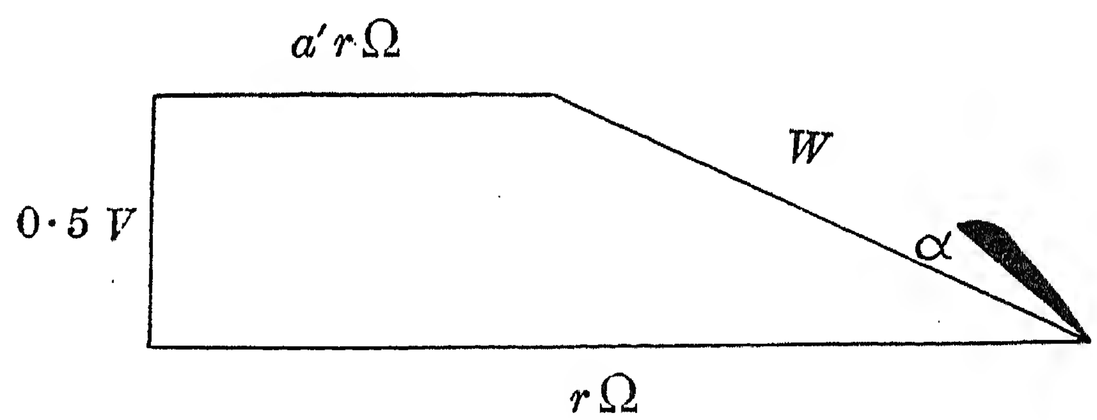

Fig. E.

Output	Air quantity	Pressure	Fan diameter
h.p.	cub. ft. per min.	in. of water	in.
110*	960	1·5	22·5
650	6 000	1·75	40·0
1 100	10 000	2·0	50·0

* Quoted by Mr. Hoseason.

Thus a machine of 10 times the output requires 10 times the air quantity, with very little increase in pressure. Nevertheless, the increase in fan diameter is such that the fan will give 5 times the pressure, if required, and it is therefore possible to pass the air through the

* R. Poole: "The Theory and Design of Propeller-type Fans," Institution of Civil Engineers Selected Engineering Papers, 1935, No. 4988.

machine at higher velocities. Moreover, the increased air quantity required by large machines permits the use of propeller fans having a greater blade pitch and increased aspect ratio, thus resulting in a higher efficiency.

In reply to Mr. Cowley, I would point out that careful attention is paid to the heat flow as well as to the air flow, in electrical machines.

Regarding the air passages through a machine, these are fixed to a great extent by the electrical design. It is generally found uneconomical to spend money upon shaping the entries to ventilating ducts, any resulting increase in machine output being more than swallowed up in the increased cost.

The effect upon output of increasing the quantity of air passing through a machine may be illustrated by a simple example. For a conductor temperature-rise of 40 deg. C. above normal air temperature, the drop in temperature through the insulation and across the iron core to the cooling surface would be of the order of 15 deg. C., whilst the difference in temperature between the surface and the cooling air might be 15 deg. C., the rise in air temperature over normal being, say, 10 deg. C. Doubling the air quantity and velocity would leave unchanged the temperature-drop of 15 deg. C. across the iron but would reduce the surface-to-air temperature-difference to, say, 9 deg. C., and the air temperature-rise to 5 deg. C. The total temperature-rise of the conductor over normal air would therefore be reduced to 29 deg. C., permitting the output to be increased in the proportion 40 : 29, i.e. by 38 per cent.

It must be pointed out, however, that it would require 8 times the power to send twice the quantity of air through a given machine. Moreover, an increased electrical loading might seriously affect the characteristics of a machine, so that the problem has to be viewed from numerous angles.

In reply to Dr. Piercy, the theory and design of propeller fans has been treated in detail in a separate paper.* The method I have adopted is a modification of the combined momentum and vortex theories.

In general, it is not necessary to vary the blade angle of a fan after manufacture, although blades mounted on spigots have been used in order to utilize the same blade pattern for various outputs and diameters of fans.

I have made use of a modern aerofoil, one of Mr. Glauert's extended Joukowski type, and the old N.P.L. sections were introduced in the paper to convey some idea of the work done in the early days of aeronautics.

The regenerative cone mentioned by Dr. Piercy could not be fitted to electrical machines generally, owing to space limitations. In the particular case illustrated in Fig. 22 (Plate 4) the motor unit drives an electric locomotive, and the overall height is limited by tunnel and bridge clearances, making it impossible to fit a cone.

Prof. Baily draws attention to the fact that the paper treats only with the quantity of air dealt with by the fans. The effect of an increase in air volume has been dealt with in my reply to Mr. Cowley. Much of the heat transfer from a well-designed electrical machine takes place in ducts through the core. The use of guide vanes, which increase the quantity and velocity of the air passing through the ducts, results in better cooling.

I do not agree that turbulence is required in the general air stream. It is well known that the cooling of a cylinder placed with its axis at right angles to an air stream is least where the turbulence is greatest; that is, immediately behind the cylinder.

Whilst the velocity through the fan may be low and below the critical velocity, that through the ducts generally exceeds the critical, so that the cooling of a machine is not necessarily improved by setting up excessive turbulence in the fan.

The final test of the efficiency of a ventilating system is the temperature of the machine. Machines of all sizes with outputs totalling over half a million kilowatts have been successfully cooled with propeller fans in the past few years. This in itself should be sufficient to convince the most sceptical of the practicability of the propeller fan for cooling electrical machines.

I do not agree with Mr. Borlase Matthews that "the top edges of the ends of the blades should be slightly more rounded in form." In the shape of blade tip lies the principal difference between the propulsive airscrew and one working as a fan against static thrust in a wind tunnel of a diameter approximately equal to that of the airscrew.

The thrust of a propulsive screw is created mainly by increasing the velocity of the slip-stream. A fan has to set up a definite difference in static pressure. The fan works almost as a piston in a cylinder. To round off the blade tips and thereby increase the tip clearance would be analogous to removing the rings from a piston; the leakage would increase and the pressure fall.

The blade form suggested by Mr. Borlase Matthews increases the eddy loss in the case of pressure fans. The close proximity of the shrouding gives the aerofoil blades characteristics approaching those of infinite aspect ratio. Any increase in the tip gap reduces the effective aspect ratio.

Regarding the suggestion of a meeting with the Royal Aeronautical Society, it is possible that in the near future a paper dealing with special static airscrew problems will be presented to the Society.

Prof. Thornton kindly draws my attention to some early and most interesting experiments on the cooling of electrical machines. He will no doubt be pleased to know that the study of the cooling of electrical machines, of which, I understand, he was one of the pioneers, has advanced so rapidly that in 25 years it has been found possible to increase the amount of heat carried away by forced convection to almost 95 per cent of the total heat generated. In present-day machines the amount of heat carried away by radiation and conduction is almost negligible, whilst in the experiments carried out in Prof. Thornton's laboratory almost 50 per cent was removed by the combined action of radiation and convection. Of recent years the total heat carried away per unit surface of machine has been considerably increased, as shown by Mr. Hoseason in a recent paper.*

* "The Theory and Design of Propeller-type Fans," *loc. cit.*

* "The Cooling of Electrical Machines," *Journal I.E.E.*, 1931, vol. 69, p. 121.

PRIVATE PLANTS AND PUBLIC SUPPLY TARIFFS.

By J. A. SUMNER, Associate Member.

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SUMMARY.

The general purpose of the paper is to make a comparison of the costs of power production for normal power requirements up to 500 kW. Power from private plants is considered first and the results are given for an investigation into power-plant costs which was made in the area of a semi-rural supply undertaking. A more generalized study of the cost of running Diesel and "pass-out" steam plant is then made, as these were found to be the most efficient forms of private plant.

The second part of the paper is devoted to an analysis of the costs of providing electricity from public-supply undertakings. The particular case of an actual semi-rural undertaking is studied (in whose area the private-plant investigation was made), and the technical design, progress, and costs, are detailed for this scheme. Private-plant and public-supply costs are then compared for this particular case.

Finally, a brief classification is made of the various types of supply undertakings, and the generalized costs for private plants are compared with the general case for the public supply. The conclusion is reached that a modern and efficient public-supply undertaking can supply power more cheaply than the most efficient private plant, except for those special cases which are outside the scope of this paper. It is found that public-supply tariffs are widely dissimilar, although the capital expenditure per consumer is shown to be approximately the same for all types of areas. These variations in cost are due in part to varying stages of developments between undertakings, but chiefly to the lack of a standard tariff basis, and it is suggested that national co-ordination of tariffs is essential if increased sales are to be obtained.

CONTENTS.

INTRODUCTION.

It is probably not realized sufficiently to what an extent the obtaining of a power load can reduce public-supply electricity-costs, owing to improvement in the system load factor and the reduction in costs per unit sold. A further point which may require some emphasis is that power loads are not confined to the urban areas but that there is a large potential load existing in the rural and semi-rural districts for such purposes as water-pumping, farm loads, etc., quite apart from those power requirements which may be indigenous to rural areas, e.g. for quarries, cement works, and coal and local mineral mining. It would seem that economic conditions such as heavy urban taxation and the desire for improved hygienic conditions of working may cause industrial plants to be moved from the towns to the rural areas, and several instances of this nature have occurred in the area of supply which is considered in the paper.

It is shown in the paper how a power load in a semi-rural area can accelerate development and the production of low costs, and it is desired to make a quantitative comparison of the costs of power produced from private plants and the public supply, respectively. The criterion which is adopted by a power consumer in deciding whether to use private plant or the public supply is that of deciding which method will provide him with the cheaper form of power. So far as private plant is concerned, the selection of the types of prime-movers which require study is narrowed down to the Diesel engine for ordinary cases, and the study is comparatively simple. Unfortunately, it is found that the costs of public supply depend upon the locality in which the power is required, and that the cost varies very widely between different areas. The paper, therefore, represents an attempt to classify and define the types of

authorized electricity areas, in order to ascertain whether there is any wide difference in the cost to the consumer of public supplies between the various types of areas, and then to make a generalization as to the possibility of standardizing public-supply costs.

The terms "urban" and "rural" are often applied, somewhat loosely, to differentiate between two main types of distribution schemes. This broad classification has been adopted in the paper to distinguish between schemes in areas with a dense population (exceeding 10 000 per square mile) where underground cables are used almost exclusively, and those areas comprising rural and urban (administrative) districts with a population density of about 200 per square mile. In the latter case it is possible to adopt a fairly comprehensive system of overhead lines. It is not generally realized that the rural scheme as defined above can very often afford to supply electricity at rates which compare favourably with those prevailing in the urban areas, if the areas are equally developed, owing to the reduced cost per mile of electrification which is possible by the use of overhead lines and the avoidance of heavy reinstatement charges, etc., compensating for the reduced load density in the rural areas. Further advantages occur, in that the competition of the gas industry is often absent or negligible, and also that a considerable power load may be developed.

The broad features which are common to various types of areas are indicated in Table 1. The first two cases refer to schemes which are entirely situated in rural districts, and the third to a scheme in an area comprising rural and urban districts, while the last two schemes relate to a small and large municipal borough respectively; the estimated figures for Great Britain are also shown so far as relates to the area now receiving supply.

The real distinction between rural and urban areas is shown in the "population per square mile" (Table 1). Even the small municipal borough has a density more than 10 times that for the semi-rural area and nearly 60 times that of the real rural areas. In the case of the larger borough, the population density is approximately 100 times more than for the rural area. The most interesting feature of the table is found in the fairly constant value of the "capital expenditure per dwelling on route of mains," from which it would appear that there is a real reason for assuming that the electrification of rural areas is not necessarily more expensive than distribution in urban areas, at least up to a 70 per cent development in the rural area.

It is also seen that the percentage of the population having an electricity supply available is higher for the urban than for the rural areas, and it is considered unlikely that more than 75 per cent of the rural population can be economically supplied under the present system of distribution financing. It should be noted, however, that less than 20 per cent of the population of England live outside the urban areas (municipal boroughs and urban districts), and that the rural area in which this small percentage of the population dwells comprises more than 90 per cent of the whole country.

A study of the data given in the last line of Table 1 leads to the conclusion that the margin of economic

TABLE 1.

Mains system	Bedford Rural Scheme*	Norwich Rural Scheme*	Scheme detailed in paper			Small borough	Large borough	Great Britain (estimated)
			No. 1 area (Semi-rural)	No. 2 area (Rural)	Total			
	Overhead	Overhead	Overhead	Overhead		Underground	Underground	Authorized areas
Area (sq. miles)	101	125	103	354	457	5·5	11·1	—
Population	15 860	14 166	56 000	40 000	96 000	33 000	130 000	—
Dwellings in area	4 083	3 500	12 500	9 500	22 000	6 600	36 000	$7 \cdot 7 \times 10^6$
Dwellings on mains	3 028	2 800	8 200	5 800	14 000	5 200	32 000	$6 \cdot 4 \times 10^6$
Distribution capital	£116 250	£100 000	£220 000	£142 000	£362 000	£160 000	£900 000	£194×10^6
Population per sq. mile	153	113	540	110	210	6 000	11 720	—
Capital expenditure per head of population	£7·3	£7·0	£4·0	£3·5	£3·8	£4·8	£6·9	—
Capital expenditure per dwelling in area	£28·5	£28·6	£17·7	£14·9	£16·5	£24	£25	£25
Capital expenditure per dwelling on route of mains	£38	£35·8	£27	£24	£26	£31	£28	£30
Dwellings on route of mains / Dwellings in area	75%	80%	65%	62%	64%	79%	89%	83%

* Data are from original estimates.

development occurs at a point which is less for rural than for urban areas; the probable margins are indicated in Fig. 1 and it would appear that the economic limit of development occurs in rural areas when approximately 70 per cent of the dwellings in the area have a supply available, as compared with 80–90 per cent in urban areas.

The analysis shows that the distinction between rural and urban undertakings is not to be found in the financial results which are ultimately achieved. It is probable that the difference is to be found in the relatively heavier and temporarily unremunerative expenditure which a rural undertaking must carry out in the initial stages, with the possible consequence of a greater accumulated deficit before financial equilibrium is obtained. An urban undertaking with a congested area can obtain a large number of consumers for a relatively small expenditure on mains, whereas the rural undertaking may have to provide an almost complete transmission scheme before being able to connect sufficient consumers to ensure an adequate return on capital.

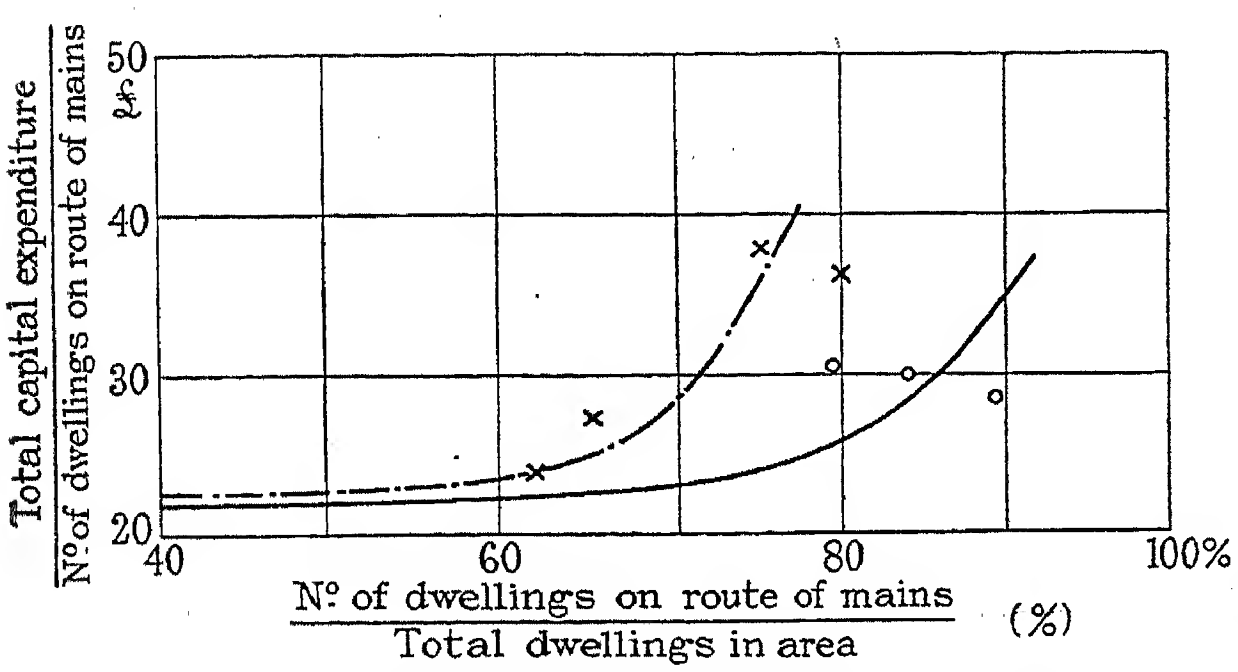


FIG. 1.—Approximate cost of connecting consumers (urban and rural areas).

Rural areas —·—·—
Urban areas ————

The policy of high prices and small sales is never justified in the later stages of development of any undertaking, but it is almost inevitable in the initial stages of a rural undertaking so long as the financial results of each undertaking are kept separate. The heavy accumulated deficit which is shown in Fig. 7 to have occurred during the first four years in the rural undertaking surveyed in the paper is a deterrent to the executive recommending tariff reductions. In the subsequent consideration of the cost of supply in rural and urban areas, therefore, this fact should be kept in mind.

The analyses made later bring to light a number of points which the scope of the paper precludes from full discussion. In the first place the generally accepted opinion that rural distribution is more expensive than urban distribution appears to be fallacious. It follows that the tariffs which require to apply in rural areas need not be greatly different from those applicable to urban areas, and in the analysis which is made in the paper this presumption is confirmed. It would seem that there are very good grounds for stating that it is possible to impose a standard national charge for electricity without disturbing national distribution finances.

Part 1. PRIVATE PLANTS.

(1) Summary of Investigation into Private Power-Plant Costs in Detailed Supply Area.

General.

It is probably correct to say that the majority of owners of private plant have no knowledge as to the cost per kilowatt-hour of power production from their plant; in some cases annual costs may be kept, but there is no knowledge as to the annual power output. A plant owner generally becomes interested as to the cost of his power production for one or both of the following reasons:—

(1) He may desire to replace existing, or to install additional, power plant.

(2) A public supply of power may become available and he may be led to ascertain the comparative cost of the public supply and his private-plant costs.

In either case, it will be necessary to determine two factors before any choice can be made, i.e. the amount of power which he requires and the load factor at which his plant runs.

The total cost of power is made up of standing and running charges respectively; generally, if the capital expenditure on plant is increased, the running costs will be lower and high efficiency may be attained. A point is reached, however, where the additional capital cost required to produce lower running costs will result in additional standing charges which will more than outweigh the saving obtained in operation. In correlating these two variables it is vitally necessary to determine the annual load factor at which the plant can be operated.

The need for stressing the importance of a knowledge of the load factor of a plant is the chief point in the paper and is essential to knowledge of the cost per kWh when electricity is sold on a 2-part tariff or when the cost per kWh is being compared for private plant and the public supply.

Purpose of Investigation.

The main purpose of the investigation was that of determining the actual cost and load factor at which each of the various private plants in the area was being run and the actual power requirements in units of power, with a view to submitting to each plant owner a detailed report showing the relative cost of the private and public supplies. It was immediately apparent that very few of the private-plant owners had any knowledge as to their real costs; the most usual method adopted was to keep total annual costs and to compare them year by year, but these costs were seldom related to the annual output, either in mechanical or electrical units or in manufactured articles.

The second point which arose was that the private-plant owners generally affirmed that they were running more cheaply than they could if they were to take a public supply of power, despite the lack of knowledge as to their annual output and the cost at which a public supply could be given; this assertion was most pronounced when Diesel plant was being used. Careful tests and investigations were accordingly made for each plant, and detailed reports were submitted to each plant

owner, showing the annual demand and output. In the majority of cases it was found that the cost of public supply was less than that of the private plant, and subsequent change-over to public supply has verified this conclusion.

Probably the most interesting result of the investigation has been to show the annual saving to the industries, in the geographical area defined later in the paper, which may possibly be effected by changing over to a public supply. It may be said that the introduction and use of a public supply has had the following results:—

(1) To re-organize the industries in the area and permit of cheaper production by virtue of the annual saving in cost and the greater facilities for working.

(2) To permit of future plant extensions at a cost of approximately 12s. 6d. per kilowatt (additional transformer capacity) instead of approximately £20 per kilowatt for additional private plant.

Before proceeding to the actual costs, the following points should be considered as relating to the comparison between private-plant and public-supply costs:—

A prolonged period of trade depression will involve the private-plant owner in the continuation of heavy standing charges which may be eliminated when the public supply is in use in the factory or works; even for normal running these standing charges may be greater than the running costs.

A manufacturer is only interested, primarily, in the total annual cost of his plant, until a public supply becomes available; e.g. if his 50-kW plant costs £1 000 at, say, 15 per cent, the total annual cost of £150 is of more interest than the cost per kW of maximum demand. But when a public supply is available and a comparison is being made of the private and public supply costs, then the cost per kW of maximum demand on the plant *must* be taken for comparative purposes, i.e. if the average of the maximum demands on the 50-kW plant is only 25 kW, the cost per kW of maximum demand for the private plant, which is comparable with the public-supply cost, is £150/25 kW, or £6 per kW, and it would be economical to take the public supply at this price.

A comparison between costs for private generating plant as compared with public-supply costs, particularly where it is proposed to initiate a power supply, should be projected into the future for a sufficient period to consider the possible trend of:—

(*a*) Relative fuel costs—chiefly for heavy oil. For political and economic reasons the price of fuel oil is likely to rise in the future, and an increase in price may neutralize any improvement in efficiency or reduction in first cost of internal-combustion plant.

(*b*) The "economic life," or obsolescence, of plant. Where a private plant is installed the whole plant must be written down in proportion to its economic life. If some change were to occur which produced a much more efficient prime mover, or if public-supply charges for power fell rapidly, the equivalent cost of running existing private plant would be increased at once. This increase would occur because of the need to write off the private plant more quickly than mechanical depreciation would require, owing to the "economic life" of the plant being the determining factor.

(*c*) There is greater flexibility of use and extension of power consumption when a factory is supplied from the public supply.

The last point demands further explanation. It was found that the "fleshing" machine in a certain tannery had a normal load of 12 h.p., rising for a period of 15 seconds each minute to 60 h.p. Similarly, an engineering works containing butt-welders and large presses had a normal maximum demand of approximately 1 000 kW, which rose to 1 800 kW for as long as 20 seconds. The necessary kW capacity of the private plant was, therefore, considerably higher than the recorded maximum demand on the public supply, which was based upon the averaged continuous demand over 30 minutes. The effect of this temporary overloading is of much greater moment when Diesel plant is proposed to be used than if steam plant exists, owing to the smaller overload capacity of the Diesel engine.

(2) Averaged Cost of Running, and Derivation of Averaged Tariffs.

(*a*) 12 Miscellaneous Private Plants.

The investigation represents the results of inquiries made over a period of 2 years for approximately 30 private industrial plants installed in engineering and industrial works and small collieries. The types of plant in use varied in age from 3 to 20 years and included steam, gas, and heavy oil plant. In a number of cases no costs were available, but 12 cases where fairly accurate costs were available have been chosen as representative of the type of plant in the particular semi-rural area of supply. The costs shown in Table 2 are those which were given by the plant owners, and it is reasonable to expect some variation due to different methods of costing; it is also unlikely that the costs are excessive in view of their origin.

The annual plant load factor of each plant examined is set out in Table 2 (line 6) and will be observed to average 17·1 per cent. The effect of this low annual load factor is of very great importance in determining the annual cost of running and in examining any comparison which may be made with public-supply costs. The average small factory or works runs for 48 hours per week, which would provide an annual load factor of 28·5 per cent if the plant could be run at continuous full load. In actual fact this latter condition is never realized and it is usual to find that the averaged load throughout the year is between $\frac{1}{2}$ and $\frac{3}{4}$ full load for the average class of works considered, giving an annual plant load factor of between 14 and 21 per cent.

It was found that the total kW capacity of the private plant installed was always considerably higher than the highest maximum demand (30-minute basis) which was registered on the public-supply meter after the change-over. The ratio between these capacities is shown in line 8 of Table 2, and the average for all the works in the area is found to be 1·65. The annual charges for interest, depreciation, etc., as also the annual load factor on private plant, must be based upon the total capacity of the plant installed, whereas the fixed charge per kilowatt for the public supply will only be in respect of the highest annual 30-minute demand. The effect of the resulting discrepancy, where comparisons are made

TABLE 2.

Item	Summary of Costs for 12 Private Plants												
(1) Types of Works	Mine and crusher	Small mine	Quarry	Foundry	Foundry	Tilery	Engineering works	Foundry	Foundry	Foundry	Foundry	Colliery	Averaged figures
(2) Year of installation, and type* of private plant	1930 PG	1912 S (NC)	1928 DS (C)	1910 PG	1909 and 1918 S (NC)	— TG	1914 S (C)	1916 PG	— PG	— PG	1918 PG	Mixed-pressure turbine	—
(3) Units required per year, kWh ..	1 089 360	280 500	416 300	180 000	640 000	51 000	2 361 000	185 000	243 000	88 760	281 000	998 000	(Total) 6 813 920
(4) Installed private plant capacity, kW	510	180	315	186	343	21	1 400	172	375	93	273	700	(Total) 4 568
(5) Highest (30-minute) maximum demand, kVA (includes conversion, power factor, and transforming losses)	230	100	280	100	303	21	800	123	250	56	183	295	(Total) 2 741
(6) Private plant load factor— $\left(\frac{\text{Item 3}}{\text{Item 4}} \times \frac{1}{8\,760}\right)$	24%	17·8%	15%	11·1%	21%	27·8%	19·3%	12·3%	7·4%	11·0%	11·8%	16·3%	17·1%
(7) Public-supply load factor— $\left(\frac{\text{Item 3}}{\text{Item 5}} \times \frac{1}{8\,760}\right)$	54%	32%	17%	20·6%	24%	27·8%	33·8%	17·3%	11·3%	18·2%	17·7%	38·8%	28·4%
(8) Weighting ratio for comparison with public-supply costs (Item 4 ÷ Item 5)	2·2	1·8	1·13	1·86	1·14	1·0	1·75	1·4	1·5	1·67	1·5	2·38	1·65
Private-Plant Costs.													
(9) Annual cost per kW plant installed, £	3·1	1·12	3·4	1·1	1·6	1·1	1·62	4·3	2·2	—	4·0	—	2·16
(10) Running costs per unit, d. ..	0·91	1·27	0·56	1·31	1·04	2·1	0·62	0·53	1·6	—	0·71	—	1·06
(11) Total cost per unit, d. ..	1·26	2·97	1·15	1·66	1·26	2·21	0·85	1·58	2·7	1·63	1·64	0·83	1·41

* S = Reciprocating steam (condensing—C, non-condensing—NC). PG = Producer-gas plant. TG = Town gas. D = Diesel.

NOTE.—The depreciation has been based on 15 years' use of plant, or on actual time of installation where this exceeded 15 years.

between private-plant and public-supply costs, will be to weight the fixed annual charges for private plant by a certain proportion. The weighting ratio adopted later in the paper, where comparative costs are considered, has been taken at 1·65, i.e. the averaged figure obtained from line 8 of Table 2, and this ratio has also been taken to include the amount of spare private plant which a works must install for breakdown and maintenance contingencies.

The chief result of the investigation was to show that the types of plant which achieved the lowest costs were the Diesel engine and steam power plant which passed out steam for works heating. The costs of running these particular types of plant will be accordingly studied now in some detail.

(b) Diesel-Engine Plant.

The Diesel engine is probably the most efficient type of private plant for installation in works, except for special cases where a demand for steam exists for heating or process work. The thermal efficiency of the Diesel engine is high and the present price of fuel oil permits of low running costs being achieved. The main factor in determining the cost per unit of power (kWh) produced per year, which is the basis of comparison adopted in this paper, is the annual load factor at which the plant can be run. As has been shown, the load factor is very low for small works running 48 hours per week, so that the annual standing charges on the high initial cost per kW of Diesel plant are the cause of a high price per unit of power produced. When a works is running continuously the Diesel engine would show better results, except that the cost of cessation of output from the factory is more serious and the time available for plant maintenance is much less. For these reasons more spare plant must be installed than is required for the

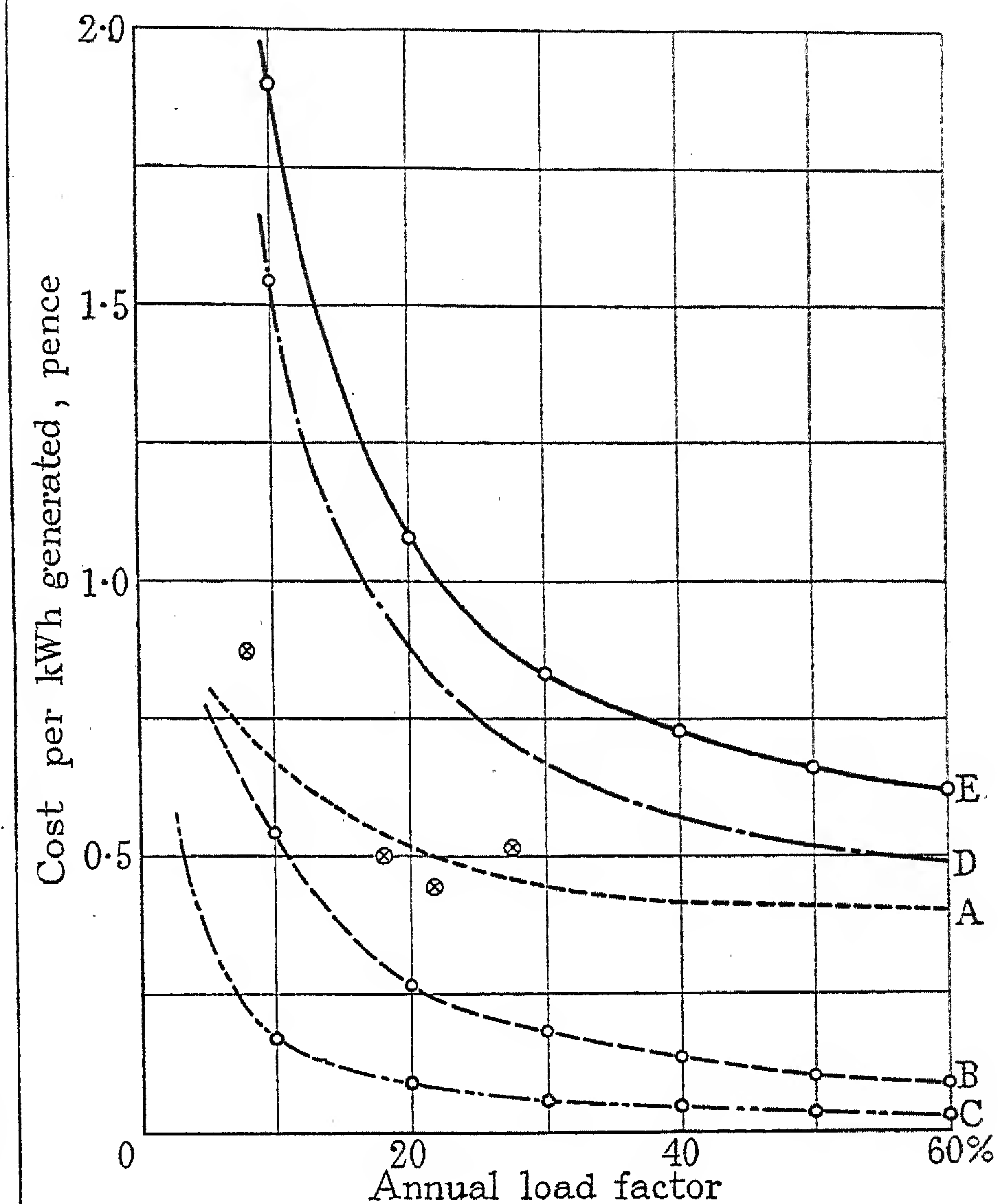


Fig. 2.—Cost per kWh generated by heavy-oil-engine plant. Figures of cost in curve A from D. E. U. A. Report, 1931–2.

Curve A.—Actual running costs for fuel oil, water, wages, repairs and maintenance. Fuel oil at 62s. per ton.
Curve B.—Interest and obsolescence £12 per kW; 4 per cent interest; 8 years' life.
Curve C.—Rent, rates, taxes, insurance—5 per cent per year on plant cost.
Curve D.—Total cost with no spare plant, i.e. sum of curves A, B, and C.
Curve E.—Total cost with spare plant, 65 per cent in excess of averaged max. demand, i.e. sum of curves A, 1·65 B, and C.

Table 3.

Cost of Running Small Diesel-Electric Plant (Direct-Coupled).

Capacity, kW ..	14	18	20	25	32	45
Capital cost of engine and generator ..	£340	£390	£420	£475	£575	£675
Price per kW ..	£24	£22	£21	£19	£18	£15
Working Costs, pence per hour.						
Fuel ..	6·9	8·9	9·9	13·2	15·95	22·44
Lubricating oil ..	0·4	0·5	0·6	0·68	0·95	1·3
Working Costs, pence per kWh.						
Total running costs (fuel and lubricating oil), pence per kWh ..	0·52	0·52	0·52	0·56	0·53	0·53
Fixed Charge (£ per kW).						
10 per cent depreciation, 5 per cent maintenance, 5 per cent interest } engine and generator ..	4·8	4·4	4·2	3·8	3·6	3·0

Running costs are based on 2 400 hours per year at continuous full load, i.e. 28 per cent load factor.
Fuel at 5·5d. per gallon = £5 2s. 4d. per ton.

Summarized Costs.

Average running costs (fuel and lubricating oil) = 0·53d. per kWh.
Average fixed charges, £4 per kW per year.
Total annual cost = £4 per kW of installed plant capacity + 0·53d. per unit generated.

works running intermittently, and the advantage of running for a greater number of hours in the year may be neutralized.

The averaged costs of running Diesel plant is now given, based partly upon tests made by the author on actual plant, and partly upon the averaged costs of running which are published in recent reports of the Diesel Engine Users' Association.

Fig. 2 shows the averaged costs for large Diesel engines; the fuel costs in Curve A relate to tests taken on a number of engines each running at varying annual load factors. A second example relating to the cost of running smaller plant (14–45 kW) is shown in Table 3.

Cost of Diesel plant per kW of Installed Plant Capacity.

The generalized cost of running Diesel plant, based upon installed plant capacity, is therefore:—

Large Diesel-electric plant:—
£3 per kW of plant installed + 0·4d. per unit generated.

Small Diesel-electric plant:—
£4 per kW of plant installed + 0·53d. per unit generated.

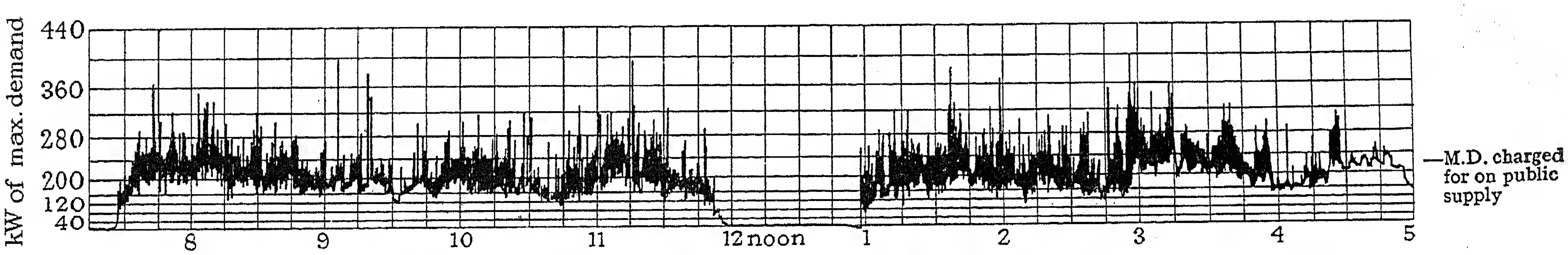


FIG. 3.—Works daily load curve showing relation between averaged maximum demand (220 kW) charged for on public supply and private plant capacity required.

Cost of Diesel Plant per kW of Averaged Maximum Demand.

This cost is of interest only when private-plant costs are being compared with public-supply costs, where the charges are generally based upon the " kW (or kVA) of maximum demand " based upon the average of all the demands which occur in a given period (generally 30 minutes). Some indications of the quantitative nature of these demands will be observed in Fig. 3, which is a reproduction of the actual load curve taken under ordinary working conditions on a December day at an engineering works with a normal type of load. The highest kW demand for which this works was charged over the period of 3 years that it has been taking the public supply is 220 kW. Previous to taking the public supply 400 h.p. of Diesel plant had been in use.

Where private-plant costs are being compared with public-supply costs, therefore, the basis for determining the fixed charge will generally be the cost per kW of maximum demand. It was shown, as an average, in Table 2 that the kW of installed plant capacity was 1·65 times greater than the kW of maximum demand on the public supply. Hence, for purposes of the comparison which is made later in the paper between public supply costs and Diesel costs, where the latter are based on installed plant capacity, it is correct to increase the private plant fixed charges by 1·65 to permit of a direct comparison with the ascertained kW charge for the public supply, provided the higher load factor which results on the public supply is adopted when determining the cost per kWh. The costs for Diesel plant then become as follows:—

Large Diesel-electric plant:—
£4·95 per kW of maximum demand + 0·4d. per unit.

Small Diesel-electric plant:—
£6·6 per kW of maximum demand + 0·53d. per unit.

(*c*) STEAM PLANT FOR COMBINED POWER AND HEATING.

Use of Heat rejected from Steam Engine.

An increase in the overall efficiency of the steam cycle, by using, for works-heating or process purposes, the heat in the steam exhausted from the prime-mover, will generally permit of a low cost for power production, given favourable conditions of use. There must, however, be continuous use of this rejected heat if high efficiency is to be maintained, whereas the average works only requires heating for approximately one-third of the year.

The efficient use of pass-out steam plant is dependent upon the following variants:—

(*a*) Cost of alternative forms of energy available for use in place of the rejected heat.

(*b*) The efficiency of utilization of the rejected heat.

(*c*) Possibility of controlling and finding continuous use for the rejected heat.

This latter is probably the most important variant; as an example, building heating is only required for about 35 per cent of the plant working days, and the plant must run for 100 per cent of the time. It is only in exceptional cases that any approach can be made to the continuous use of the rejected heat.

The most efficient use occurs where a pass-out turbine is used, and a case which was recently examined by the author illustrates the cost of steam power very completely.

The case relates to an investigation recently made in respect of an industrial engineering works which utilized the exhaust steam for factory and office heating and small processes; the engines and heating system were installed before the War. It can probably be considered as illustrative of the typical case for the class of power production now under consideration where reciprocating steam plant is still in use.

Highest annual maximum demand of power load	513 kW
Units required per year	949 000
Annual load factor of power load ..	20 per cent

Building heating required for 50 per cent of working hours, i.e. 2 572 × 50/100 or 1 286 hours.
Additional heating work consisting of canteen and ablution requirements, works processes, etc.
Evaporation factor 6·5/1.
Compound expansion reciprocating engines exhausting to heating system.

The costs submitted to the author for the existing plant were very carefully detailed and unusually accurate, but excluded capital charges. The plant had been in use for more than 20 years so that its value was assumed to have been written off. The total annual costs were as follows:—

Fuel, oil, and water (boiler efficiency 55 per cent)	£5 842
Other running costs	£1 714
Total annual running costs	£7 556

An investigation was made to ascertain the annual cost of supplying steam direct from the boiler to the heating and process work; this was found to be £1 478. The annual cost of the power production (excluding fixed charges) was thus shown to be £7 556 — £1 478, i.e. £6 078 or 1·54d. per kWh generated. It appeared probable that the fixed charges were not less than £2·5 per kW, representing a total cost of £2·5 per kW + 1·54d. per unit.

Substitution of "Pass-out" Turbine for Reciprocating Engines.

The installation of a 350-kW pass-out turbine was considered, running on the base power load and using part of the reciprocating plant to carry the winter lighting peak load, and the data for this plant and the economy achieved are as follows:—

Annual Standing Charges.

Interest (4 per cent), obsolescence (10 per cent), and insurance (3 per cent) on £3 950 (new turbine)	£671
Standing charges on 71 per cent of 800-h.p. boiler plant	£382
Standing charges on reciprocating engine	Not given
Total fixed annual costs	£1 053
Wages, maintenance, etc. (excluding fuel) ..	£1 700

Annual Running Costs.—The total amount of steam required per year for power and heating was found to be 38 000 000 lb., and the annual running cost for fuel in respect of this steam generation was £1 766. The steam for heating was only required during the winter months, and it was found that it could be supplied direct at an annual cost of £1 478 so that it was considered justifiable to charge the annual excess of £288 direct to the running cost of steam for power production. For an annual demand of 513 kW and an annual consumption of 949 000 kWh, the respective fixed and running charges were therefore

$$\frac{£1\ 053}{513} \text{ or } £2{\cdot}05 \text{ per kW}$$

and
$$\frac{£1\ 700 + £288}{949\ 000} \text{ or } 0{\cdot}55\text{d. per unit.}$$

When considering these later figures for the purpose of making a comparison for general cases, it should be noted that no capital charges have been included for the reciprocating steam plant. Again, it is possible that the cost of works heating could have been carried out as cheaply, or more cheaply than by the direct steam method, by means of electricity, with a certainty of better control and utilization. If the existing reciprocating plant which was used on the peak had not been available, the cost of power, assuming only 350 kW maximum demand, would have been as follows:—

$$\text{Fixed charge per kW, } \frac{£1\ 058}{350} = £3$$

$$\text{Running costs per unit, } \frac{£1\ 700 + £1\ 766 - £1\ 478}{949\ 000} = 0{\cdot}55\text{d.}$$

(*d*) Small Private-Residence Plants.

In general, the alternatives available are a small prime-mover and electric generator combined with a storage battery, the power requirements being met from the battery, or a prime-mover and generator without battery storage may be used, the prime-mover being automatically started whenever supply is required. The size of this last type of plant commonly in use varies from approximately 1 kW to 2 kW capacity. This type of plant is chiefly used for small power and lighting for residential purposes.

Inquiry has shown that the small residence gives less diversity than the larger residence, and that the smaller plant runs for a greater length of time at a point nearer to full load. The average load at which various sizes of plant are run is shown in Fig. 4A, which refers to an automatic petrol-electric plant. The diagram shows the cost of fuel consumption for varying loads, as stated by the plant makers; the makers' figures for the component and total annual costs are also shown in Fig. 4B.

The automatic plant referred to in Figs. 4(A) and 4(B) costs about £150 per kW to install; maintenance is carried out for a fixed annual sum of £7, and running costs apart from fuel are almost negligible. The component costs and the total annual costs of running the plant, also the summated cost in pence per unit generated, are indicated in Fig. 4B; the 2-part public supply tariff which corresponds to the total-cost curve in Fig. 4B is £22 per kW + 7·0d. per kWh.

The results of a number of tests and investigations

made on the various types of small private plants installed in farms and various classes of private residences are shown in Table 4. In many cases it has been necessary to accept the plant owners' allocations as to fixed

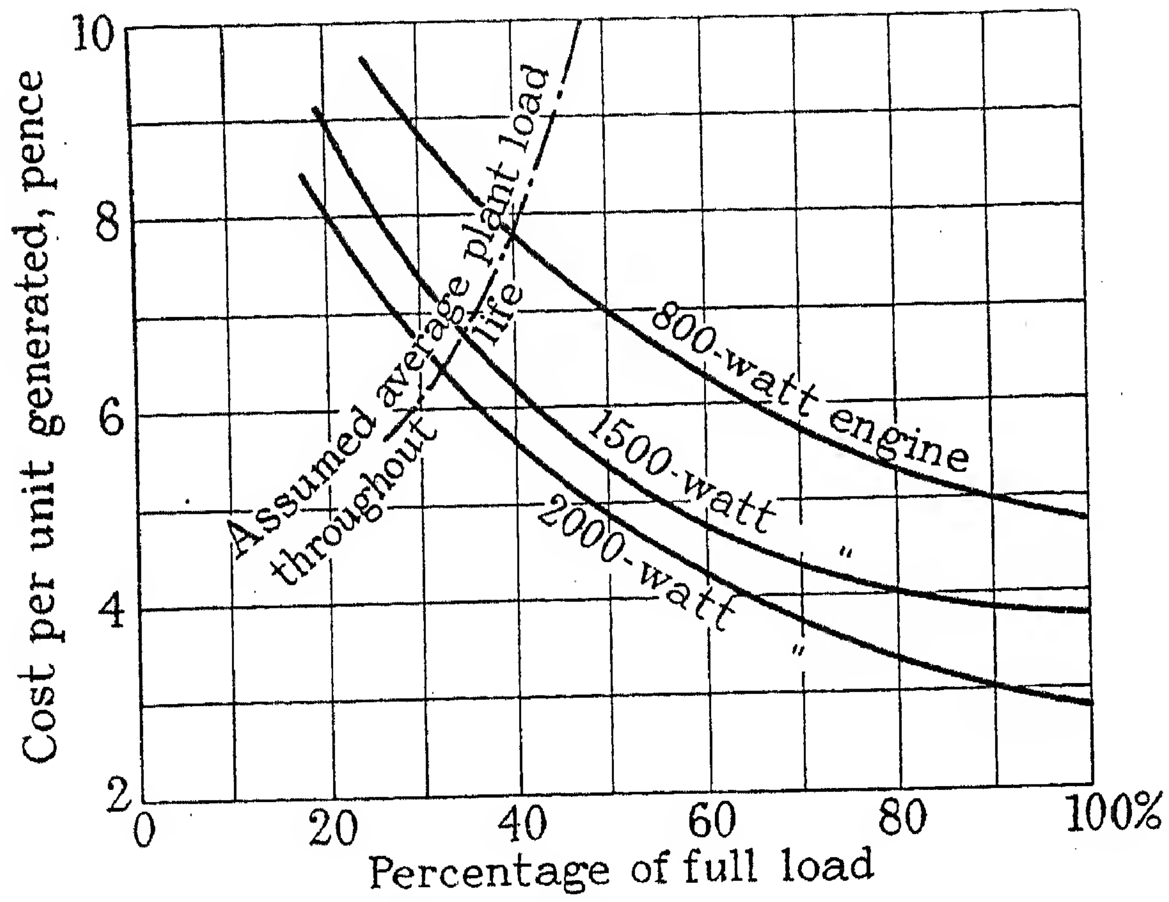


FIG. 4A.—Fuel costs at varying loads for small automatic petrol-electric plants. Manufacturer's figures.

and running charges, but in all cases the "Total annual costs" and the "Total cost per kWh used" have been ascertained fairly accurately. A large number of tests have been made on other, similar, plants in the area, and the costs agree very closely with the seven representative cases shown in Table 4.

Comparison of Private-Plant Costs.

The various costs which have been derived for the private plants considered, have been collated and set out in a graphical manner in Fig. 9, where the cost per

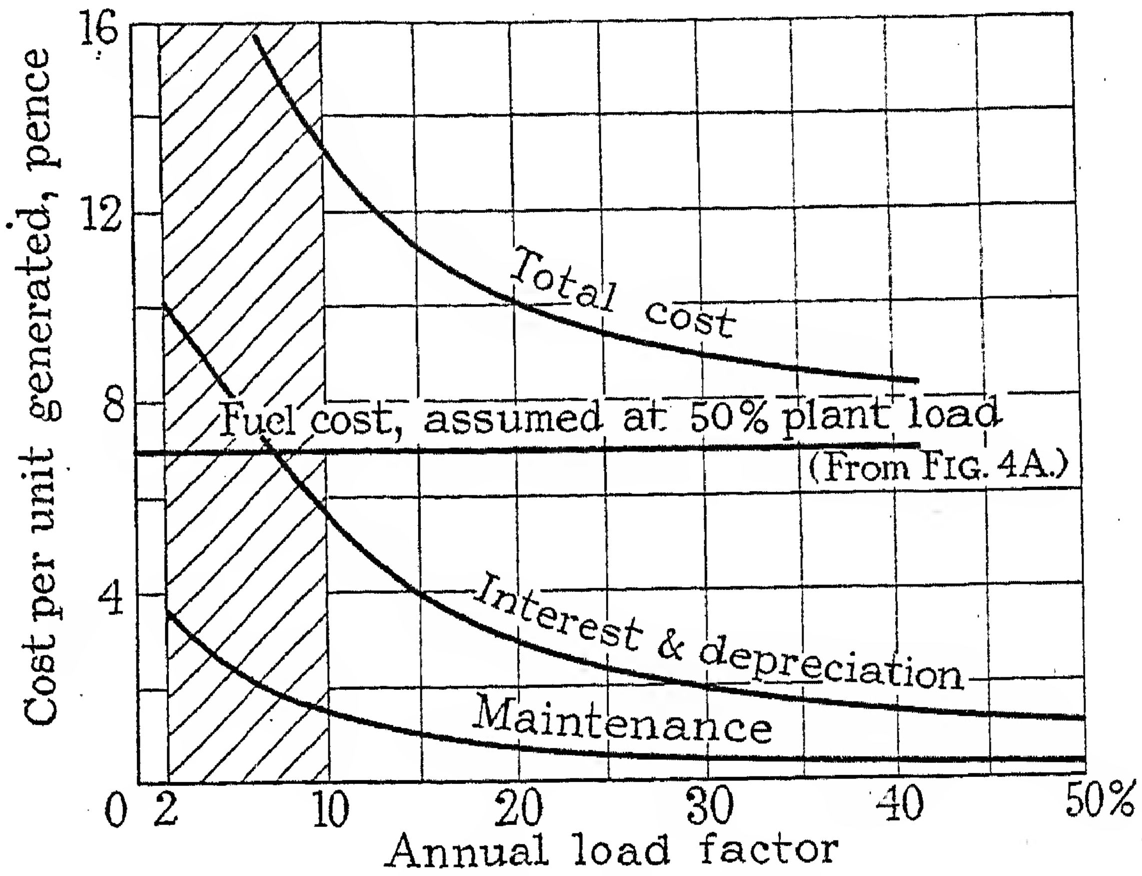


FIG. 4B.—Total and component cost per unit for small automatic petrol-electric plants. Two-part tariff: £22 per kW + 7·0d. per kWh. Normal range of load factor (2–10 per cent) is shaded.

unit is shown for an averaged annual load factor of 28 per cent. These costs are reproduced later in Figs. 9, 10, 12, and 13, when the cost of supply from various classes of public supply undertakings has been ascertained and a direct graphical comparison between private-plant and public-supply costs then becomes possible.

TABLE 4.

		Private house 1	Private house 2	Hall 3	Manor house 4	Small farm 5	Large farm 6	Cottage hospital 7	Averaged figures
Size of plant, kW		3·75	6·75	6	20	2	30	2·8	(Total) 71
Units generated per year, kWh		1 100	3 960	3 270	3 850	270	6 600	732	(Total) 19 780
Annual load factor		3·5%	6·7%	6·3%	2·2%	1·5%	5·2 %	3·0 %	3·2%
Annual standing charges (at 17½%), £	Total, £	45	25	30	10	2·5	—	18	—
	£ per kW	2	3·7	5	5	1·25	—	6·4	4·0
Annual running costs (per kWh)	Total, £	10	85	53	128	15·5	—	12·8	—
	per unit, pence	2·2	5·2	3·9	8·0	13·7	—	4·2	6·2
Total annual costs, £		55	110	83	139	18	24	31	—
Total cost per kWh used, pence		12	6·7	6·1	8·7	16	10	10·2	9·8

Averaged tariff . . . . £4·0 per kW + 6·2d. per kWh

Part 2. PUBLIC SUPPLIES.

(1) Detailed Description of a Particular Distribution Area.

(*a*) Geographical Features—Distribution of Loads and Population (Fig. 5).

The area, which is 456 squares miles in extent, is shown divided into two portions. No. 1 Area includes four small urban districts and has a fairly considerable industrial load in the form chiefly of small works with demands ranging from 30–350 kW; details of the class of works are given more fully in Table 2. The population density is 540 per square mile (see Table 1). No. 2 Area comprises the remainder of the scheme and has a population density of 113 persons per square mile, i.e. the same density as the average for the rural areas in Great Britain. Certain power loads are available, and one case is of particular interest as indicating the growing tendency to establish new industries in rural areas. It refers to an old-established business with an annual power consumption of 400 000 kWh which transferred plant and staff from a large town in the Midlands into the heart of a rural district, building a housing scheme for the staff and workmen. Another works which has an annual consumption exceeding 1 000 000 kWh is engaged in crushing and preparing a mineral which is mined locally and which only occurs in large quantities in the rural district indicated. In both instances modern private plant (Diesel and producer gas) was in use when the public supply was commenced.

The distribution area comprises four urban districts having populations of 8 185, 7 388, 3 677, and 3 439 respectively, and a municipal borough which has an area of 8·8 square miles and a population of 7 831; the remainder of the distribution area consists entirely of rural districts. The scheme was commenced in 1930 and there are now 4 500 consumers taking supply.

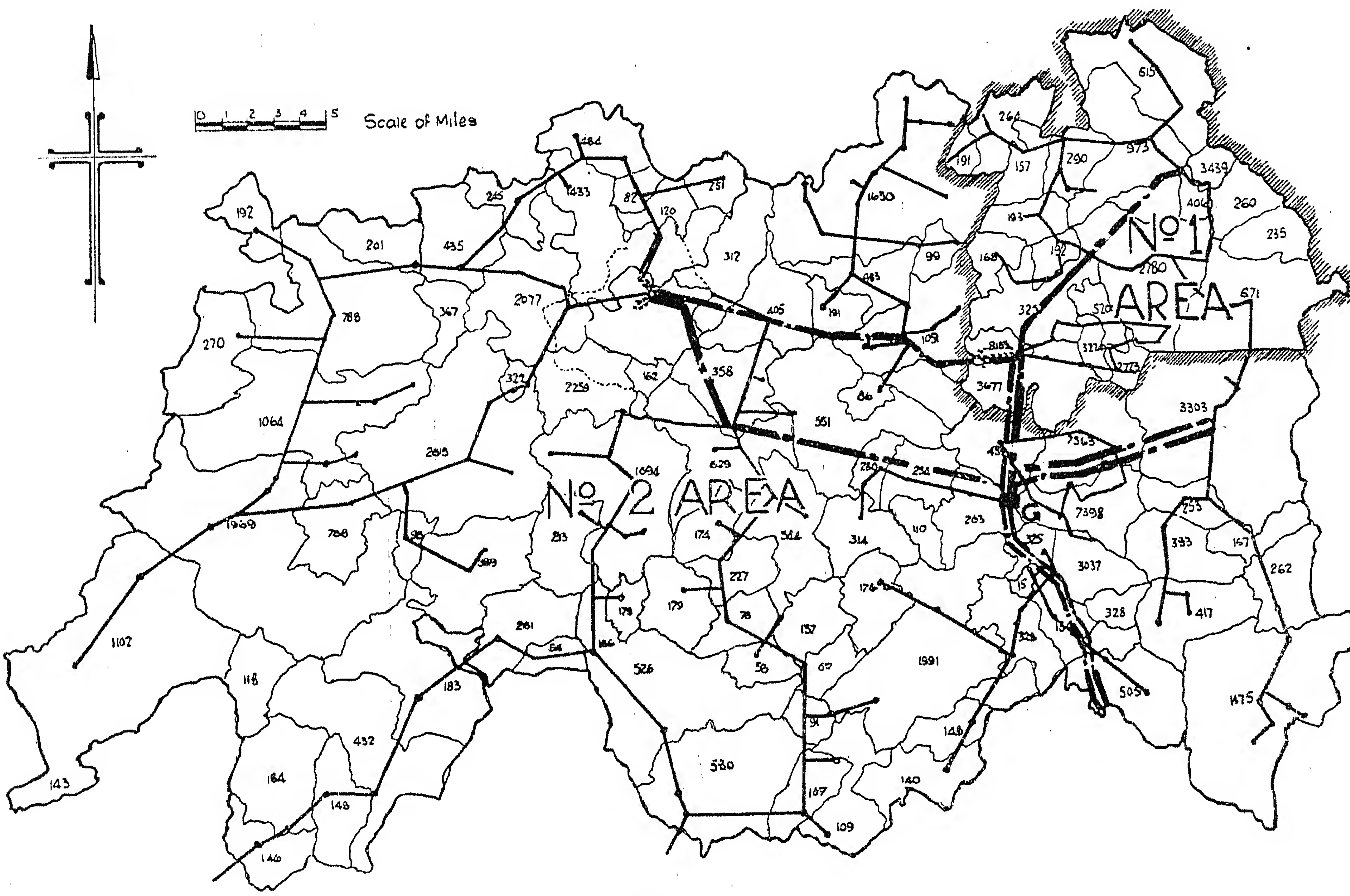


Fig. 5.—Plan of areas and lines (7th year of development).

Steel-tower 33-kV lines — Wood-pole 11-kV lines

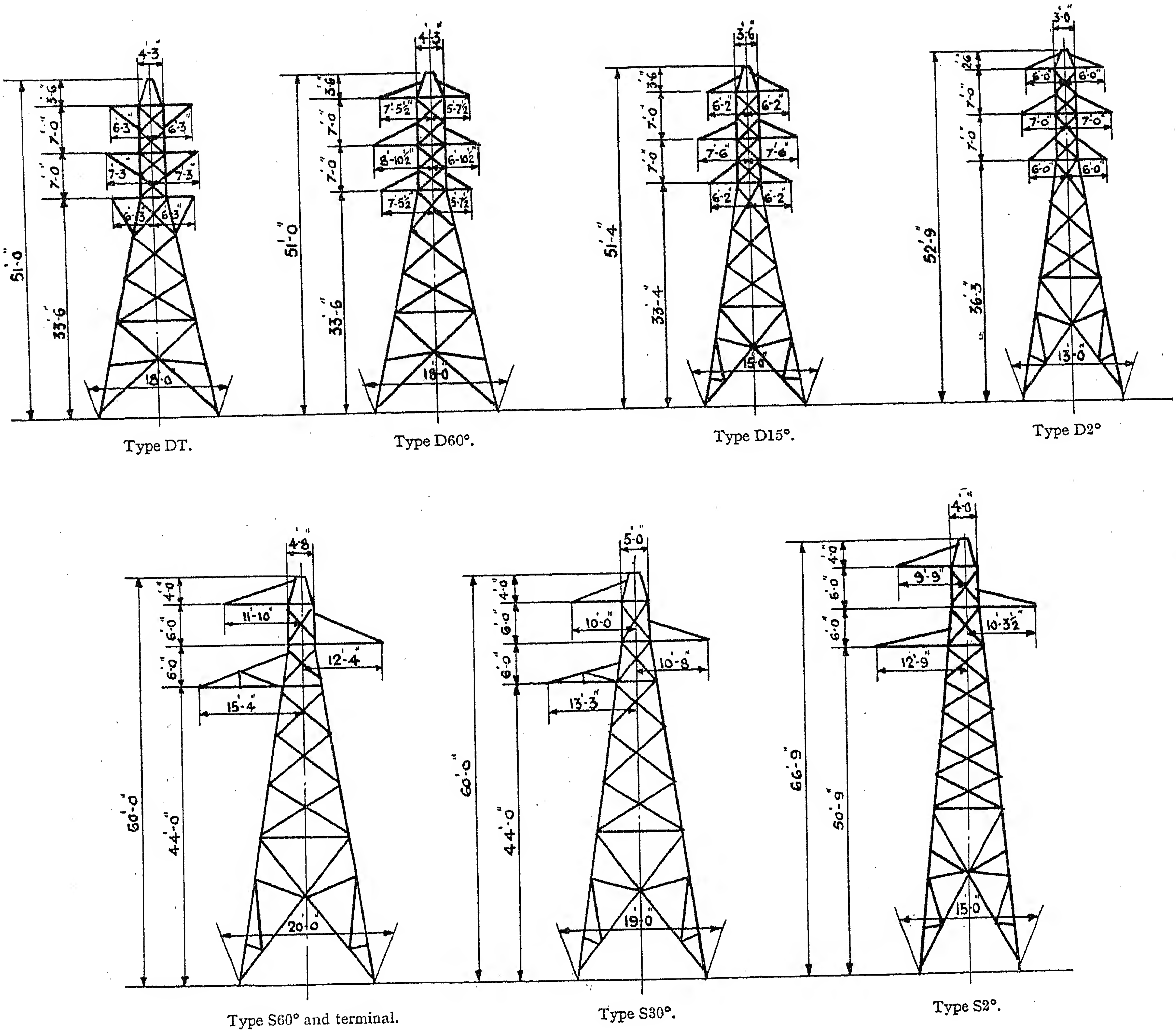
Development proceeded exclusively in No. 1 Area during the first two years, and a gradual development commenced in No. 2 Area at the end of the second year.

(*b*) Technical Features of Scheme and Lay-out of System.

The supply is taken from the grid at the point " G " indicated on the map in Fig. 5, and 33-kV mains were installed at the commencement as shown by thick chain-dotted lines; reactors are provided of sufficient impedance to limit the short-circuit kVA to 500 000.

Main 33/11-kV transforming points are thus provided at sufficiently wide intervals throughout the area to allow of a general system of secondary (11-kV) and low-tension (400/230-volt) lines being installed, sufficient to

33-kV SUPPORTS.



L.T. and 11-kV SUPPORTS.

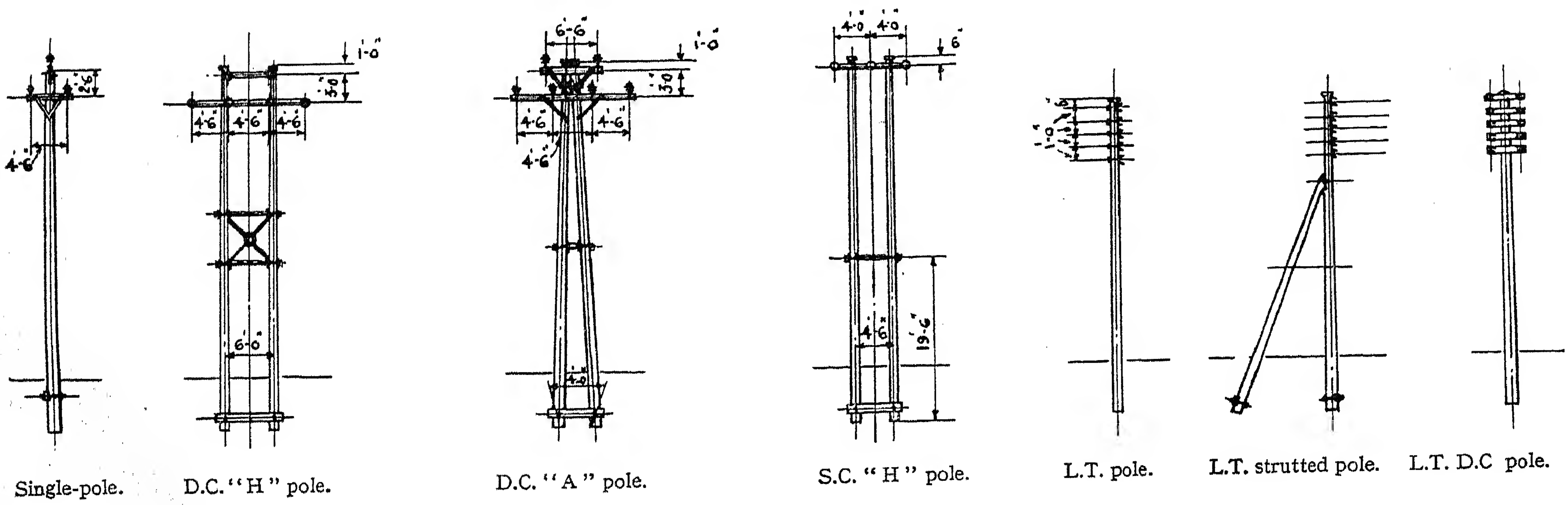
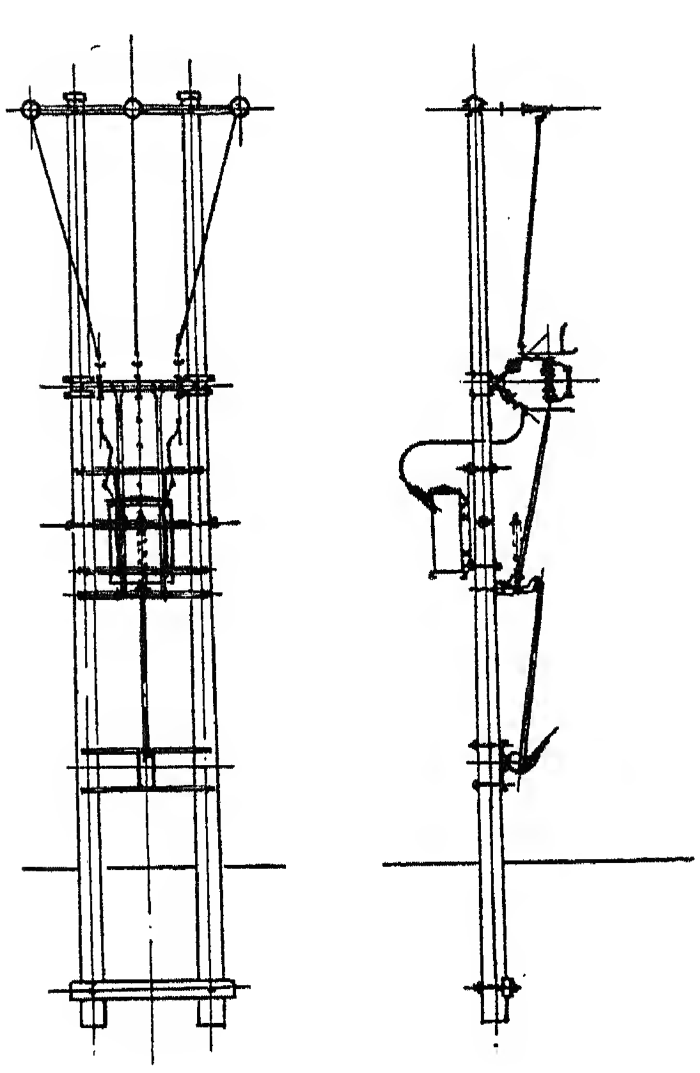
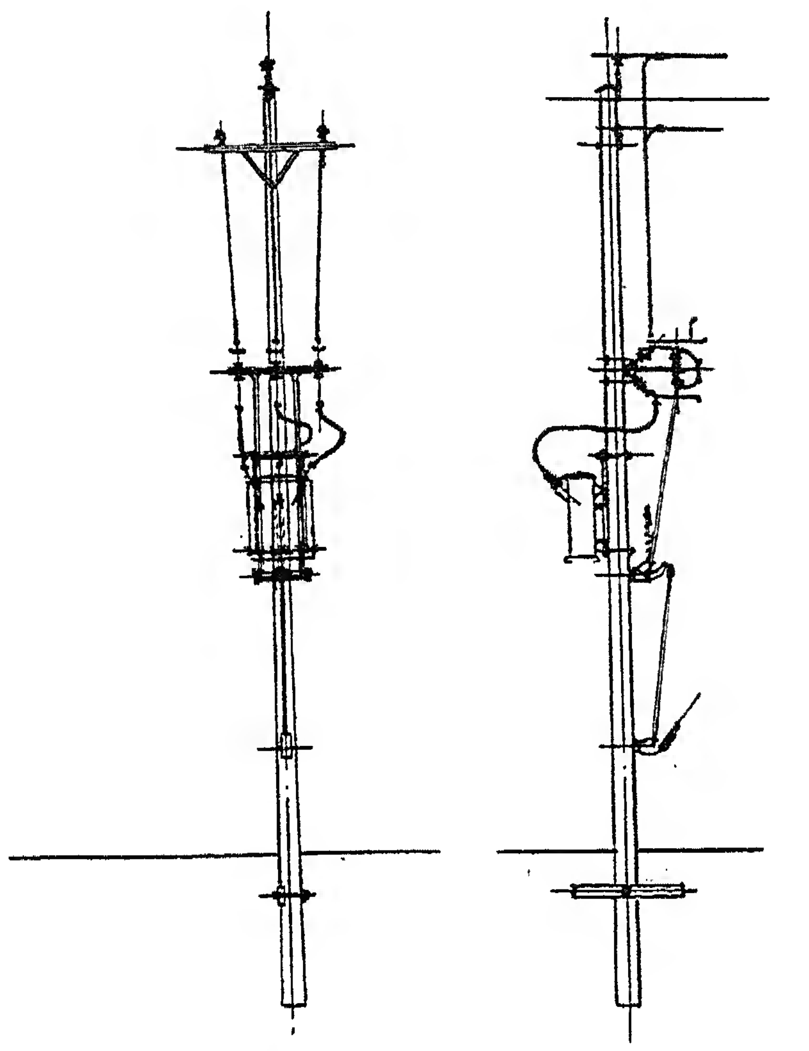


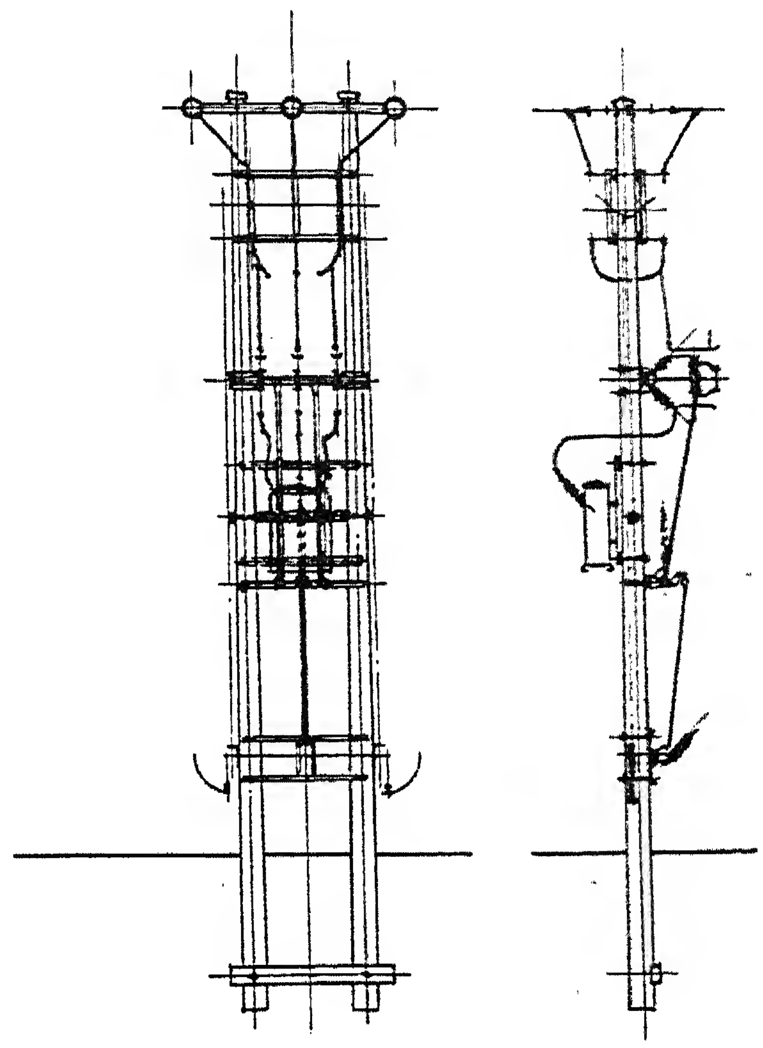
FIG. 6A.—Supports.



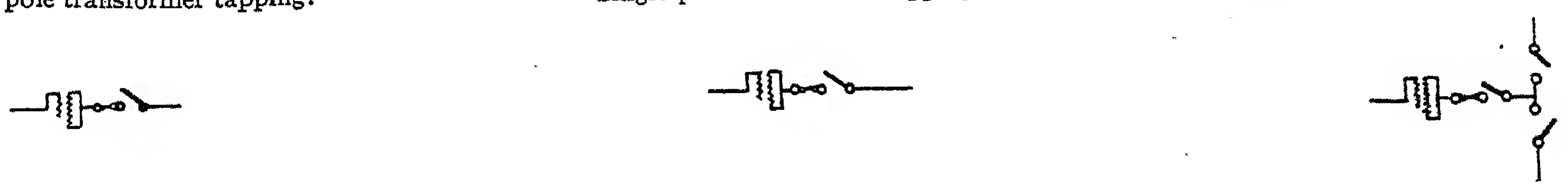
" H " pole transformer tapping.



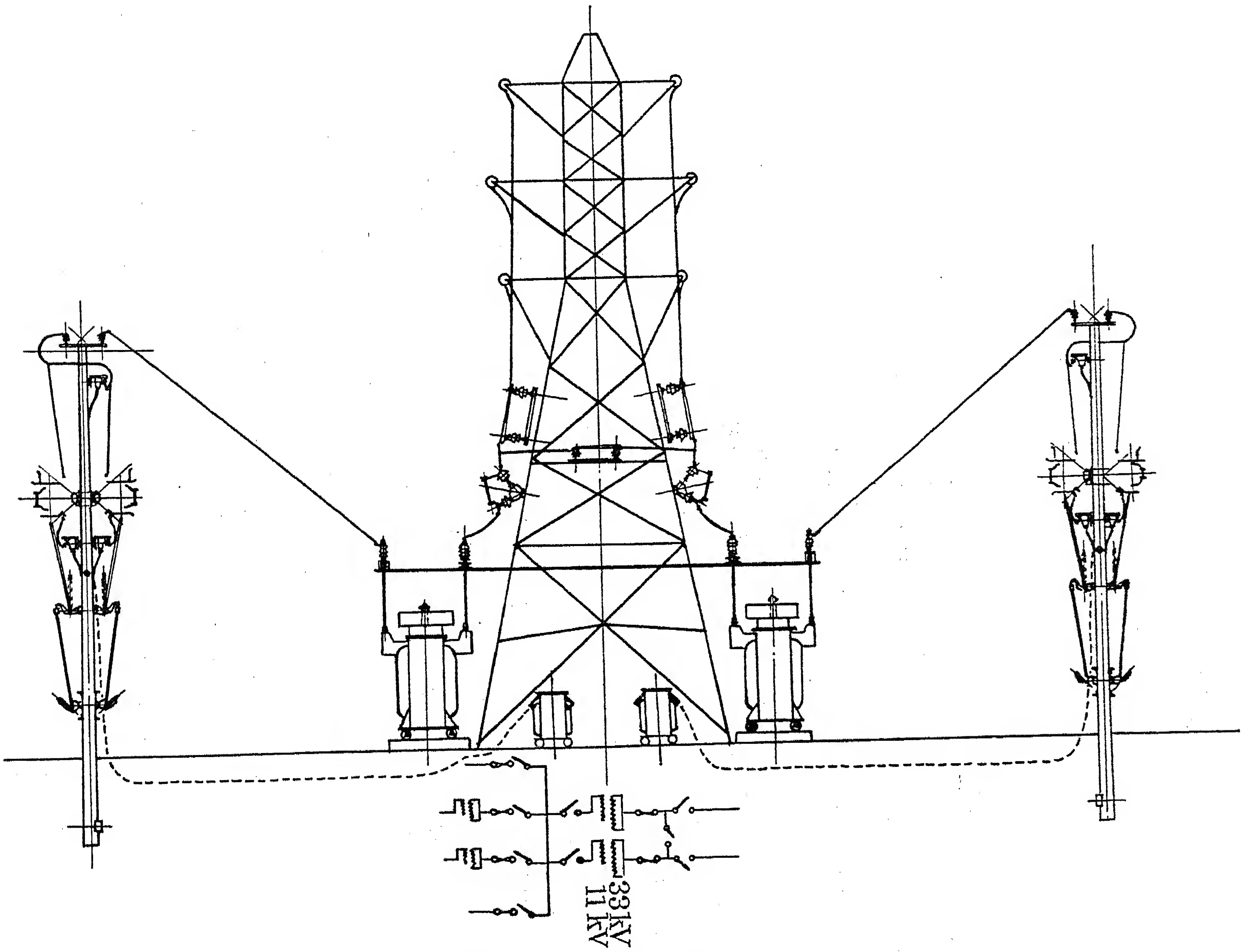
Single-pole transformer tapping.



" H " pole isolating and transformer tapping.



11-kV switching and tapping points.



33/11-kV switching and tapping points.

FIG. 6B.—Switching and tapping points.

meet the load likely to develop in the first 10 years. The system is designed to be capable of meeting a maximum demand of 20 000 kW, 10 000 kW of which it was anticipated would occur after 10 years, by which time the estimated total capital expenditure would be approximately £400 000.

The sizes and lengths of the various classes of mains and distributors are shown in Table 5.

It will be observed that one of the conclusions reached by the author is to the effect that h.t. power can be supplied from a rural or semi-rural scheme at less cost than if modern private plant were installed. If this conclusion is correct it is reasonable to infer that new industries may be established in the country, so far as the consideration of cost of power is concerned. The trend of economic circumstances during the past few years has definitely encouraged the tendency to develop industries in rural areas, and there appears to be every possibility that it will be maintained, particularly as more new trades and industries are brought into being. It should be noted that the technical lay-out of the scheme would permit of new industrial demands being met in the most rural portions of the area. The use of 11-kV closed-ring systems in No. 2 Area using conductors of 0·025 sq. in. (equivalent copper) cross-section, permits of low initial cost and gives a conductor large enough to deal with even more than the anticipated rural demand. If an industrial load were to arise within these rings, a diagonal feeder could be installed quite cheaply for the purpose of giving a direct feed to the new load and to permit of tapping-in to a point in the ring which is most distant from the feeding points.

The major portion of the scheme is carried out by means of overhead construction having designs and characteristics as shown in Figs. 6A and 6B. The design is standardized and unit construction is adopted wherever possible; the latter feature is to be seen in the case of the 11-kV line tappings where the development may be seen from the single tapping point into a combined isolating and tapping point. The rather large clearance between 11-kV phase conductors (3 ft. 6 in. minimum), combined with the use of long pin insulators, is a further feature which has proved of great value in the elimination of bird faults or conductor short-circuits during high winds. The length of each class of main on the system is indicated in Table 5.

Main Transmission.—A voltage of 33 kV was adopted for main transmission, and all 33-kV lines are on broad-base, lattice-steel towers (Fig. 6A) with **37/·102-in.** steel-cored aluminium conductors having an equivalent copper section of 0·15 sq. in. The immediate demand of the distribution area does not justify the use of the excellent but expensive steel-tower lines, but consideration was given to the fact of the heavy cost of line losses and the difficulty in voltage regulation which would ultimately be experienced with smaller conductors at or near full loading. The use of this type of support and conductor was accordingly standardized throughout the scheme for all main transmission. The single-circuit line supports are to the same design as for the single-circuit 132-kV lines of the Central Electricity Board, and it is interesting to note this instance of low costs due to standardization. In **1931**, when tenders to the specification of the undertaker were invited, alternative tenders were called for, based upon the 132-kV tower design specified by the Central Electricity Board. It was found that a reduction in price of £300 per mile was obtainable by using the 132-kV type of tower; the greater clearances which were obtained made possible an improvement in the line design as well as effecting a considerable saving in cost.

Secondary Transmission.—Secondary transmission is at 11 000 volts (3-phase) as this was considered to be the future standard voltage likely to apply throughout the country. The system of earthing at each pole was adopted owing to the saving in cost, as compared with the alternative method of erecting a continuous earth wire above the phase conductors. It will not be possible to state whether the saving in cost has been justified, until a number of years' experience of lightning trouble

TABLE 5.

Lengths of Mains on the System (7th Year of Development).

Class of main	Conductor sizes (sq. in. of area)							Totals (miles)		
	0·25	0·20	0·15	0·10	0·075	0·05	0·025	No. 1 Area	No. 2 Area	Totals
33-kV double-circuit steel-tower line ..	—	—	18·6	—	—	—	—	10·3	8·3	18·6
33-kV single-circuit steel-tower line ..	—	—	18·2	—	—	—	—	6·6	11·6	18·2
33-kV underground cable	2·2	—	—	—	—	—	—	0·2	2·0	2·2
11-kV double-circuit wood-pole lines	—	—	—	—	3·1	—	—	1·8	1·3	3·1
11-kV single-circuit wood-pole lines ..	—	—	—	2·6	53·4	25·3	202·2	51·3	232·2	283·5
11-kV underground cable	—	—	1·1	3·7	1·5	5·4	—	7·1	4·6	11·7
L.T. wood-pole lines	—	—	—	2·6	32·6	79·3	5·4	32·7	87·2	119·9
L.T. underground cables	—	0·2	7·6	19·2	—	0·5	—	20·6	6·9	27·5
Service cables and lines	—	—	—	—	—	—	70·0	40·0	30·0	70·0
								170·6	384·1	554·7

has been obtained and analysed. There is no doubt that the use of a continuous earth wire above the phase conductors is a valuable safeguard against lightning surges, and that the effect of these surges may prove expensive in certain parts of the country; unless the earth wire is placed above the phase conductors there is no safeguard against lightning troubles.

Copper and steel-cored copper conductors are used throughout the 11-kV system. Two sizes of conductor only have been used for 11-kV lines throughout the whole scheme, i.e. 7/·117-in. copper with a cross-sectional area of 0·075 sq. in., and steel-cored copper conductor consisting of 12 strands of 0·052-sq. in. copper surrounding 7 strands of 0·052-sq. in. steel; the equivalent copper section of the compound conductor is 0·025 sq. in. It was found that only a very small saving in line cost was achieved if copper conductors of less than 0·05 sq. in. were used in place of the 0·075-sq. in. conductor, owing to the increase in pole height, or reduction in span length necessitated by the greater sag that occurs with the smaller section. Where small loads had to be carried, the 19/·052-in. steel-cored copper conductor was considered to be the most economical size, with span lengths averaging 440 ft.; the span lengths on the larger lines average 375 ft. It is concluded that single-phase extensions are uneconomical and unsuitable, and in the few cases where a single-phase transformer and supply may be required in the initial stages the complete 3-wire 3-phase line is always run. Secondary spur lines are generally required for farms where a power load will ultimately develop, and the use of 3-phase working then becomes essential.

Low-Tension Distribution and Services.—This work is carried out by means of overhead lines, with the exception of the distribution in two small urban districts where the streets were very narrow and tortuous. It was considered that the minimum size of conductor should be 0·05 sq. in. copper, and that the small saving per mile of line which was effected by using a smaller section was not justified in view of possible future load developments.

It will be seen from Fig. 6A that D-iron brackets are used as a standard for all l.t. lines. In certain cases single-phase overhead mains are run, but in these cases the supports are always drilled for a 3-phase line, and the medium wood support and span lengths of the 3-phase line are adopted. Provision is made for one pilot wire on the overhead lines, and all underground cables are provided with two pilot cores.

Post Office and Railway Crossings.—The cost per mile of line may be considerably influenced by the number of Post Office and railway crossings which occur and are usually difficult to avoid. The practice which has been adopted in carrying out the detailed scheme is to arrange for the Post Office to place their wires underground wherever this is found to be practicable. In cases where a Post Office trunk line occurs, the Post Office will not agree to run their line underground, and in such cases the power wires have been run overhead and surrounded by the rather ugly guard which is required. This latter practice has been found to be cheaper than to run the power conductors underground. The cost of various methods of crossing are given in Table 6; these are the averaged costs for a number of cases.

TABLE 6.

	Voltage of power circuit			
	11 kV		33 kV	
	Per yard	Total	Per yard	Total
Post Office Communication Circuits.				
Power circuits run underground (estimated cost only)	£1·5	£73	£7·5	£750
Power circuits and P.O. guard over P.O. circuits (cost of guard)	£0·57	£39	£3·3	£260
Post Office circuits run underground	£0·366	£25	£0·366	£37
Railway Communication Circuits.				
Power circuits overhead across railway (guard provided by Railway Company)	£1·05	£70	£1·5	£100
Railway Company's circuits run underground	£2·0	£100	£3·0	£150
Power circuits run underground across railway	£1·8	£93	£7·5	£750

General Procedure during Erection.

All high-tension lines are surveyed and a 25-in. scale longitudinal section and route plan of the route prepared before quantities are taken out and ordered. It has been found that the saving in quantities effected by this method more than repays the cost of surveying work. This saving is particularly noticeable when spans exceeding 250–300 ft. are being used. The section and route plans are then issued after the quantities have been taken out, ordered, and delivered to the nearest unloading site; in this case also the preparation of the precise details which are issued to the constructional engineer permits the work to proceed with a minimum of delay and cost of erection.

The construction of the steel-tower lines was carried out by contractors owing to the special and temporary

nature of this class of work, which requires specialize labour to obtain economical erection. The constructiod of the wood-pole lines, however, can be carried out i a very economical manner by the undertaking, who are bound to keep a certain staff available for maintenance and fault work. The obtaining of wayleaves and statutory consents also is of such a hazardous nature that satisfactory erection can only be achieved when the undertaking employs the staff necessary for all branches of the work, and is able to co-ordinate their activities.

(*c*) Financial Development and Growth of Load.

The following main tariffs were adopted initially:—

H.T. power:—£4 10s. per kVA of highest annual maximum demand (30-minute increments).
Unit charge of $\frac{1}{2}$d.

Lighting:—Flat rate 7·2d. per unit (net).

Heating:—Flat rate 1·8d. per unit (net).

L.T. power:—$2\frac{1}{4}$d. per unit.

Domestic 2-part tariff:—Fixed annual charge of 7s. per 100 ft. of occupied area of dwelling.
Running charge of 1d. per unit.

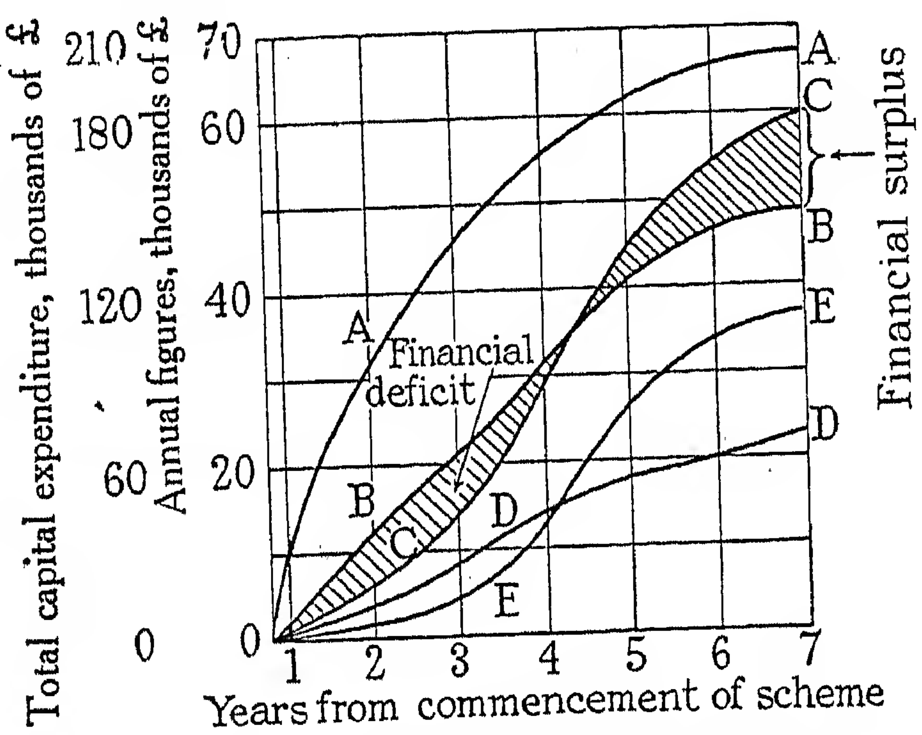


Fig. 7.—Financial and development data. No. 1 Area only (to 7th year). The 6th year is 1935–36.

A = Total capital expenditure.
B = Annual costs (including cost of current).
C = Revenue (all purposes).
D = Revenue (l.t. supplies).
E = Revenue (h.t. supplies).

Special rates are available for off-peak and continuous loads and all special cases of consumption which improve the system load factor.

Development of No. 1 *Area* (*Fig.* 7).

The curve of capital expenditure (A) begins to fall off rapidly after the fourth year, due to the secondary and main transmission having been completed; curve (B), the annual costs, reflects the tendency of the capital expenditure. In considering curve (C), which indicates the annual income from consumers, several points of interest occur. To the end of the first year only a very small income was being obtained from h.t. power consumers, and it is clear that the normal revenue from l.t. consumers (chiefly domestic and shops) would not permit of a condition of financial equilibrium being established before the 6th or 7th year of operation, at the tariffs which were formulated initially. The need for developing the power load, so as to increase the annual income and reduce the working costs per unit sold, was realized to be a vital matter, so that during the second year an investigation was commenced in the industrial works and a commencement was made in connecting them to the h.t. mains, the process being continued for three more years, by which time 85 per cent of the possible h.t. load had been obtained and connected to the mains. The effect of obtaining this load is very obvious from curve (C) and has enabled a surplus to be achieved at the end of the third year of operation. At the time that the surplus was achieved it was considered that the need had passed for intensive effort on the industrial load, and these efforts are being diverted to the agricultural power and the l.t. power and domestic load; the effect of this development cannot yet be estimated.

Development of No. 2 *Area* (*Fig.* 8).

Referring to Fig. 8, at the end of the third year development was commenced in the more sparsely populated No. 2 Area. The effect of this second extension will be seen to react unfavourably on the whole

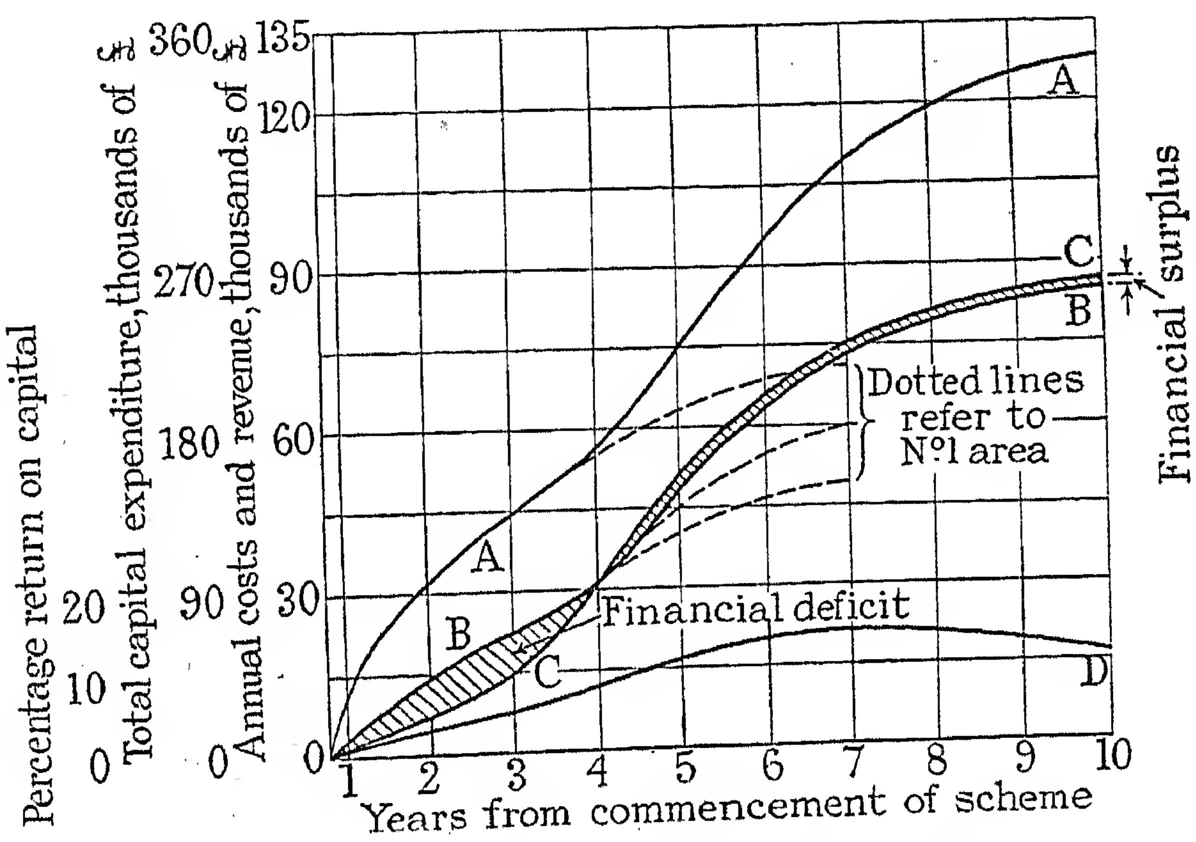


Fig. 8.—Financial and development data. Nos. 1 and 2 Areas (whole scheme) to 10th year.

A = Total capital expenditure.
B = Annual costs (including cost of current).
C = Annual revenue.
D = (Annual costs)/(Total capital expenditure), per cent (excluding cost of current).

scheme, at least so far as the immediate position is concerned. If, however, the position is considered in its true perspective, and continuing development of demand is postulated, it will be found that the larger scheme will be more remunerative than the smaller scheme, and, in addition, the moral obligation which is with any undertaking, i.e. to give the benefits of the service of electricity to as many persons as possible, has largely been complied with.

Curve D (Fig. 8), which represents the percentage return required on capital, is of some interest, especially as the time at which financial equilibrium in annual costs is achieved coincides with the achievement of a ratio of 12 per cent between annual charges (exclusive of cost of current) and capital expenditure. It is estimated that the return on capital which will be required after the 5th year should fall to an average of 12 per cent or less as the revenue and administrative charges will remain fairly constant beyond the time that financial equilibrium occurs.

TABLE 7.

Loan Charges and Method of Deriving Tariffs for Detailed Public-Supply Scheme.

	7th year	10th year
(*a*) Total maximum demand, kW	4 300	7 850
(*b*) H.T. maximum demand, kW	3 200	5 050
(*c*) L.T. maximum demand, kW	1 100	2 800
(*d*) H.T. consumers' diversity (on above demand)	1·55	1·58
(*e*) Total units sold per annum	15 900 000	23 500 000
(*f*) Consumers	5 500	8 200

Item	No. 1 area (7th year) Annual costs (Loan charges only)	No. 1 area Cost per kW of M.D. H.T.	No. 1 area Cost per kW of M.D. L.T.	Whole area (10th year) Annual cost (Loan charges only)	Whole area Cost per kW of M.D. H.T.	Whole area Cost per kW of M.D. L.T.
	£	£	£	£	£	£
33-kV transmission	4 120 ÷ 4 300 kW	0·958	0·958	4 680 ÷ 7 850 kW	0·603	0·603
33/11-kV plant and substations	888 ÷ 4 300 kW	0·206	0·206	1 144 ÷ 7 850 kW	0·146	0·146
11-kV secondary mains	4 230 ÷ 4 300 kW	0·983	0·983	11 800 ÷ 7 850 kW	1·52	1·52
kW losses (at H.T.)	410 ÷ 4 300 kW	0·095	0·095	815 ÷ 7 850 kW	0·104	0·104
11-kV consumers' substations, etc.	760 ÷ 3 200 kW	0·237	—	926 ÷ 5 050 kW	0·183	—
11 000/400-volt plant and substations	905 ÷ 1 100 kW	—	0·828	3 436 ÷ 2 800 kW	—	1·21
L.T. mains and switchgear	3 520 ÷ 1 100 kW	—	3·2	6 830 ÷ 2 800 kW	—	2·45
L.T. services and meters	3 920 ÷ 1 100 kW	—	3·564	5 360 ÷ 2 800 kW	—	1·92
kW losses 11 kV/400 V	610 ÷ 1 100 kW	—	0·554	1 100 ÷ 2 800 kW	—	0·41
Total system cost per kW	—	2·50	10·663	—	2·55	8·36
Cost per kW of grid supply	—	3·50	3·50	—	3·50	3·50
Total cost per kW if no diversity		6·0	14·163	—	6·05	11·86
Cost after reduction for diversity	H.T. = 1·55 L.T. = 1·22	£3·87	£11·6	H.T. = 1·58 L.T. = 1·35	£3·83	£8·92
		Cost per kWh			Cost per kWh	
Remainder of annual costs (including unit losses)	£12 490	—	—	£16 900	—	—
Cost per unit sold	£12 490 ÷ 15 900 000	0·19d.	0·19d.	£16 900 ÷ 23 500 000	0·17d.	0·17d.
Cost per unit from grid	—	0·20d.	0·20d.	—	0·20d.	0·20d.
Total unit cost	—	0·39d.	0·39d.	—	0·37d.	0·37d.

Total Costs (including Grid Charges).

	No. 1 area (7th year)	Whole area (10th year)
H.T. supplies	£3·87 per kW + 0·39d. per unit	£3·83 per kW + 0·37d. per unit
L.T. supplies	£11·6 per kW + 0·39d. per unit	£8·92 per kW + 0·37d. per unit
Average cost per unit sold	0·88d.	1·08d.

(2) Analysis of Costs of Scheme and Derivation of Tariffs.

The annual costs for interest and sinking fund for each stage of transmission and distribution of the detailed scheme are set out in Table 7. The cost per kW of maximum demand (at the time of the highest annual peak) is also shown at each stage. After adding the cost of the supply from the grid, an allowance is made for the diversity of the h.t. power load and the l.t. load.

Observations regarding Table 7.

First consider No. 1 Area. Supplies taken by power consumers at 11 000 volts will be seen to cost £3·87 per kW + 0·39d. per unit, whereas the cost of transmitting and distributing beyond this point to l.t. consumers is much more expensive, as the cost to consumers taking l.t. supply is £11·6 per kW + 0·39d. per unit. The average total cost per unit sold will be observed to be 0·88d.

The effect of extending into the more sparsely populated No. 2 Area, where there is a preponderance of domestic load, is to reduce the system load factor and to increase the average cost per unit sold although the increased l.t. demand permits of a lower kilowatt charge being achieved, owing to the greater use of the main and secondary systems. The potentialities are much greater with the larger area, however, for increasing the "off-peak" load and thereby improving the system load factor. The efficiency of use of the system would be improved by the increased number of units sold, and the cost per unit sold would then be reduced.

It was stated earlier in the paper that the 11-kV system was designed for a maximum load of 10 000 kW, and it is interesting to note that the average cost per unit sold would be reduced to 0·65d. for a fully loaded system at 40 per cent load factor. If it is assumed that the 11-kV power load remained constant and the l.t. load were built up so that the maximum system load mentioned above was achieved, the cost of the 11-kV supplies would then become £3·6 per kW+0·34d. per unit, and the l.t. supply would cost £4·2+0·34d. per unit. These figures assume an additional capital expenditure of £120 000 and an addition of £10 000 per year to the running costs; they are necessarily very approximate.

The costs which have been derived are shown to a hyperbolic scale in Figs. 12 and 13, where the cost per kWh is indicated at various load factors.

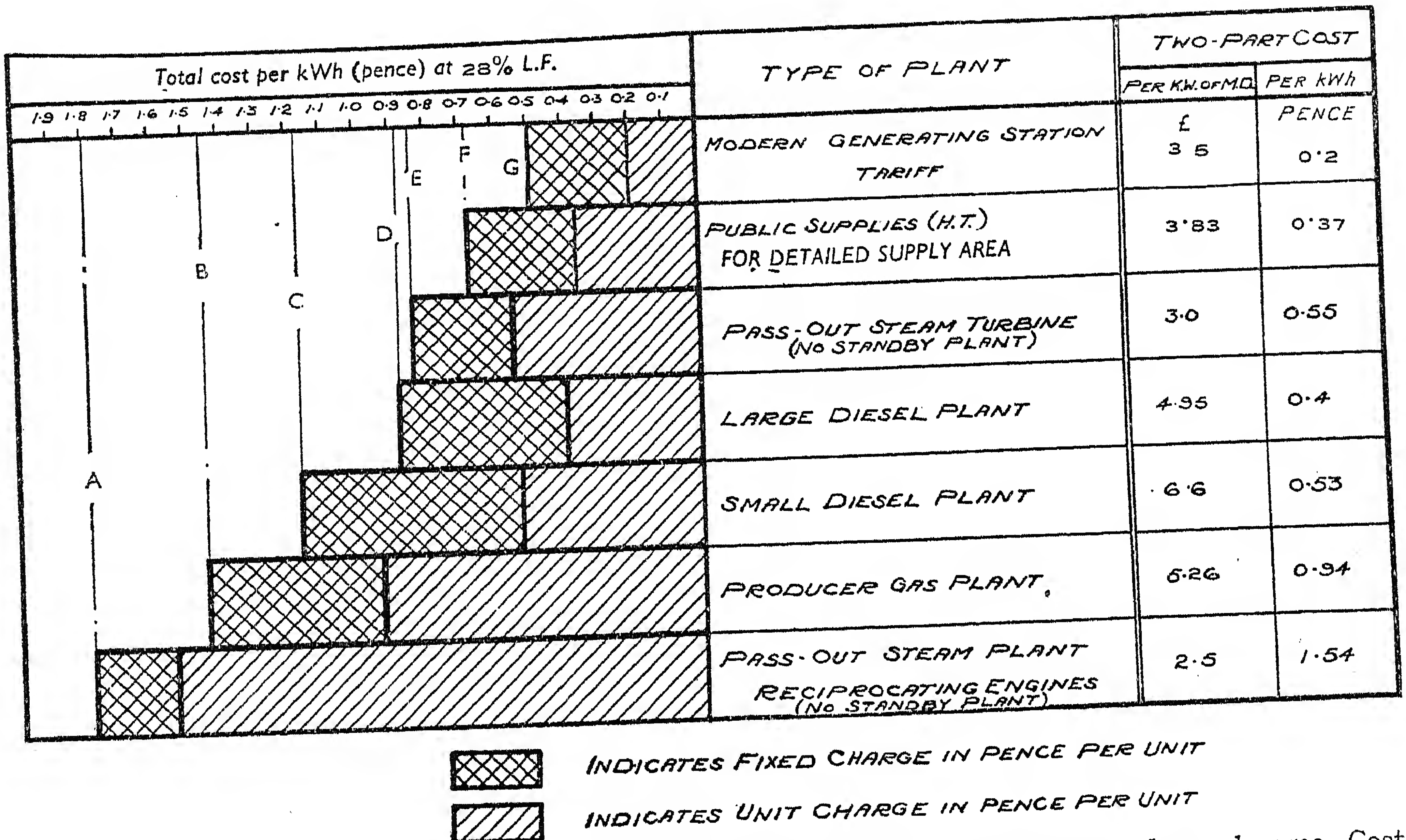


Fig. 9.—Cost of supply for private plants and public supply undertakings in a particular rural supply area. Cost per unit at 28 per cent annual load factor and averaged tariffs obtained in paper.

A = Pass-out steam reciprocating engines:—1·79d. B = Producer-gas plant:—1·45d. C = Small Diesel plant:—1·18d. D = Large Diesel plant:—0·87d. E = Pass-out steam turbine:—0·84d. F = Public supplies:—0·745d. G = Modern generating stations:—0·5d.

Method of Allocating Component Charges.

The allocation as a running charge of "the remainder of the annual costs" except loan charges, i.e. all annual charges except interest and depreciation, is quite arbitrary, although it follows the method of costing adopted for private plant earlier in the paper. A number of these remaining revenue charges partake of the nature of a "fixed" charge but are clearly not a "kilowatt" charge.

Of the two broad methods which are commonly used for allocating to fixed and running costs, it seems preferable to place as many items as possible into the running charge so as to keep the fixed charge to practical competitive figures and to permit of a greater sharing of charges between consumers; this method has accordingly been adopted in the paper.

In the author's view there is a grave danger that the 2-part method of costing may be carried to an extent which is not justified, having in view the purpose for which the costing is being carried out. It would seem

to be necessary to sell electricity to each class of consumer at a price which is competitive with other, alternative, forms of energy, rather than at the actual 2-part cost of supplying the consumer, since the 2-part cost is hypothetical and varies with each consumer.

The proper division of costs between the various classes of supply can be carried to an interminable degree of refinement so as to permit of a consumer paying only his proper proportion of the costs; but in the author's view this refinement is seldom justified or appreciated by the consumer. It even seems doubtful whether electricity should be sold at its actual cost to the undertaking, but rather in accordance with its value to the consumer. Consideration of these two qualities is undoubtedly difficult, since the "value," just as the "cost," will be peculiar to each customer and will even vary in accordance with the amount he uses, as there is a certain amount that is essential and will have a higher value than succeeding amounts.

It is obviously impossible to do more than divide consumers into broad classes, to allocate the annual costs of the undertaking between these classes, and then to ensure that the allocation will permit of electricity being sold at prices which are competitive with alternative forms of energy.

(3) Comparison of Private Plant and Public Supply Costs (at High Tension) for Detailed Supply Area.

This completes the study of the particular semi-rural area, and a graphic comparison is now given in Figs. 9 and 10 for the various costs of supply which have been derived. In order to obtain more generalized figures for public-supply costs the previous method of analysis has been applied at a later stage in the paper to other types of areas. It may be of interest at this stage briefly to summarize the data which have now been acquired.

The comparative cost of power, available from private plants or the public supply respectively, is shown in Fig. 9, and in order to bring all the costs to a common basis it has been assumed that the power will be taken in each case at the annual load factor of 28 per cent, which was the average for the power demands from the public supply in the detailed area. The method used for constructing the diagram is the one commonly used for the purpose of indicating to prospective plant consumers the "cost per unit or b.h.p.-hour" of oil-engine plant, except that the essential ingredient of load factor has been included in Fig. 9.

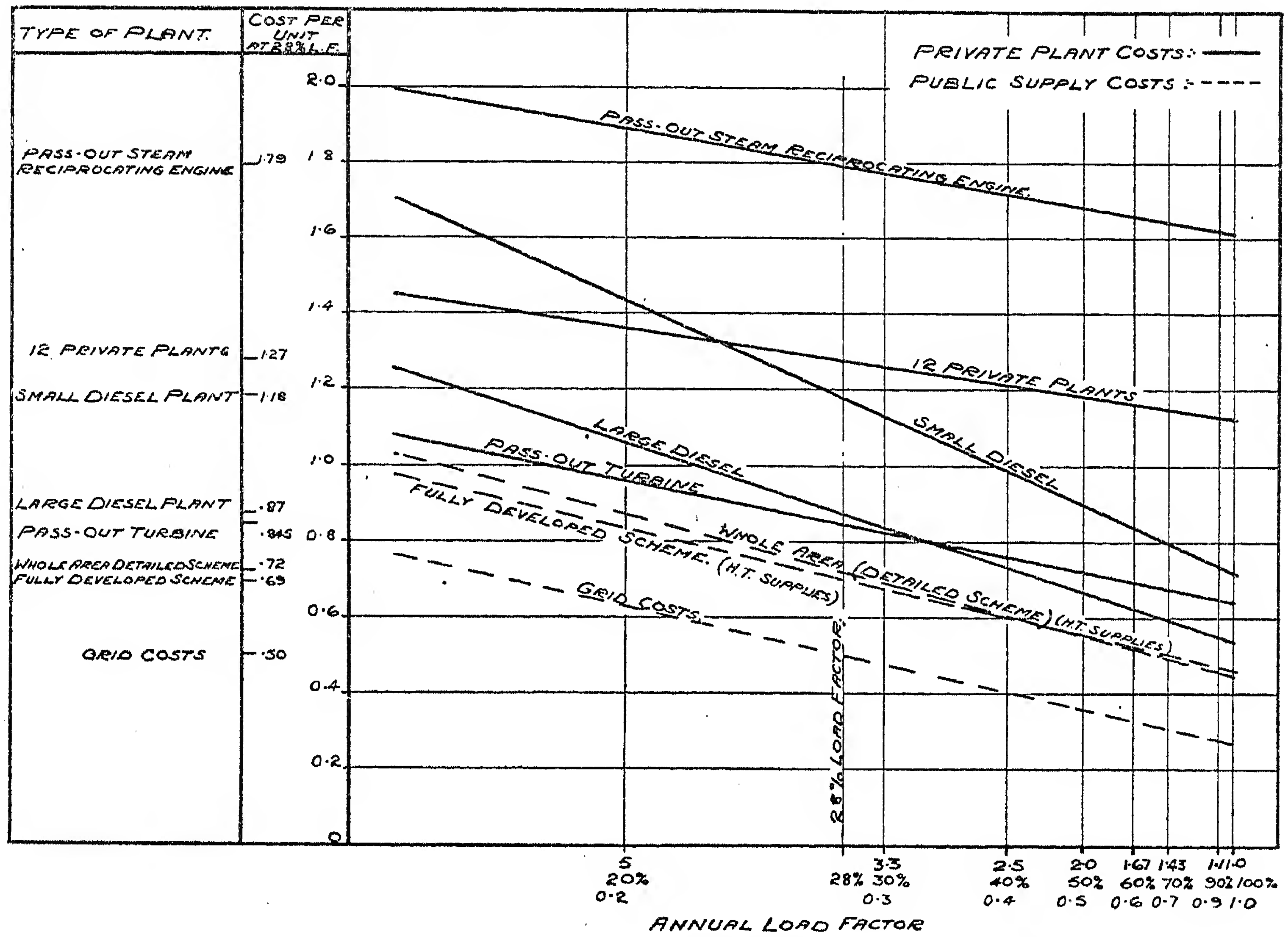


Fig. 10.—Cost of supply from private plants and public supply undertakings in a particular rural supply area. Cost per unit at varying load factors and at 28 per cent load factor.

The 2-part costs previously derived can be shown as a "total cost per unit" (C) at a given annual load factor (L) by using the formula

$$C \text{ (pence per unit)} = \frac{k \times \text{cost per kW of maximum demand (£)*}}{\text{Annual load factor (100 per cent = unity)}} + \text{running cost in pence per unit}$$

e.g. for a 2-part cost of £3·83 + 0·37d. per unit the total cost per unit (pence) at 28 per cent annual load factor would be

$$C = \frac{0\cdot0274 \times 3\cdot83}{0\cdot28} + 0\cdot37\text{d.} = 0\cdot75\text{d. per unit.}$$

* At a cost of £1 per kW and an annual load factor of unity (100 per cent) the cost per kWh (in pence) will be $\frac{£1 \times 240\text{d.}}{8\,760} = 0\cdot0274$d. per kWh. At an annual load factor of less than unity, the cost per kWh will be increased proportionally.

This formula has been used to set out the costs per kWh shown in Fig. 10, at various load factors; the base gives load factor plotted to a hyperbolic scale.

The first point to notice is that the semi-rural public-supply undertaking can afford to supply power at secondary voltage at lower prices than that power can be obtained from the private plants which were existing, and that the public-supply charges are lower than those which would be achieved if Diesel plant were to be installed. It is to be noted that even collieries have found it economical to change over, and as the public supply becomes developed the discrepancy in the charges will become more pronounced.

It will also be observed that, contrary to general opinion, an undertaking of a semi-rural nature can supply power at rates which are comparable with those for an urban undertaking. The effect of the power load has been to accelerate development just at the time when the domestic and l.t. supplies are slow in developing. It is pointed out later that a new undertaking has immediate overhead charges for staff and administration which are necessary but productive of an annual loss until reasonable development has taken place; in the case of the detailed scheme the power load has justified these initial charges and has actually advanced the time when a condition of financial equilibrium can be achieved.

It has not been possible at this stage to make a survey

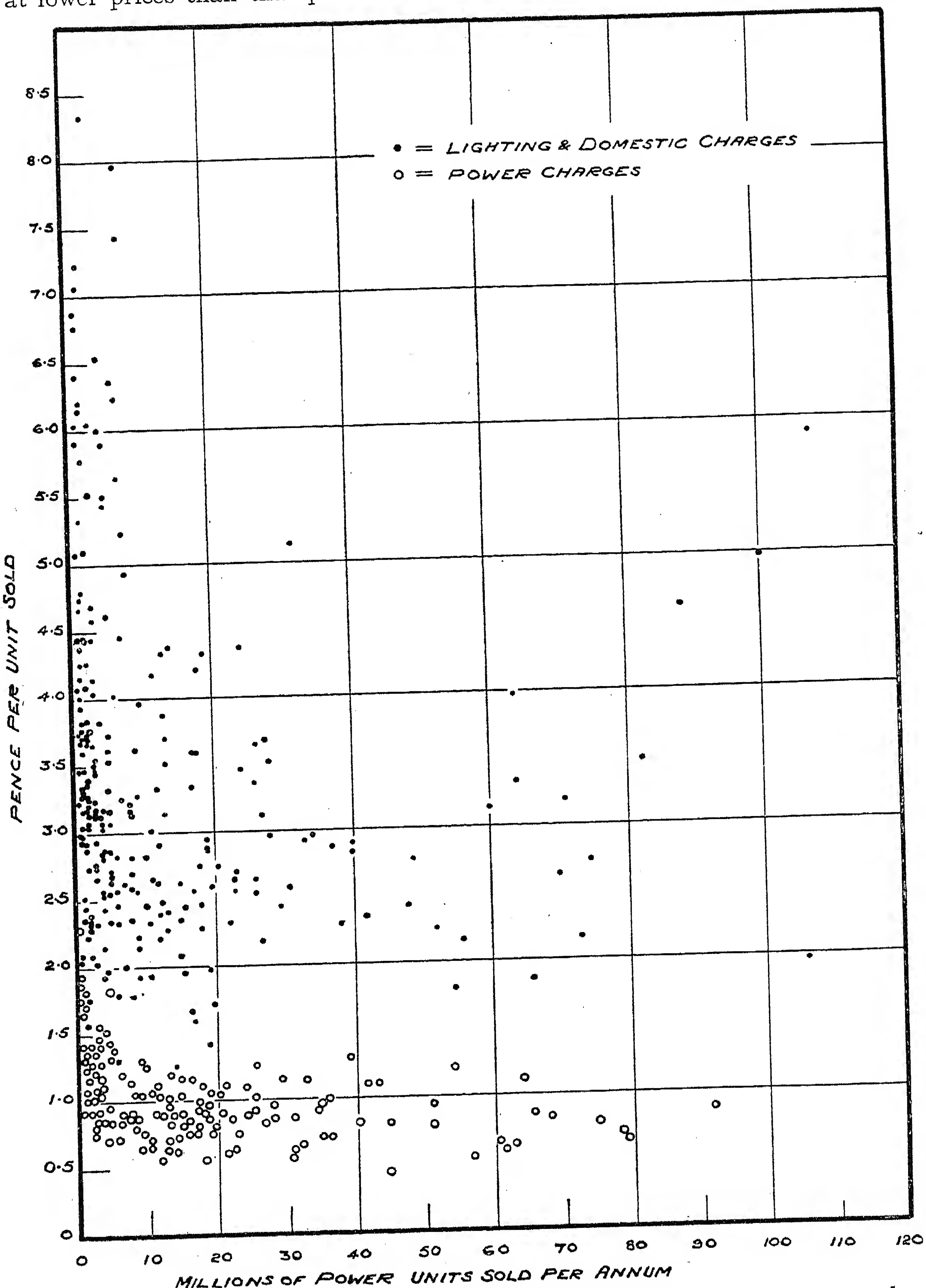


FIG. 11.—Relation between charge per unit (average prices, power and domestic) and annual power output.

of the domestic and general l.t. load requirements, and only the usual routine work has been carried out towards obtaining and developing this l.t. demand. In view of the power load having been developed fairly completely, preparations have now been made for diverting the specialized staff and their special experience to the small power, pumping, domestic, and water-heating, loads, and it is anticipated that two further years of development will be devoted to this matter. By this time it is anticipated that the increase in load will have enabled the cost of supply to be reduced to such a low figure as to permit of a successful domestic campaign being made. The whole of the special development work is being carried out by the technical distribution staff, i.e. the surveyor, wayleave officer, etc., in conjunction with their normal duties, and the work is then co-ordinated by the distribution engineer. It was realized that these officers have unusual facilities for getting in touch with prospective consumers, particularly with farmers and domestic consumers. Upon receiving an inquiry in the course of their normal duties, this inquiry is dealt with on site as far as possible and if

TABLE 8.

Type of area	Population	Area (square miles)	Maximum demand (kW)	Units sold per year	H.T. supplies (See Fig. 13)	L.T. supplies (See Fig. 12)	Average cost per unit sold
(1) Urban (no h.t. system or power load)	33 000	5·5	2 200	$3 \cdot 6 \times 10^6$	Nil	£7·1+1·2d.	2·2d.
(2) Urban	130 000	11·1	25 000	93×10^6	£3·4 +0·35d.	£6·7+0·35d.	0·88d.
(3) Urban	265 000	19·5	70 000	256×10^6	£3·85+0·24d.	£6+0·24d.	0·64d.
(4) Semi-rural (detailed) scheme, 7th year	56 000	103	4 300	$15 \cdot 9 \times 10^6$	£3·87+0·39d.	£11·6+0·39d.	0·88d.
(5) Rural (detailed) scheme, 10th year	95 500	457	7 850	$23 \cdot 5 \times 10^6$	£3·83+0·37d.	£8·92+0·37d.	1·08d.
(6) Assuming full development	95 500	457	10 000	35×10^6	£3·6 +0·34d.	£4·2+0·34d.	0·85d.

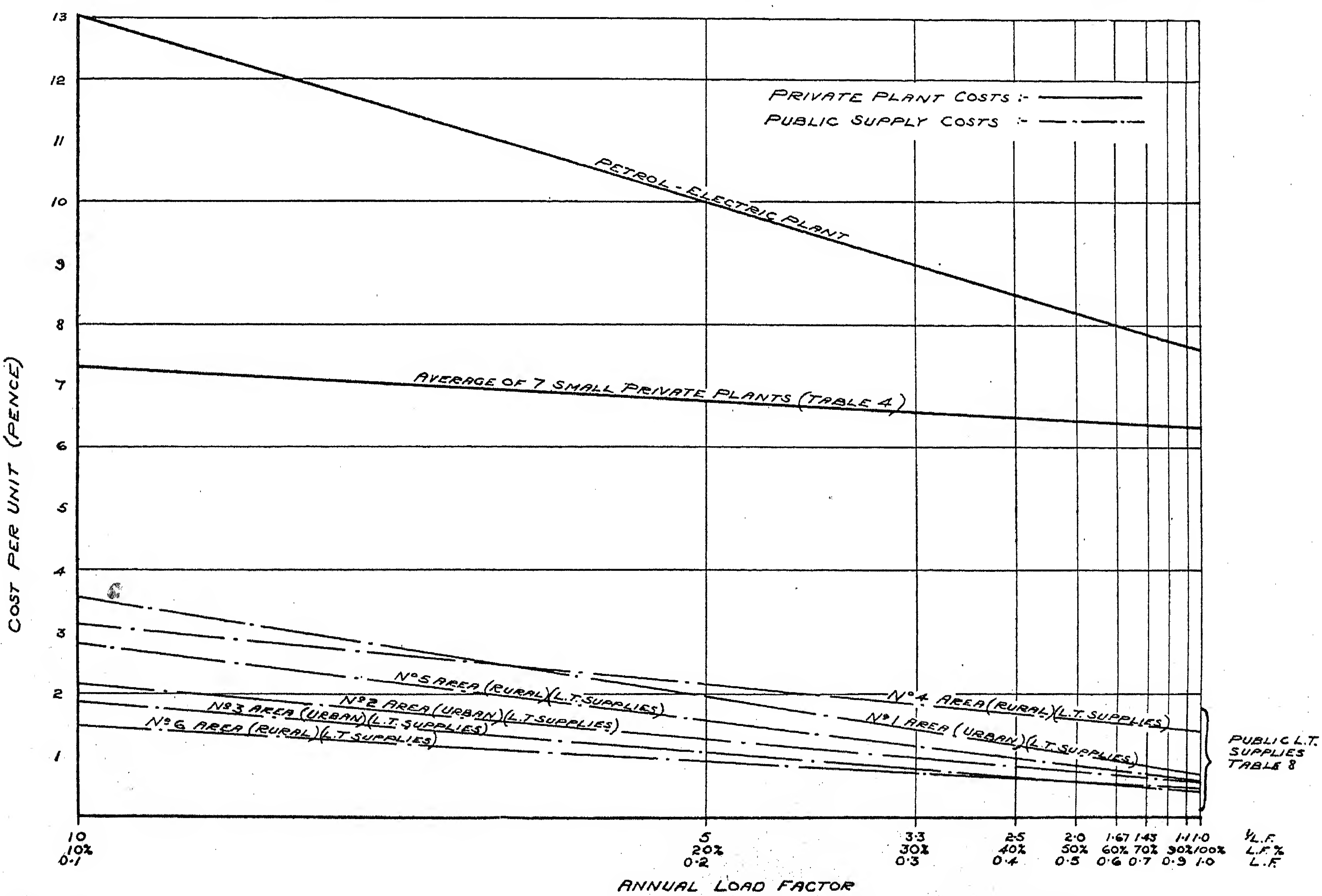


FIG. 12.—Cost of supply at various load factors for small-residence private plants, compared with cost of low-tension supplies from representative public-supply undertakings.

further details are required or a special visit is needed for the purpose of submitting a detailed report to the prospective consumer, the matter is passed forward for further attention.

(4) Comparison of Costs of Public Supply in Various Areas.

It is reasonable to consider that the public-supply costs previously derived are particular to a given under-

(3) To ascertain whether consumers in rural areas are charged at a higher rate than the urban consumer.

Cost of Providing Supplies in Various Types of Areas.

These costs have been derived and are given in Table 8 for three urban areas, two of which had a large power load, being industrial towns; the costs are then shown graphically in Figs. 12 and 13.

The data given in Table 8 would suggest that the cost

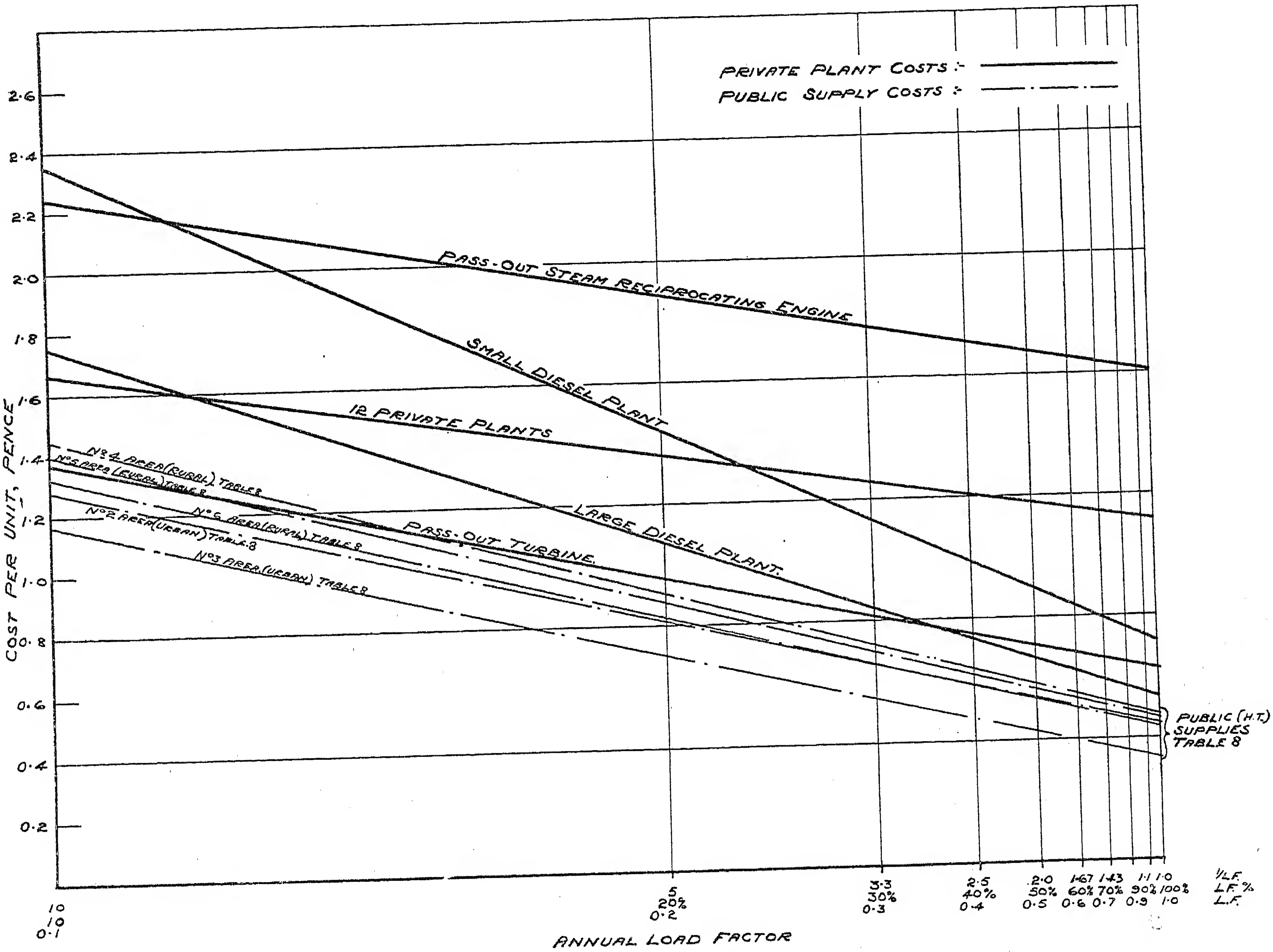


Fig. 13.—Cost of supply for large private plants at various load factors compared with cost of high-tension supplies from representative public supply undertakings.

taking and that they are not, therefore, representative of public-supply costs in other types of areas. To meet this criticism and before making a more complete comparison between private plant and public supplies, a brief examination has been carried out for other types of supply areas, with the following purposes:—

(1) To ascertain the costs of providing supplies of electricity in urban areas, by adopting the same method of analysis as for the detailed semi-rural area; and to make a comparison of the cost of supply in various types of undertakings.

(2) To show the effect of the power load upon the charges made to h.t. power and l.t. domestic consumers.

of giving supplies is not materially different between rural and urban areas, but that the effect of a power load is to reduce the average cost per unit sold. This feature is particularly marked in the case of the municipal undertaking which has no secondary transmission and practically no power load (Item 1). When it is considered that the variation ranges from the very largest size of urban area to the small rural area of the detailed scheme, and that the differences in costs are only 0·2d. for h.t. supplies and approximately 1d. for l.t. supplies, it would seem that there is some other factor which accounts for the wider differences in charges which are shown in Fig. 11.

Effect of the Power Load upon the Costs of Public Supplies (*see Fig.* 11).

In order to ascertain, by means of examining the actual charges made to consumers, whether the power load which is almost exclusive to urban areas had the effect of reducing the cost per unit sold and thus giving an advantage in lower charges to urban consumers, Fig. 11 was prepared; the average revenue obtained per unit sold for power and domestic purposes respectively is set out for most undertakings in the British Isles (1930–31) against the power units sold annually by each undertaker. There would appear to be no clearly defined relationship between charges and output. The general tendency is for power charges (per unit) to fall gradually from averages of 1·5d. to 0·8d. as the annual output increases to 10 million units, but from this point the charges remain fairly constant in spite of the greater output. There is an indication that the charges for domestic supply actually rise with a greater power output, and so far as any relation can be observed between the charges for the two classes of supply the results would appear to confute the result which was expected, i.e. that large sales of power would result in a progressive decrease in power and domestic charges. One reason for the lack of correlation is that the tariffs of most undertakings are based upon expediency instead of upon a co-ordinated basis which relates to each class of demand, e.g. in certain cases it was found that where the power charges were below the average the domestic charges were higher than the average for undertakings with a similar output, this being especially noticeable in several of the larger industrial towns. In the author's view the need for standardizing tariffs throughout the country is a paramount one if sales are to be radically increased.

Despite the conflicting evidence obtained from Fig. 11, it still remains true to say that the characteristic feature of most urban areas is the power load that is generally available, and that this load will result in a better system load factor, a greater number of units sold per connected consumer, and reduced distribution charges; these advantages will combine to reduce the average cost per unit purchased from the grid as well as to provide a lower running charge per unit sold.

The power load which is existent in rural areas is often to be found in the agricultural requirements for ploughing, threshing, etc., and it is very probable that if these requirements were fully exploited the increase in sales would permit of the lower price features, which should prevail in urban areas, being reproduced in the rural districts.*

Differences in Charges to Consumers in Rural and Urban Areas.

A further indication of the relative costs of providing supply in rural, as compared with urban, areas can be obtained in a more generalized manner than has been possible in the previous analyses, by considering the results of an analysis made by Messrs. Dickinson and Grimmitt† to determine the average charge made for electricity by 221 rural undertakings and 355 other undertakings which were principally urban in nature (Table 9).

* J. A. Sumner: "Electro-Farming."
† *Journal I.E.E.*, 1932, vol. 70, p. 189.

The charges derived in Table 9 take no account of the demand of the consumer, and the "service charge" must not, therefore, be considered to be a "kilowatt charge." But within the limits of the method of analysis which those authors adopt, the figures for the first two cases may be taken as representative of the cost of supply for the two types of areas, as they are derived from the actual tariffs adopted by the 576 undertakings. The similarity of these charges, which are generalized and independent of the costs shown in Table 8, strengthens the inference made in Table 1 (line 9) that the cost of extension is the same in rural as in urban areas, for equal stages of development.

The fixed annual charges for the last two undertakings have been obtained by dividing the annual fixed charges by the number of consumers and by adopting the running charges which were derived earlier in the paper. It is suggested that here is a basis for a standard tariff, i.e. to impose an annual fixed charge which is not based upon the consumer's demand but upon the average cost of providing the supply, in addition to a running charge

Table 9.

Analysis of Charges to Domestic Consumers for Rural and Urban Undertakings.

Type of undertaking	Fixed (or "service") charge per consumer	Unit charge
Rural (221 undertakings) ..	£4·75	0·3d.
Urban (chiefly) and partly rural	£5·05	0·93d
No. 2 Urban area (Table 8) ..	£7·0	0·35d.
No. 4 Rural area (Table 8) ..	£4·25	0·35d.

ascertained from the actual running cost per unit for each year. In order to ensure that small consumers should not be penalized it may be necessary to make two or three divisions of consumers with the fixed charge varying from a minimum of, say, £2. The main feature of the suggestion is that a consumer should not be charged with his kilowatt demand, as, in the author's view, such a course is becoming less practicable each day.

A more extensive investigation which was carried out confirmed the general facts shown in Tables 1 and 9 as to the similarity of cost and charges for rural and urban areas. It was ascertained, however, that where an undertaking with a developed nucleus extended into a surrounding area, a higher average price per unit was obtained from the extension area than from the central area. It would be true to say, therefore, that the rural consumer pays more for his electricity than the urban consumer. The reasons suggested to account for this apparent anomaly are as follows:—

(1) A rural undertaking is expected to "pay its way" almost from the commencement, and it is usually considered that if high charges are imposed initially

this condition will be realized. It seems doubtful whether such a policy does not defeat its own purpose. The history of electricity supply shows that tariffs are generally reduced in proportion to the stage of development of the undertaking, and since most rural undertakings are of fairly recent origin high tariffs prevail.

(2) Although the average revenue required to be received from domestic consumers is approximately the same for rural and urban areas (Tables 1 and 9), the minimum revenue received from the small-consumption consumer in rural areas is greater than from the small-consumption urban consumer. This would appear to be a correct course to follow, as the tendency of many urban undertakings to adopt a low and uneconomical flat rate for lighting favours the small, low-load factor consumer at the expense of the consumer with the better demand.

(3) The power load, which usually occurs in urban areas but seldom in rural areas, improves the system load factor and reduces the overhead charges per unit sold to the urban consumer, although the lower costs are not always available to the domestic classes of consumer, as they are often used to subsidize the power consumer and to sell power at a rate less than that which can be achieved by power plants.

The examination which has been made of the cost of supply in various types of areas may be summarized by stating that it is difficult to make any clear determination as to which type of area is supplying at the lower cost. The wide variety of tariffs which are in existence, with so many different bases, as also the variation in prices even between those undertakings with similar tariff bases, showed that a classified comparison was not possible. In the author's view this extraordinary state of chaos is one of the most serious hindrances to the future development of the supply industry. Even if it be admitted that the charges in a given undertaking must be proportional in their decrement to the period of operation and development, it seems reasonable to state that the constancy of the figures of capital cost per consumer given in Table 1 should permit of the same price being charged for all undertakings which are at a similar stage of development.

Part 3. CONCLUSIONS.

(1) Comparison of Previous Costs. Generalized Conclusions as to Relative Costs of Public and Private Supplies.

The various costs of supply which have been ascertained for the detailed area of supply are set out in Figs. 12 and 13 so as to permit of a direct comparison being made.

Large Private-Plant and Public Supplies taken at Secondary Voltage in Particular Supply Area.

This comparison is made in Figs. 9 and 10.

Comparison of More Generalized Public Supply Costs derived in the Paper (Fig. 11).

These comparisons are made in Figs. 12 and 13. A comparison of the various curves in Fig. 13 indicates that the fixed charges have the effect of increasing the cost per unit at low load factors more rapidly when Diesel plant is installed than when the public supply is taken. With respect to the miscellaneous private plant in use in the detailed area, it will be noticed that this plant becomes relatively cheaper to run at the lower load factors. This is because of the low fixed charges which prevail owing to the majority of the plant having been in use for a number of years, but it will be seen that these reduced fixed charges are neutralized by the higher running costs. The cost for the pass-out steam turbine has been shown in Figs. 8, 9, and 13, but as this is a particular case of use it is not possible to make a comparison with the public-supply costs in the same manner as for the more generalized costs of Diesel plant.

It has to be observed that the comparison shown in Fig. 13 is a financial one, and certain items which are not financially computable would appear to favour the public supply to a still greater degree.

Small Private Plant (Domestic Use) and Public Supplies taken at Low Voltage.

A graphical comparison is made in Fig. 12 between the derived public-supply l.t. costs and the costs for seven small representative private plants (Table 4) in use in the area, as well as for the generalized case of the automatic petrol-electric plant. The extremely low load-factor of the average private-plant owner who uses the supply for lighting only is illustrated very clearly in Table 4.

Conclusions as to Comparative Costs.

The conclusions which are reached as a result of the comparisons shown above are as follows:—

(1) An efficient public-supply undertaking can supply untransformed h.t. power at a less cost than that at which the power can be generated by any modern form of prime-mover installed on the consumers' premises; the public-supply cost is also more likely to decrease in the future. An exception may arise if a factory requires steam primarily for process work and secondarily for power production, and if the process requirements are coincident in time.

(2) The averaged public-supply tariff which will enable power to be obtained as economically (at high tension) as it could be generated by means of a Diesel engine is estimated to be £4·95 per kW of maximum demand (30-minute average) and 0·4d. per unit for large plant, rising to £6·6 per kW of maximum demand + 0·53d. per unit where smaller plant is installed.

(3) The cost of power generated at small residential plants exceeds the cost at which power is available from an efficient and modern public-supply undertaking.

(2) Suggestions regarding Public Supply Costs and Tariffs.

The primary purpose of the paper has been to establish some relation between the costs at which private industrial plants are run in an average type of supply area, and the cost at which the supply can be obtained from a public undertaking. It has been demonstrated that a modern undertaking can supply power at less cost than that at which it is likely to be produced by modern private plant, except in special cases of use, and that

the annual load factor is the main feature which determines the cost per unit of power required. The comparison has been based only upon the computable financial items and takes no account of certain other advantages which are peculiar to the public supply.

A further purpose was that of obtaining some discussion of the merits of public-supply or private plants and also of the figures for the costs of running which are given in the paper. The author has found that in a large number of instances there is a belief that electricity can be obtained from privately installed Diesel plant at ½d. per kWh, but so far he has not found an actual instance where generation is being carried out at this figure when all the real cost components are taken into account. From the point of view of the electricity supply industry the real cost of private plant running should be openly discussed and the broad limits of cost should be published. In many cases supply undertakings are being induced, by the implied threat of the installation of Diesel engines, to sell electricity at costs which are far below the real costs of running private plant, and the domestic consumer is not always receiving his proper quota of the reduced costs which ensue by virtue of the power load that is available.

The increase in sales of current which is so necessary to the success of the electricity supply industry is not to be obtained, however, merely by the demonstration that the public supply can provide the cheapest form of energy; it is necessary to ensure that a reasonable national standardization of tariffs should be made.

It is suggested, firstly, that the capital expenditure required to give supply to a consumer is the same for all types of areas of equal development. As there is no great difference between areas in the characteristics of the domestic and general l.t. demands, the cost of giving supply should therefore depend only upon the stage of development of the undertaking and, to a far less degree, upon the nature of the system load. The effect of obtaining a power load is, however, to improve the load characteristics of the undertaking and to reduce the average price at which electricity can be sold, and it is reasonable to expect that urban undertakings, which generally have a large power demand, will have lower charges for all purposes than rural undertakings, where the power load does not usually occur. It is not found that this relation exists between increased power output and reduced charges to the domestic consumers.

A study of the tariffs which prevail throughout the country shows that many undertakings are selling electricity for power purposes at a lower rate than is required to compete with the real costs of running private plant, and in most of these cases the domestic consumers are not receiving the benefit of the reduced tariffs which are justified because of the reduction in overall costs brought about by the power load.

Although it is outside the scope of the paper to attempt to formulate detailed tariffs, the author is definitely of the opinion that it would be financially possible to achieve an approach to one tariff, common to all undertakings throughout the country, provided some co-ordination were obtained between undertakings. It is considered that the effect of the power load is to reduce the cost at which electricity can be sold, and any consideration of the merging of supply areas should have this feature in view. By merging areas in this way it is probable that the cost per unit supplied by any undertaking would not exceed 1d. and in many cases would be as low as 0·7d. Is it not possible that the method of charging of the future may involve a return to the "flat rate" and that all consumers, both power and domestic, may be charged at one basic price, with reductions for off-peak loads, etc., i.e. the average cost, to the undertaking, per unit supplied for all purposes? In the author's view, in all cases of power consumption except the very special ones it would be found economical for the consumer to pay up to 1d. per kWh, and it is beyond question that the domestic consumption would be greatly stimulated if a flat rate below 1d. per kWh were available for all domestic consumption.

After a careful effort is made to study the manner in which an improvement in diversity or system load factor can cause reductions in tariffs, it does not seem unreasonable to forecast that electricity will be available within the next few years to all consumers at a rate of 0·5d. per unit.

A further inference which can be made as a result of the analyses is that the merging of electricity areas into much larger single districts than at present is necessary for reasonable standardization of tariffs. In this way the inevitable deficit of a newly developed area can be balanced against the surplus from the older areas and a uniform tariff can be kept throughout the single large administrative district. It is only by this means that the present "see-saw" effect in tariffs can be eliminated.

It can hardly be disputed that one of the most serious hindrances to the proper development of the public supply is the extraordinary inequality of the charges made for electricity in different but contiguous electricity areas. It is contended that the most urgent need in the electricity supply industry is that of deciding upon the basis of the standard tariff to be adopted, and then for this tariff to be adopted at once by all undertakings, so that each consumer throughout the country shall pay on exactly the same basis. The author ventures to put forward the opinion that if this step were taken and suitable publicity given, the result would be immediately noticeable in a large increase in the sales of electricity and in the number of connected consumers.

ACKNOWLEDGMENTS.

Acknowledgments are due to a number of persons who have assisted the author in preparing this paper, particularly to Mr. F. Morgan, who was responsible for taking a number of the tests on plant and engines, and to Mr. R. A. Woods, B.Sc., Associate Member, who assisted in checking the proofs and made a number of suggestions.

[The discussion on this paper will be found on page 334.]

Discussion before The Institution, 28th February, 1935.

Mr. E. F. Hetherington: I agree with the author that the average factory owner often has but the flimsiest idea of the operating costs of his plant. In many cases these are merged in his general factory costs, and in others the capital on the plant has been entirely wiped off and his power production is therefore carrying no capital charges at all, in spite of the fact that the time is maybe near when either heavy maintenance charges or complete plant renewals will be necessary.

It is quite true, as the author says, that Diesel-engine plants are highly efficient, and the figures quoted by manufacturers will often be achieved, but only if these plants can be operated at about 100 per cent load factor and at their most economical rating. Usually, however, the actual load is less than the maximum rating of the plant, which is operated at anything but its most economical load. Where the private-plant owner is induced to permit the supply authority to investigate his present costs of operation, making due allowance for load factor, capital charges, and running costs, it is generally found that a supply from the public mains is competitive. That this is the case is demonstrated by the fact that, in the area for which I am responsible, it has been found possible to change over approximately 85 per cent of the private plants to public supply at a tariff as high as £4 10s. per kVA plus ½d. per unit, and in practically all these cases the owners of the plant are satisfied with the results.

The author has not been particularly happy in his selection in Table 2, because the plants he instances are not all of a very high degree of efficiency. I should like to mention one factory which is not included in Table 2. It belongs to a very up-to-date firm having a refined system of costing, and which therefore knew exactly what were the generating costs of its private plant. The station was equipped with thoroughly up-to-date Diesel engines, the operating costs of which were well over 1d. per unit. This figure was slightly improved upon after the change-over to public supply, in spite of the fact that the tariff previously referred to cannot be considered to be a very low one.

I cannot go all the way with the author in suggesting that we should base our tariffs purely on the competitive cost of operating private plants. If an undertaking is in a position to quote lower rates than those which might be considered strictly competitive, I think it should do so, as such action tends to a lowering of production costs, and therefore contributes to national prosperity.

The "typical rural scheme" described in Part 2 of the paper is in fact the distribution area of the West Midlands Joint Electricity Authority. This body administers electricity supply in an area of about 1 000 square miles, rather more than half of which is supplied by local authorities purchasing their energy in bulk from the Authority. Before the West Midlands Joint Electricity Authority commenced operations, the remainder of the area was absolutely barren of any electricity supply. The Authority proceeded to develop this area, which is of a semi-rural character and contains only one or two small towns of 3 000 to 4 000 inhabitants, about 4 years ago. The costing for this distribution scheme is kept entirely separate from the main bulk-supply costing, so that detailed operating costs can be very accurately obtained. The difficulty in deciding upon a tariff for an area of this kind is to fix prices which are low enough to obtain business and yet sufficiently high to ensure the balancing of the accounts within a prescribed number of years. The tariffs finally decided upon approximate to those set forth on page 324, and it is interesting to note that the flat-rate figures are those which obtain after a discount of 10 per cent for cash in a month has been allowed. This discount is a little unusual, but it has been found to save a great deal in collection costs.

The capital outlay on the distribution scheme up to the present time is over £200 000, and, as was expected, a deficit (see Fig. 7) resulted from the first 4 years' operation. This deficit, however, was in accordance with estimates, and the scheme has now reached a point when a surplus is anticipated.

Dealing with the figures set out in Table 7, I think the author has been a little courageous in basing his argument on the figures for the 7th and 10th years of development in the area, whereas the 4th year only has been reached. I disagree, moreover, with his suggestion (page 326) that "there is a grave danger that the 2-part method of costing may be carried to an extent which is not justified, having in view the purpose for which the costing is being carried out." Having said that, the author proceeds to eliminate from the costs—in working out his kW charge—everything but the capital charges, and this, I think, is quite wrong.

Turning to his statement (in the Summary) "it is suggested that national co-ordination of tariffs is essential if increased sales are to be obtained," I think he very much overrates the extent to which the existing condition of things is affecting sales. That tariffs should be co-ordinated on a national basis, I am prepared to agree, but I very much doubt whether a national tariff as suggested on page 333 is a practical solution. I am inclined to agree that some rearrangement of boundaries is desirable, if not urgent, and that some generally agreed tariff basis is needed, but a national tariff on a flat rate would be uneconomical and would not help very much, particularly as it would reflect unfairly on good-load-factor consumers.

Referring to Table 1, the author makes the suggestion (page 311) that rural electrification can be carried out as cheaply, from the point of view of capital expenditure per dwelling on the route of mains, as town electrification. I do not think that Table 1 supports that point of view. The author says "The most interesting feature of the table is found in the fairly constant value of the 'capital expenditure per dwelling on route of mains,'" and yet in Table 1 he shows the considerable divergence represented by the difference between £28 and £38 per dwelling, a difference of nearly 50 per cent. Later he asserts that for the same degree of development the costs are the same; yet Table 1 gives the cost per dwelling in a large borough as £28 with 89 per cent development, whereas in the Bedford scheme the cost is £38 for 78 per cent development. In spite of what the author says, therefore, I still share the general opinion that rural electrifi-

cation does indeed cost more per consumer than urban electrification, and it must, moreover, be borne in mind that in rural areas the revenue per consumer must be less than that obtaining in towns. In determining the cost of giving a supply to the purely rural area shown on the west of the map, reproduced in Fig. 5, it was actually found that the ratio of annual revenue to capital expenditure could not possibly exceed 7 or 8 per cent. This is a fairly clear indication that the electrification of purely rural districts is still a very difficult financial proposition, in spite of the optimism shown by the author.

Mr. R. A. S. Thwaites: I can testify to the author's statement as to the great value of the power load in developing the domestic and other loads in rural areas. Fig. 1 seems to bear out the general view that, provided one is content to supply only some 60 per cent of the dwellings, a rural area can be developed nearly as economically as an urban area because of the very much greater use of overhead mains. When, however, we come to 75 per cent of the dwellings, it costs 50 per cent more to develop the rural area than the urban area; and if we try to reach saturation, then the upper curve in Fig. 1 " goes off the map." As the author said in presenting the paper, supply authorities in rural areas are in somewhat of a dilemma as to whether they should supply, say, 60 per cent of the domestic load at low prices, or endeavour to supply almost the whole at much higher prices.

I am glad the author has brought forward the question of costs; and to those who, like him, are endeavouring to obtain all the power load possible, I would recommend the acquisition of the annual Tables published by the Diesel Engine Users' Association, which give the operating costs of a large number of Diesel plants. The difficulty, as the author points out, is that very often a manufacturer does not know what it is costing him to generate his own power. I do not agree with the author that we should obtain what price the market will stand; I think we ought to sell at an economic figure. Some municipalities in the West Riding of Yorkshire are offering a supply for power at about 20 per cent below the grid price; thus the more profitable loads are, in effect, subsidizing the power load.

Referring to Table 7, I agree with Mr. Hetherington that the author is a little illogical in putting a large proportion of what ought to be regarded as fixed charges in with the running costs. In the right-hand portion of the table (dealing with the 10th year) the sum of £16 900 is given as a running charge. Assuming that £10 000 of that is attributable to the high-tension consumers mentioned in the last column but one, and treating it as a fixed cost, we get approximately £8 per kW plus 0·2d. per unit as the cost without diversity; then the author introduces a diversity factor of 1·58, which will bring the cost down approximately to £5 per kW plus 0·2d. per unit. This is on the assumption that the diversity is always 1·58, irrespective of the load factor; but such is not the case. For instance, at 100 per cent load factor the diversity is nil. At 2 or 3 per cent load factor the diversity is very high, and the cost of £5 plus 0·2d. can be corrected for this in an approximate manner by reducing the fixed charge and increasing the running charge. By making certain assumptions as to the manner in which diversity varies, it can be shown that a more equitable tariff would be almost £4 per kW plus 0·3d. per unit, which, curiously enough, is in fairly substantial agreement with the author's calculated costs of £3·83 per kW plus 0·37d. per unit, but which he arrives at by a rather illogical method. As the cost without diversity is £8 plus 0·2d. per unit, such an undertaking as that dealt with in the paper should give any bulk supplies to authorized undertakers at this same price, because with bulk supplies the diversity is substantially nil, whatever the load factor.

Mr. A. C. Sparks: The paper itself appears to refer to private plants in general, but the Summary makes it clear that plants above 500 kW are excluded; I think it unfortunate that this point is not more clearly brought out by the author.

The interest and depreciation rate in Table 3 indicates 8 years' life for Diesel-electric plant, which appears moderate having regard to the substantial allowance for maintenance.

On page 317, £2·5 per kW appears to be added without justification, as the author assumes that the capital charges have been written off, and the deductions are mainly dependent on the accuracy of the figure of £1 478, which cannot be commented upon from the information given. Further, the author only allows a life for turbine plant of approximately 9 years, and if this is increased the cost will go down. He also allows 3 per cent for insurance, which seems very adequate.

If a supply undertaking shuts down a private plant, the private consumer may save some fuel and labour costs, but any capital charges on the plant shut down and on the new arrangements for the public supply should be added to the supply undertaking's charges in order to get a true comparison. No account appears to be taken of this in the paper, and as an instance it would mean, broadly, that the fixed-charge portion in examples A to E inclusive (Fig. 9) should be omitted.

I am surprised that the author's undertaking should have found it cheaper to use a grid 132-kV tower than their own design on 33 kV (incidentally, I note that the double towers are not of grid design, as stated). I should have thought that a still greater saving would have resulted if the grid 33-kV tower or some similar design had been adopted.

On page 330 the author finds that the costs for giving rural and urban supplies are little different. This is partly due to the grid system, whereby the urban areas subsidize the rural areas. As an example of this I would refer to the recent comments of the chairman of the London Power Co.*

The conclusions on page 332 referring to comparative costs are sweeping, as paragraphs (1) and (2) are based on working on one particular load, namely 65 per cent of the full load of the engine. In paragraph (1) the author refers to untransformed h.t. power, which is not previously mentioned in the paper. If he means power at 11 000 volts, this appears quite impracticable for these very small private plants. Perhaps he will explain what he has in mind.

I do not share the author's optimism as to the cost of

* *Electrical Review*, 1935, vol. 116, p. 222.

public supply in general in the future, one reason being the question of obsolescence and efficiency versus cost per kWh, which the author mentions on page 313 as difficulties in connection with private plants, but which I submit apply equally to plants for public supply.

The figures in paragraph (2), page 332, for small sets are based on the 6 engines in Table 3 on which I have commented, the largest of which is of 45 kW. Therefore, if correct, they would only apply in particular cases.

Mr. D. J. Bolton: There are two or three figures in Table 7 which are very difficult to understand. For instance, the cost per kW of grid supply is put at £3·50 in three cases and £3·25 in the fourth case. As these supplies presumably come from the same point, it is difficult to see the reason for the difference in price. Again, the diversity factor for the high-tension supply is given as 1·55, that for the low-tension as 1·2, although the low-tension supply itself forms one of a number of high-tension points. Thus if we supply three or four private consumers at 11 000 volts, and also one sub-station serving low-tension consumers, it is difficult to see how the former can have the higher diversity. If there are a number of consumers on the low-tension side, the diversity there ought to be higher.

The author rather despairs of 2-part tariffs, and says that consumers should not be charged with their kW demand. Naturally, this is impossible at £11·6 per kW; but does the whole of the demand which is to be assessed at £11·6 per kW actually represent effective demand on the grid point? I suggest that the diversity factor is much too low, and therefore the figure of £11·6 is much too high. The author shows a fundamental lack of faith in 2-part costing, and he lumps a number of costs which he admits are fixed in character into the running charge simply in order to get a not too impracticable result. He could have got the same result by adjusting the diversity. The time has come when we must face the fact that costs and tariffs are two entirely different matters; we should be scientific about costs and politic about tariffs.

With regard to the 10 per cent allowance for depreciation (page 315), possibly this covers some insurance; otherwise it would represent a life of only $8\frac{1}{2}$ years at 5 per cent interest. If 60 per cent of the total were due to the Diesel plant, it would represent a life of only 6 years on this, allowing 20 years' life on the electrical plant.

Mr. A. F. Webber: On page 332 the author states that the cost for the pass-out steam turbine shown in Figs. 8, 9, and 13, is a particular case of use, and therefore it is not possible to make a comparison with the public-supply costs. In view of that statement, with which I heartily agree, I think the whole reference to this particular pass-out turbine might well have been eliminated. The conditions at the factory quoted do not seem to be very good; the boilers are run at 55 per cent overall efficiency; and certainly electric supply for private power has more formidable competitors than this.

I made an attempt to find out from the paper whether in this case something better could not be done with a pass-out turbine on theoretical estimates, and discovered that not nearly enough data are given by the author to enable any sort of estimate to be made. It therefore seems that no good purpose is served by incorporating this individual instance in the paper. I do not deny that far too optimistic estimates are often made for the cost of electric power obtained from combined pass-out plants, chiefly owing to a tendency to assume that the power and the heating loads will fit in a great deal better than in fact they do; but the author goes to the other extreme and, by implication rather than by direct statement, condemns the pass-out combined system on evidence which, though no doubt quite reliable, has only been obtained from a particularly bad instance. It should be remembered that although the possibility of controlling and finding continuous use for the heat rejected from the power unit is necessary in order to produce the highest thermal efficiency throughout the year, nevertheless, if a considerable heating load has to be met during the winter and possibly a small heating load during the summer, the boiler plant has to be installed anyway and the capital charges for it have to be borne throughout the year. Thus there is a considerable inducement to extend slightly the plant in order to be able to use the fuel and boiler plant throughout the year for combined power and heating.

Mr. W. A. Crocker: The author and previous speakers have given the impression that the owners of private plants do not know what their costs are or what they are going to be; in fact, that they are simply using their plant in a rough-and-ready manner. This may apply to plants of 500 kW or less, but in my experience, which is mainly of larger installations, I find that the owners do know exactly what their present plant costs them for electrical energy, by-pass steam, and so on. Further, I think that, in the past, supply undertakings have been wont to send representatives with insufficient knowledge of works costs to confer with the directors of manufacturing firms as to the advantages of changing over to public supply.

Mr. Forbes Jackson: May I add one word to bear out rather more fully what the author has said about the variety of tariffs? I am responsible for buying electricity in a number of different places, amounting each year to 20–25 million units. We buy from 50 different undertakings, and each has about 6 different tariffs. I feel that a serious effort must be made in the industry to standardize tariffs, so as to make matters more easily understandable to the lay mind. If the present position is difficult for us, with technical people at our command, I am sure it is very much more so for consumers who are more or less guessing what price they should pay. I am frequently asked to explain why in two neighbouring areas we should pay what are apparently widely different tariffs for the same service. I agree that sometimes these apparently different tariffs produce approximately the same price per unit, but this only makes it more difficult to explain why the two different tariffs are necessary. The lay mind does not, and cannot, appreciate that there are genuine differences of opinion among engineers as to the fairest way of determining the cost of a given service, and assumes, not unreasonably, that the only explanation is that one supply authority is more grasping than the other. I suggest, from my experience as a buyer of electricity, that the standardization of tariffs should be the first concern of the industry, and that it is even more important than the standardization of supply voltages and systems.

Mr. G. W. Molle (*communicated*): I can support the author's statement as to the ignorance of owners of private plant regarding the true cost of their own power production. In a very recent case with which I had to deal, an old d.c. plant (some of it 22 years old) was supplying power to a large works. There were several small producer-gas sets and a much larger heavy-oil set. The switchboard meters were of a very obsolete pattern, and, so far as anyone could tell me, had never been checked or tested in 20 years. None of the fuel was bought against specifications. It took days of consultation with several accountants to arrive at figures for capital cost of plant, depreciation allowances, etc. No fund had been established for plant replacement. The only records available were the actual running costs for wages, fuel, and repairs. In the case of the heavy-oil engine set, in the seventh and eighth years after installation the cost of repairs and replacements on the engine totalled about one-fifth of its capital cost. This is a good illustration of the fallacy of working out the alleged cheap cost of generation by such plant for two or three years ahead only. The plant may give many years' good service, but if, for instance, the makers' instructions are not properly carried out, repairs and replacements can be heavy after 5 years. In the case under consideration, the question of an alternative bulk supply arose, and, after arriving at the proper figures for the cost of private generation, this was found to be just about twice the actual "running-costs" as given by the returns of fuel, wages, and maintenance. Incidentally, it corresponded closely to the average for seven cases of producer-gas and Diesel plant, given in Table 2 of the paper. From experience on several large private-plant schemes, I agree with the author's figure of £20 per kW for private generating plant; and the fact that extensions can be carried out so cheaply where a public supply is used is one of the most powerful arguments in its favour, especially where there has been little thought for the future and the owner of a private plant is suddenly faced with the fact that all his engines are about to shut down simultaneously. Such lack of foresight may sound absurd, but does happen in practice, especially where the ultimate executive control is in the hands of officials, directors, etc., who have no engineering knowledge. The crux of the whole question, private plant versus public supply, is, of course, a matter of policy and the degree of control and reliability required, rather than a mere matter of comparative costs. For instance, I have known an important repair-shop, dependent entirely on a bulk supply from a hydro-electric source, to be shut down for three weeks through floods. Again, there are cases where, owing to the difficulty of obtaining the right skilled attention to plant, it is advisable to accept a bulk supply. The best solution, bulk supply plus local stand-by plant, is usually impossible on economic grounds.

[The author's reply to this discussion will be found on page 346.]

North-Western Centre, at Manchester, 5th February, 1935.

Mr. A. B. Mallinson: To my mind the best part of the paper is contained in the closing words, where the author pleads for a unification of tariffs among supply engineers. If that can be accomplished, supply engineers will find it much easier to negotiate with consumers.

It is as well, perhaps, that the scope of the paper is limited to plants up to 500 kW, since it is common knowledge that in Lancashire there are straight condensing plants of 500 to 1 200 kW which are 50 or more years old, and yet are driving cotton mills much more cheaply than can be done with power from a supply undertaking at 0·5d. or 0·6d. per unit.

The Introduction indicates that as a supply engineer the author is desirous of attracting the power load. Public supplies started with electric light, extended to power, and later spread to heating and cooking, the power supply tariffs in many areas being kept considerably higher. All power users will be glad to learn of the author's interest in the power load, because for some years past there has been a tendency to put domestic requirements first.

Dealing with Part 1(1), I agree that the majority of private-plant owners may not know their costs per unit, but why should they? It is the cost to run the works per year for power and (if used) steam or heat that counts. The remark of the author that only in some instances do the owners know the cost may hold good in a few rural industries he has investigated, but it certainly cannot be substantiated as worded for plants up to 500 kW all over the country. Even if the private-plant owner knows his cost per unit it is of no help to him in deciding whether the supply undertaking's charges are lower. He has to wait till the end of the year (assuming the general 2-part tariff) to find out what the actual cost is. The author makes a good point in emphasizing that a supply engineer ought to satisfy himself regarding the load factor before making statements to prospective consumers as to what it will cost to take a supply from the mains. Two-part tariffs, banking on the 30-minute peak, do not always turn out so well as the author indicates, as the proprietors of many cotton mills have found, owing to the Monday-morning peak affecting the whole week's running.

Turning to the question of relative fuel costs, the author's remarks about oil no doubt express the wish of supply engineers, but with so much over-production I think they will have to be patient.

His arguments in respect of the economic life of plants are ingenious but weak. May I give one indication of the contrary? One of the most successful process steam plants I have installed was made out of two 500-kW triple-expansion high-speed engines, which had then —4 years ago—been running condensing for 28 years. The low-pressure valve was taken out, and the engines then ran on 15 lb. per sq. in. back pressure, each giving 320 kW. That plant is now saving over £9 000 a year in coal, wages, and bought electric power. The grid stations operate at 22–27 per cent thermal efficiency: if the losses are anything approaching what they are in France—if we take the word of Mr. Flandin* 1 kW sold may mean 2 or 3 kW made—the thermal efficiency of

* *Electrician*, 1934, vol. 113, p. 800.

the grid station at the consumers' meters is 9–13½ per cent. With any ordinary process steam plant, using the steam after it has gone through the prime mover, the thermal efficiency is 60 per cent, and the owner need not worry about obsolescence so long as the plant continues to function.

Turning to page 313, what type of plant is it that costs £20 per kW? The average figure for steam plant and Diesel plant in all sizes up to 500 kW is nearer £10 per kW. I do not agree with the author's figure of 12s. 6d. per kW for the additional transformer capacity; I think this refers to a big transformer, and perhaps leaves out the switchgear.

On page 313 the author refers to the effect of trade depression on standing charges. The same argument applies to the public supply. The maximum-demand charge has to be paid, whether the plant is run for 1 hour or 8 720 hours in a year. With the private plant, if trade is bad the owner knows he makes a direct saving when he is shut down; but with many supply authorities he has, by his agreement, to pay for so much electricity per quarter whether he uses it or not.

Table 2 gives 12 cases which show the author's arguments to the best advantage. The figures in line (11) are not comparable with those for public-supply 2-part tariffs. Either we must consider the running cost only of private plant, when comparing it with the running cost on a public-supply 2-part tariff, or, if we are persuading the owner to change over to public supply, we must add all the standing charges on the private plant which have yet to be liquidated.

The "weighting of private-plant figures by provision of spare plant" is the old red herring of the supply industry. It would be interesting to know why the private-plant owner must be so saddled when, no matter how many grid stations there are, he has to depend on overhead lines, at the mercy of lightning, wind, snow, etc.; unless he is a bulk-supply user, he finally depends on one service, and, if it is a small plant, on one motor. Last week-end I was at a hydro in Matlock where I installed a generating plant over 25 years ago. The lights flickered when the lift was worked, but when I suggested that the lift should be put on the public supply (which is now used for radiators) the reply was, "We dare not do it. The public supply is off too much to risk running our lift from it."

In Wilmslow we have had in 11 months 7 total failures of the public supply, totalling 6 hours 48 minutes.

Under the heading, "Steam Plant for Combined Power and Heating" the author considers works heating as a typical process steam load, and says that only in exceptional cases can any approach be made to the continuous use of the rejected heat. Such conditions are practically universal in bleachworks, dyeworks, calico printing works, paper mills, most chemical works, soapworks, laundries, public institutions, and large hotels. I never advise a process-steam power scheme unless I can show, after all charges, at least a clear 15 per cent overall saving. A plant in which the process heat is used for only 35 per cent of the running time must obviously be inefficient; the fact that the author takes the boiler efficiency at only 55 per cent is proof of this.

On page 316 he remarks that pass-out turbines are the most efficient type of process steam plant. He is wrong. The back-pressure engine is the most efficient heat engine, as it ensures that all the heat units are utilized.

In Fig. 9 the figures for the reciprocating engine are absurd. They are evidently based on process steam being used for only 35 per cent of the running time. The figures in Fig. 9 are dangerous to the user, the cost of bought power being made attractive by the smallness of the unit charge compared with the kW charge.

In Figs. 10 and 13 the dotted lines represent what may possibly be the public-supply charges of the future, not those in force to-day. The private-plant costs are all bolstered up by the excessive plant charge.

I have worked out the figures for a 250-kW back-pressure plant, and taking a very liberal basis, including a high-pressure boiler, the cost per unit with a 28 per cent load factor is 0·185d., and the plant charges are 0·229d., making a total of 0·414d. A line on Fig. 9 for the back-pressure set under proper conditions starts at 0·2d. (100 per cent load factor) and rises to 0·414d. (28 per cent load factor). The pass-out plant line depends on the percentage of pass-out steam; the maximum pass-out line will follow very closely above the back-pressure line, because of the loss of heat in the condenser.

Turning to the "Conclusions as to Capital Costs" (page 332), and dealing first with (1), the author has only attained his figures by excessively high plant costs; they are certainly incorrect when the plant cost has been wiped out, as it will be in time. Conclusion (2) is possibly correct if the supply is provided at the rates he gives, but not if the plant costs are what the ordinary manufacturer would tender. He will put one set in, not two. As regards (3), I agree that a house lighting plant cannot compete with the supply that can be got from the grid lines.

The author's general conclusion seems to be a pious hope that sooner or later the users of the cheap supply in the congested areas will have their price raised so that consumers in outlying districts may get the benefit; but what the users in the congested areas will say when their cost goes up is another matter.

Mr. W. E. Swale: I agree that the average small-plant user has only the most sketchy knowledge of his actual works power costs. Frequently these can only be obtained by thrusting into his hand a blank schedule detailing the actual costs and saying, "Will you be good enough to fill that in conscientiously for 12 months?" At the end of that period he often says, "My power is costing twice as much as I thought it was."

With reference to page 313, col. 1, I cannot help thinking that if the cost of a private plant is £20 per kW it is not quite fair to compare it with 12s. 6d. per kW, the cost of the transformer alone. The public-supply user has to pay an extra kilowatt charge in his tariff, corresponding to his increased demand.

Table 2 is instructive, but in one point it mystifies me. The public-supply load factor, item (7) is obtained by dividing item (3) by item (5), but the former is expressed in kWh and the latter in kVA. The note to item (5) says "includes conversion, power factor, and

transforming losses," from which one may infer low-voltage metering. If the figures in item (5) really are the measured kVA, then all the load factors in item (7) are some 25 per cent too low; but they are, in fact, already much higher than we should expect to find in Lancashire. An explanation of this apparent anomaly would be appreciated.

I was puzzled, at first, by the ingenious method of weighting the cost of private plant by multiplying by a factor of 1·65. The information in Table 2 was possibly collected 3 or 4 years ago during the trade depression, and that might account to some extent for the high weighting ratio. Obviously, if a plant was running at 65 kW maximum demand then, and trade improved, and the load went up to 90 kW, the owner would not immediately put down spare plant; in other words, the weighting factor is gradually depreciating as the business of the factory increases. The author admits that the weighting factor does not include stand-by plant, and in Lancashire it is difficult to get engineers to appreciate the necessity for 100 per cent stand-by plant.

The various methods of analysis leading up to Figs. 4A, 4B, and 9, are valuable, and illustrate the significance of load factor very clearly. The author's methods are correct, but one should be careful not to use the results at the present day without corrections for fuel price and cost of plant.

In my own experience also, the method of heating buildings by back-pressure steam from the engine is most inefficient for a works. Where it is done in public institutions and hospitals, the case for the pass-out engine is better because of the far greater load factor.

In the last 5 or 6 years it has rarely been necessary in Manchester to go into a detailed analysis of operating costs. The saving effected by going over to the public supply has been so marked that it has been mainly a question of how to raise the necessary capital, and how to effect the conversion. In 5 years in Manchester 74 small engines have been replaced by the public supply.

The author speaks of the paramount importance of unifying tariffs, but in my experience the differences in tariffs are only of minor account in negotiating power business. In dealing with industrial power users who have little technical knowledge of electricity supply, it is a matter of getting into touch with the responsible man at the works and putting up a sound scheme to him, based not on kW or kWh but on the annual cost of running. One should simply talk of total costs, in pounds per annum; the mere fact that for one consumer the fixed charge is 10s. per kW more than his neighbour's, and the running charges are less, makes little difference. Whilst the desirability from a theoretical point of view of having standard conditions is obvious, I do not consider differences in power tariffs to be seriously detrimental to power sales development. A move is already on foot for reaching some measure of uniformity in regard to the type and conditions of a standard power tariff; but the question of standardizing the actual prices is so difficult that, in my opinion, it may well be left alone for the time being. With the author's plea for an aggressive campaign to obtain further power load I am in wholehearted agreement.

Mr. W. Fennell: The part of the paper that interests me most is the Introduction. In the first paragraph the author says, "It is probably not realized sufficiently to what an extent the obtaining of a power load can reduce public-supply electricity-costs. . . ." When submitting the paper this evening, however, he remarked that he was concerned to find that an increased power consumption did not reduce his total cost to any great extent. [(*Communicated*). The explanation is that the load factors of the two types, power and domestic, are not very different.]

In the same paragraph he introduces the question of the effect of rates, not merely upon the choice of a works site, but upon the electricity undertaking. The lure of low rates is not so strong as it was before manufacturers were de-rated; they now only pay on a quarter of their rateable value. They do not now save much in rates by coming into the outlying areas where rates are low. Unfortunately, the electricity undertaking has to pay full rates on a method of rating which can be termed a savage method compared with that applied to ordinary businesses. The owner of a shop in a town is rated upon the letting value of the carcase of that building without fittings and fixtures, if empty. An electricity undertaking has to pay what amounts to a second income tax. It has to pay rates based upon a hypothetical rent for an undertaking ready to work with its consumers already connected. When a supply authority competes with Mr. Mallinson for the load in his works, Mr. Mallinson's works will only have to bear a quarter of the rates on its private plant, whereas the electricity undertaking has to bear the whole of the rates on a scale 3 or 4 times as high as Mr. Mallinson's owners will have to pay per kilowatt. It is up to us to see that the position is altered very soon.

There is a very interesting point on page 311 where the author shows that the actual expenditure per dwelling on a route of mains is more or less the same in town as in country areas. He deduces that it does not cost any more to provide electricity in rural areas than it does in urban areas. I think he has been rather misled there. His figure is roughly the amount it pays to spend on this business per consumer. In the towns, as he shows in Fig. 1, it is not necessary to bring extensions into the less attractive areas to an end nearly so soon as in the country, under present methods of construction. All electricity undertakings roughly pay, within limits, the same percentage on the distribution capital, and, when it is found that this percentage is not likely to materialize on a proposed extension, it is turned down or at any rate deferred. The economic capital expenditure sets a limit beyond which no prudent person goes, and therefore one cannot deduce from its uniformity in urban and rural areas anything except that engineers are prudent and worthy of the confidence of financiers. The effect of certain proposals which carry with them compulsory detailed supplies over wide areas in advance of demand and perhaps without ever obtaining it, would be to upset this economic balance and cause over-capitalization.

In my view a policy of high prices and small sales is never justified, and it does not pay in any stage of the development. Most of our troubles have been caused

by commencing with high prices in the early days with few consumers, with the intention of reducing them later when many consumers have been secured and a profit has been made: but many consumers are not obtained quickly if prices are unduly high, and therefore the undertaking does not get into the happy state when prices can be reduced. The mistake of to-day is that we are being urged by politicians to develop the whole area completely at once, especially a rural area, and wait for some return later on. The reflex action of that would be high prices, which restrict connections to lighting. The policy I would support is to develop the more suburban and other paying areas first, with a low tariff from the commencement, and collect consumers rapidly. By that means the supply will be used for all purposes and demand will be so stimulated that the next step into the less-populated areas will be productive, because not merely lighting but also cooking and heating loads will come on at once, instead of being delayed for years.

The conclusion I have arrived at on reading the paper is that the author is on the right lines, but each supply engineer must have his own figures. One cannot compare a mixed area, such as the author is developing, with, say, the Lancashire areas. The method is there, however, and I suggest that supply engineers should use it.

Mr. J. Frith: I note that the author's own tariffs are far higher than those indicated on page 333, where he looks forward to a flat rate of ½d. per unit. The author does not show how this figure is going to be achieved. I do not know why he did not take a new works as an example; so many small advantages, which all combine to cheapen a building, are obtained by doing away with line shafting, belting, and other such things. In a Lancashire mill, where boilers are a necessity, and when the value of the boiler and engine has been written down to almost nothing, it is very hard to persuade the owners to go over to an outside electric supply.

I am greatly in accord with the suggested unification of tariffs. I can speak from many years' personal experience of the aggravation and exasperation of a client who wants to buy electricity, when those tariffs are shown to him. He can always find some point of advantage in another person's tariff. Why must we have a kilowatt charge sometimes of £8 or £9, and sometimes of £2 or £3? Is it not extraordinary how often the first figure is a guess and the subsequent ones are worked out to two or three decimal places?

I am sorry for the failure of the combined kilowatt and unit tariff; it fails because it does not take any account of when or how often the maximum demand is taken. A consumer who takes it every afternoon between 3 p.m. and 5 p.m. only pays as much as another who takes it for 1 hour during the year.

I should like to appeal to supply engineers to give a reasonable amount of lighting at power rates to power consumers.

Mr. G. T. Allcock: The figures given in Table 2 as the total cost per equivalent kWh of power from private plant seem high, in view of the claims made by engine builders and others. Nevertheless, they are, in my opinion, fairly representative of conditions as found in the average works. Engines undoubtedly can be installed to give cheaper power, but for want of proper maintenance and intelligent running, and, still more, owing to alterations in load conditions with fluctuations in trade, the maker's prophecies of efficiency are generally falsified in practice.

In all the cases quoted by the author the plant is old, and it is open to the engine enthusiast to say that a very much better performance could be obtained from modern and efficient plant. The following case is therefore interesting in showing what a modern and efficient oil engine can do as compared with public electricity supply. A certain firm drove a tar macadam mixing-plant by means of a 50-h.p. d.c. motor from the public mains. The consumption was 28 000 kWh per annum, which at 1d. per kWh cost £116. Adding hire and maintenance of the motor, and all other relevant charges, the total cost of power was £168 per annum—an average of 1·44d. per kWh. The consumer was led to install a 75-h.p. oil engine by promises of power at less than ½d. per kWh. He spent £640 on the engine, its auxiliaries, and installation; allowing 8 per cent depreciation, his total power cost is now £229 per annum—an average of 1·96d. per kWh. Until a proper cost sheet was submitted to him, and he had filled in all the items from his books, the consumer was quite pleased with the engine. He had regarded the amount spent on fuel and lubricating oil as representing the greater part of his total power cost.

[The author's reply to this discussion will be found on page 346.]

North-Eastern Centre, at Newcastle, 25th February, 1935.

Mr. W. A. Carne: The object of the paper is twofold—to encourage the wider use of electricity for industrial purposes and to plead the cause of standardization of distribution tariffs, and thus effect (in the author's own words) "a large increase in the sales of electricity and in the number of connected consumers."

In recent years the electric supply industry has become specialized, and the process of generating has become largely centralized in large and efficient power stations, some of which are controlled by joint generating authorities. The Central Electricity Board has been brought into existence to transmit energy from these super power stations and to deliver the energy in bulk at uniform tariffs throughout wide areas. It may seem logical, therefore, to plead for the distribution of this energy also at uniform tariffs in all areas.

Those sections of the supply industry whose activities begin and end with one process—either the generation or the transmission of electricity—have been quick to realize that their growth and prosperity are dependent upon the work of those who distribute and sell the energy. It is obviously no good generating, or transmitting to and fro, more energy than the distributors require. The efforts that they (the wholesalers) can make to expand their own business are limited to (1) lowering the cost of generation and transmission,

and (2) urging the distribution authorities to greater efforts, by offering advice in speeches and in the Press.

The author's advice is, in the main, quite sound, being backed by practical experience, but he allows his enthusiasm to get the better of him on page 333, where he suggests that within the next few years electricity will be available to all consumers at 0·5d. per unit; this suggestion is not worthy of consideration.

The method used in the cost analysis on page 325 is admirable for the purpose of a paper such as this, but it has limitations, of which the author is no doubt aware, from a working point of view. The simplicity of the analysis leads to complication in its application if accurate results are required; a clue to this is given in the fact that the cost per unit comes to 0·39d. per unit sold for both h.t. and l.t. supplies, in spite of the obvious difference in efficiency that must exist. Even worse complications will occur in applying the demand figures of the analysis. I do not like the author's suggestion of placing as many items as possible in the running charge so as to keep the fixed charge to a practical competitive figure. It is preferable to have the analysis correct and to make the necessary "commercial" adjustments in framing the tariffs.

With regard to Fig. 11, the fact that the author has included this diagram, and yet confesses that he cannot draw the deduction that he hoped from it, is proof of his sincerity. I suggest that it might be worth while plotting pence per unit sold against the percentage ratio of lighting and domestic units to power units. It is obvious that any mutual advantage gained by combining power and lighting must be due to a sharing of common plant. The lighting price will therefore be highest when there is no power consumption, and vice versa.

In order to segregate the variable factors, I would suggest plotting a number of graphs; the first would contain points from undertakings with outputs of from 20 to 30 million units per annum. If this gave encouraging results, further groups would be plotted, possibly resulting in a "nest" of curves from which the author could derive more accurate conclusions.

Finally, I suggest that the curves in Fig. 1 should be shown without the plotted points, as they must be based on evidence other than that provided by the points shown.

Mr. V. A. H. Clements: The latter part of the paper contains several inaccurate conclusions which make one suspect much of what has gone before. Thus, in the middle of col. 1, page 331, the author states that in many towns—he rightly stresses industrial towns—average domestic prices are higher than elsewhere, and then he jumps to the conclusion that the domestic load is subsidizing the industrial load. Has he considered that, in industrial towns, the class of dwelling supplied is such that a use of electricity comparable with that in a residential town cannot be made, and consequently the average domestic price will, even if on a lower tariff, tend to be higher than that in a residential town of a similar size?

Any attempt to average the costs of supplying different types of loads and then to apply that average in framing a tariff is not only fallacious but dangerous. For each type of supply, the costs incurred in giving that supply must be computed, and it is therefore to be hoped that no co-ordination of areas, such as the author suggests, will ever take place on the foundation he has in mind. If such were to happen and the author's further suggestion were to materialize, that all supplies should be given at the average cost to the undertaking, with a lower price for off-peak supply, only a loss of money could result, since to supply at average cost for the bulk of the load, and under average cost for the remainder, means only that the overall average is less than the cost. Further, the effect of offering all supplies at, say, 0·7d. or even 0·5d. per unit, will be that those paying more than that will, of course, accept, while those who at present pay less, and who are instrumental in making the average low, will cease to be consumers.

The statement that except in very special cases it would be economical for power consumers to pay up to 1d. per unit is, to my mind, a fallacious generalization. The author produces evidence against himself in Fig. 11, which shows that the vast majority of power supplies are given at less than 1d. per unit, and since these are average figures many of the components of the averages must be very much less than 1d. per unit. Surely he is not bold enough to assert that the suppliers of all these units are so unskilled in their business as to charge less for them than the occasion merits.

The author's general attitude that all the public supply industry has to do is to compete with private plants is surely rather a narrow one. Is not the position rather that electrical energy is a necessary raw material in present-day industry, and the cheaper this can be sold, having regard to all the circumstances, the better for the economic life of the country? I do agree with the author, however, that merely to obtain load is not an excuse for, in effect, subsidizing that load to the detriment of other supplies.

Mr. S. Burns: My interest in the subject dealt with in the paper is centred on the selling of current to collieries, and my experience in this field leads me to suggest that electrical-energy sales to this class of consumer are not so easily effected as the author would have us suppose.

I should have liked to have seen a reference made in the paper to more difficult cases, e.g. the problem met with when the closing-down of turbo-alternators necessitates another use being found for low-pressure steam available from winding engines so comparatively recently installed that their immediate conversion to electric drive cannot be justified on economic grounds; and to the further complication which arises when turbo-compressor equipment, which ordinarily might be installed to afford an alternative use for such steam, cannot be installed because of the modern, and in most cases very desirable, tendency to eliminate compressed air in favour of electrical equipment at the coal face. I suggest that careful investigation of this and similar power-supply problems might lead the author to the conclusion that there are practicable difficulties in the way of the simplified tariff arrangements he favours.

It is pleasing to note that another factor, not mentioned by the author, shows a growing tendency to play its proper part in present-day power-supply negotiations;

it is that users display a greater readiness than once they did to regard electrical energy as a commodity which they themselves are neither equipped nor anxious to produce providing it can be purchased, at a reasonable order of price, from a supplier whose experience in this department of industry obviously must be greater than their own.

Mr. W. Cross: Table 4 shows great variations; for instance, annual standing charges vary from £12 per kW to 10s. per kW, and I think that, generally speaking, these figures must have been given by individuals with differing ideas of what is a standing charge. Referring to Table 4, line 3, a figure of 350 units per annum per kW demand can be deduced from data given by Messrs. Woodward and Carne;* this is equivalent to a load factor of 4 per cent, and I notice that the latter is exceeded by some of the author's figures. I have worked out the standing charges for private plants of from 1·5 to 10 kW and find that they vary from £2·8 to £1·2 per annum, which is considerably less than the author's average figure of £4. I should like to know what he includes in his running costs. I find that the cost of fuel and lubricating oil in the case of automatic plant works out at 1d. per unit. Also, allowing a 4 per cent annual load factor, the price per unit for standing charges alone, taking my figures, varies from 19d. to 8·2d. per unit. If, however, one could use that plant for other purposes than lighting and double one's consumption of current, those figures would be halved, and it is very clear that the standing charge is the greater part of the generation costs in practically every case. Consequently I get, instead of £4 per kW, £2 per kW, and 1·2d. per unit. It is to the customer's advantage to use his plant until it is worn out, as the secondhand value is very low.

On page 317 the author gives a figure of £150 per kW as the capital cost of this plant; I should like to know what that figure includes. I find that the list price of a Diesel plant and battery varies from £160 to £68 per kW.

On page 312 the author states that he would suggest that at an early stage the price should be higher than in the later stages of development. To me it appears it would be much better to quote a fairly low price from the first, although the loss would be greater during the first few years, for surely it is better that the distribution system should have a fair load from the outset than that the price should be so high that the consumer will not take a supply, particularly if he thinks there is a chance of the price being reduced later.

Mr. H. D. Phelps: Unfortunately, the author does not state at the top of the columns in Table 4 the particular type of plant from which the results were obtained; but I take it that cols. 1, 2, and 3, refer to Diesel plants, col. 4 to petrol-paraffin, col. 5 to petrol, col. 6 to Diesel, and col. 7 to petrol. The running costs set out in col. 5 compare closely with costs I have had given me for plant of a similar output in which everything has been included. The costs of the petrol set detailed in col. 7 appear to be too favourable. The running costs of entirely automatic petrol-driven sets are very considerably higher. There is great difficulty in obtaining reliable costs, because few owners keep detailed records. Their usual reply to a query on this point is "a gallon or possibly two of petrol per week in the winter, and practically nothing in the summer."

There is usually considerable difficulty in finding a market for secondhand plant; but if the battery is in reasonably good condition £20 to £30 may be obtainable. This sum will cover the cost of lamps, alterations to switches, etc., that are necessary when changing over to a public supply.

Turning to page 313, the success of supplies in rural areas depends mainly on the cost of the h.t. and l.t. overhead lines and transformers, and I am pleased to see that the author advocates 3-phase supplies for even small villages. Often a single-phase supply is instituted on the score that there is no power-load to be obtained; but experience shows that requests for a supply for power come from unexpected quarters, and with them come the starting difficulties with single-phase supplies, on account of voltage-drop; the necessity for friction clutches or very expensive motors arises. The cost of the latter falls on the consumer, and is often an obstacle to the installation or full use of electrical energy for power purposes.

I am sorry the author omits to give the cost of transformers and also of simple 3-phase overhead lines, as he has had considerable experience in this matter and no doubt has reduced these costs to reasonable proportions. Could he state the cost of 0·05-sq. in. 3-phase 11 000-volt spur lines in, say, $\frac{1}{2}$-mile lengths; also of 15-kVA and 25-kVA 11 000/440/230-volt 3-phase pole-mounting transformers, together with high-voltage remote-operated fuse isolators?

There is very considerable diversity of opinion as to the amount of switchgear (usually installed in the l.t. switch cupboard at the foot of the pole) that is necessary to meet the requirements of the regulations when supply is afforded to two or more consumers. Perhaps, therefore, the author would state what apparatus is installed, and at what cost, in areas under his control.

Regarding tariffs—which are always a source of difficulty—I cannot see how one based on acreage is workable. For instance, on a dairy farm of, say, 80–100 acres the consumption might easily be 6 000 units; whereas on another farm of, say, 600 acres the consumption might not exceed 600 units. Another drawback to this class of tariff is that conditions of farming and stocking vary from year to year, depending on the cost of grain growing and of imported feeding stuffs, as well as on the prices obtained in the home market for farm produce. All these items have a very definite bearing on electricity consumption, and affect the working of the tariff.

Mr. J. W. Jackson (*communicated*): It would appear that in Fig. 2 the author unduly favours the claims put forward by the crude-oil engine manufacturers. The probability is that crude-oil engine plants running at load factors up to 60 per cent are very rare exceptions. In the case of a factory with only one engine installed, a much more likely condition would be that of a plant load factor of about 25 per cent and a load varying between $\frac{1}{2}$ and $\frac{3}{4}$ full load. The final running load factor would therefore be reduced to approximately $12\frac{1}{2}$–17 per

* *Journal I.E.E.*, 1932, vol. 71, p. 879, Table 4.

cent. The actual running costs for fuel oil must therefore be considered to be those nearer to the left-hand side than those on the right-hand side of the curve in Fig. 2. The overhead charges per unit generated will be increased accordingly.

If it is desired to run the engine up to full-load conditions, the safe course to take is that of installing a similar engine as stand-by, with consequent increase in capital, because of the total incapacity of this type of engine to carry a material overload, even for a short period.

Mr. J. M. Smillie (*communicated*): The figures of cost of power generation by Diesel-engine plants set out in the paper should, I think, be acceptable as a general basis to public supply engineers. They are set out in a novel manner, and give a ready means of approximating the equivalent tariff necessary to obtain any business of which the general particulars are known.

The North-Eastern Electric Supply Co. have hitherto made use of the information collected during recent years, the object of which is to help engineers in their negotiations to displace Diesel engines, or to keep them from being installed. This information covers a range of plants up to 150 kW, and the present paper extends that range to 500 kW. The figures in the paper appear to confirm those we have collected, due allowance being made for a reduction in capital cost of plant during the intervening period between 1932 and 1934, a period which has been remarkable in respect of developments in Diesel engines.

These operating costs are, however, not always readily agreed to by the owner of a private plant when a purchased supply of electrical energy is being negotiated. In trying to whittle down these costs he has the full backing of the Diesel-engine salesmen, and it is well to be prepared for this in the negotiations, despite the confidence expressed in the paper. Power sales engineers should accordingly endeavour to give individual attention to every case; in actual practice two cases are rarely found to be identical; there are usually differences in regard to the age or capital cost of the plant, whether it has been purchased new or secondhand, and whether it has been covered by insurance. For these and other reasons it may be found not only necessary but profitable to quote a tariff appreciably below some of the estimated costs in the paper, more especially when the negotiations are designed to displace an existing plant. In this way of detailed examination and quoting the appropriate tariff, it should be possible to obtain some of the business that even now goes to the engine manufacturers.

[The author's reply to this discussion will be found on page 346.]

Mersey and North Wales (Liverpool) Centre, at Liverpool, 8th April, 1935.

Mr. P. M. Hogg: No modern industry can be run without power from some source, and, sooner or later, every industrial engineer is faced with the question of whether, given the conditions existing in his own plant, this power should be generated by a private plant or purchased from a supply undertaking. In this connection it is of interest to note that there are still over 6 million h.p. of prime movers and nearly 3 million kW of private generating plant in the industrial concerns of this country. Now, the correct answer to this power question can only be determined by a very careful investigation into each individual case, and the basis of the investigation should be a comparison between purchased power and power generated by efficient and suitable plant. If the existing plant is either unsuitable or inefficient it should not be considered. In many cases steam is still required when electrical energy is purchased, and when the comparison is made it is the total cost of all power required, in any form, that counts, and not merely the cost of the electrical power alone.

To come to the paper itself, it is rather disconcerting to learn from the Introduction that in a rural area the economic limit of distribution is reached when a supply is available to only 70 per cent of the dwellings in that area.

According to Part 1 of the paper, the author's experience has been that the majority of owners of private plants have no knowledge of their costs. This may be true of the smaller plants, with which the paper is chiefly concerned, but to-day many owners of private plants have a very accurate knowledge of their own costs.

On page 313 the point is made that during a long period of trade depression the capital charges on private plants may be very heavy, and that these charges may be eliminated if a public supply is taken. I think the word "may" is correct, because in many power contracts there is a clause which states that a certain minimum annual payment shall be made whether energy is taken or not. That minimum annual payment may be as much as £4 per kW of normal maximum demand.

Coming to the section on Diesel plant, curve A, Fig. 2, gives the cost of fuel oil as 62s. per ton, while the report of the Diesel Engine Users' Association for 1933–34 gives this figure as 77s. 4d. per ton, which represents an increase of 25 per cent.

As regards the power costs of large Diesel plant given on page 316, the figure of £3 per kW of plant installed is converted to £4·95 per kW of maximum demand because in the cases which the author has investigated the ratio of the capacity of plant installed to the maximum demand happens to be 1·65. I do not think the author is quite justified in using this figure of £4·95 per kW in comparing the cost with that of purchased power, because these engines may have been installed in order to meet possible future extensions.

I find it a little difficult to understand the basis of the calculations on page 317. In the case of the pass-out turbine the author debits the cost of steam for power production with only 16·3 per cent of the total coal bill. Apparently he then debits the power costs with all the wages, namely £1 700, and also with 71 per cent of the standing charges on an 800-h.p. boiler plant. I should like him to explain this a little more fully.

So far as small private-residence plants are concerned (page 317), I do not think anyone would install or replace one if a public supply at 7d. per unit was available.

Coming now to the technical features of the author's scheme, there are one or two questions I should like to

ask. On page 322 mention is made of long pin insulators as a feature which has proved of great value in the elimination of bird faults. I should like to ask whether these insulators were really long, or whether standard insulators were mounted on long pins?

On the same page the author mentions that the system of earthing at each pole was adopted in order to eliminate the use of a continuous earth wire. I should like to know whether the earth resistance at each pole has been measured, and, if so, with what result. Also, it would be very interesting to have details of the experience of lightning trouble.

One point which is not raised in the paper, but which I should like to emphasize, is the growing importance of continuity of supply to the industrial consumer. With the very rapid development of the continuous-process principle in industry, shutdowns are undoubtedly growing more costly. A very short interruption of supply often entails great loss of material, quite out of proportion to the length of time for which the supply is off, and sometimes means damage to the plant. The cost of electrical power in industry, except in very exceptional cases, is seldom more than 4 per cent of the cost of the manufactured article. Sometimes it is very much less, even less than 1 per cent, so that a difference of 25 per cent in the cost per unit really makes a difference of less than 1 per cent in the total cost of the article. It may well be that the reliability of supply rather than the cost per unit will be the deciding factor when the decision of the industrial consumer as to whether he should generate his own power or get it from a public supply, has to be made.

Mr. J. E. Nelson: In this area we have not been very much troubled with competition from Diesel plant, and in no case has Diesel plant displaced the public electricity supply.

It seems to me that the author is right; there is a distinct danger of selling electricity at too cheap a rate. Electric supply undertakings are expected to sell at unremunerative rates, and this has been particularly marked since the grid tariff came into being. The grid tariff does give us a measure of the actual cost of obtaining electricity, which is fairly uniform throughout the country, and I think it should be fairly easy, as time goes on, to obtain some measure of equality in the selling price of electricity. I hope, however, that the price will be based on real values. The tendency to sell at a cheap rate is rather carrying electricity undertakings away. In my opinion the figure of ½d. per unit for domestic supply is too low. I do not think that the cooking and heating load, which in this area is very largely charged at ½d. per unit, is paying its way. The ultimate effect will be, in my opinion, to move the peak of the load from the evening (where it is now) to the morning, and when this happens the cooking and heating load will really have to look after itself: it is now living on the lighting load.

Mr. Hogg hinted that it was only economical to supply 70 per cent of the houses in a rural area; perhaps the author will agree that a higher percentage than this may be expected.

The author remarks that the importance of load factor is not properly appreciated. This may be so with regard to the user, but not with regard to the supplier; and it is his business, of course, to try to educate the users in the same direction.

Another point I should like to emphasize is that the intending user frequently thinks that the maximum demand will be considerably higher than it proves to be; consumers are disposed to think this unless they have had actual measurements made on their own plant, and reliable figures are very seldom obtainable before the supply is connected.

Mr. R. A. Harrison-Watson: I have not been able to follow from the paper exactly how the costs of energy from the public supply compare with those of generation by a small private plant. In the last 30 years I have had an opportunity of installing a number of private plants, mostly paraffin-driven, and from records have usually found that—after making all allowances for fuel, depreciation, upkeep, and repairs—the generation costs are about 3d. per kWh as against the usual charge of 8d. or 9d. made by the supply authority in rural areas. In Table 4 the author gives the average cost per kWh used as 10d., and in Fig. 12 as 7d. Both of these seem to me to be high. The values which I believe to be correct are given in Fig. 4A, which shows the manufacturers' figures for small automatic petrol plants, which are very inefficient owing to their so rarely working at full load. Would the author tell us what percentage he allows for depreciation, and how many years it is spread over? From some of the figures it appears as if he only allows the plant a life of about 6 years.

I have recently had occasion to work out the generating costs of a 36-kW Diesel-engine plant. The manufacturers give the generating cost at full load as 0·37d. per unit and at half load as 0·44d. per unit, with fuel at 5¼d. per gallon and oil at 3s. per gallon. Allowing 3 per cent interest on capital and 5 per cent depreciation with a plant generating about 52 000 units per annum, the generating costs are at full load 1d. and at half load 1·6d. per unit. I think these figures agree fairly well with the curves in Fig. 2. The plant is used entirely for lighting, and in this particular case the supply authority cannot supply at less than 4½d. per unit. Since working out these figures I have found that on making a contract the oil company will supply the fuel oil at 4½d. per gallon, which will further reduce the cost of generating.

The author's views on the desirability of a common tariff and a flat rate will be welcomed by those of us who have to try to sell installations to the public. It is very difficult to explain the various rates available where there are six or seven different scales, and it is quite exasperating to the ordinary public who do not understand them. Electrical contractors and all those who have to deal with the consumer direct would be glad if a flat rate could be arranged, and I suggest that a sliding-scale rebate according to the number of units used would be the most suitable arrangement. If this system were adopted, both the large and the small consumers would know exactly what rate they had to pay.

Mr. T. Hodge: I should like to make a few remarks on the subject of the systems of charging for electrical energy now in use by supply undertakings. As regards the maximum-demand system, there is no doubt it is a

great advantage to have electricity "on tap," as in many cases only a small part of a factory is required to do some small job or even to complete an urgent order. When, however, there is a rush on, it does seem hard on the consumer that such a little thing as exceeding for 30 minutes the declared maximum demand of the whole plant should penalize him for the rest of the month or quarter, because in all probability the demand will not reach nearly that value again throughout the remainder of the period. On the other hand, some authorities who employ a standing-charge system penalize the consumer because he has not consumed enough energy.

I feel sure that if power consumers were shown a more practical way, and if group driving were encouraged, many of these difficulties could be surmounted, to the mutual benefit of both producer and consumer. It might even be possible to revert to a flat rate with discounts.

We do not seem to make very much progress with the cheapening of electricity; for, as far back as 1907, consumers of over 20 000 units per month could get a 25 per cent discount, with a further $2\frac{1}{2}$ per cent for monthly accounts.

Mr. W. A. Hatch: The importance of continuity of supply to some industries is such that a reduction in the cost per unit is of comparatively minor interest; for instance, a machine can produce in 24 hours a product worth £300 and yet only consume electrical energy costing 6d., but if a failure of supply occurs it may take 24 hours to restore the process to saleable production again.

Quite apart from financial loss, mention must be made of the manual and physical difficulties in restarting a process dealing with molten metal at about 1 450° C., and the mechanical damage which may be caused to water-cooled and air-cooled rollers, etc., when the electric supply fails.

It is therefore of interest to compare the results given by a local municipal power station before and after it became linked with the grid. During the 15 years ending October, 1933, only one complete shutdown occurred; this took place in September, 1925, and was of 15 minutes' duration. Since the station was linked with the grid, however, the failures set out in Table A have occurred. This record does not take into account many surges sufficiently serious to interrupt various sections of the works fed by synchronous machinery, or voltage-drops causing the no-volt release operation of motor starters.

TABLE A.

Date	Duration	Remarks
6th December, 1933	8 min.	Complete shutdown; grid lines tripped owing to fog
28th January, 1934	5 min.	Complete shutdown; heavy surge on grid lines
12th February, 1934	15 min.	Complete shutdown; grid lines tripped
13th October, 1934	14 min.	Complete shutdown

Mr. W. H. Howard: With regard to private plants in large residences, the general tendency amongst owners is to take a supply from the public mains immediately the opportunity becomes available. Some people, however, are a little difficult to persuade, especially when their plants are in good running order. The main argument put forward is "I already possess the plant for generating my own power and if I sell it now it will bring me a very small sum; capital charges and depreciation do not, therefore, concern me, and I can, in effect, generate for the cost of fuel plus a small allowance for maintenance." Costs arrived at in this manner can very easily be below the undertaker's flat-rate tariff.

Private generation of this kind is mostly for lighting purposes only, and the type of consumer who can afford it is often very anxious to modernize his household by adopting electricity for heating and cooking as well. This desire, together with the inconvenience of his own plant, nearly always leads to an early change-over, and in some cases considerable contributions are paid towards long service lines and special high-tension tappings.

The importance of good load factor has been mentioned already, but it seems to me that diversity is the factor which has the greatest influence on the domestic tariff. A low-tension network fed from a single transformer of 50 kVA capacity can often deal with about 100 small consumers with a fairly well-developed cooking and heating load and one or two small motors, the individual maximum demands being of the order of 4 or 5 kVA. If each separate consumer were to be charged at the grid tariff there would be some formidable annual fixed charges. Diversity is of similar importance, though in a less degree, in connection with the large power demands supplied direct from high-tension networks, and is an argument in favour of large distribution areas. An undertaking should certainly be many times larger than its largest power consumers for the fullest advantage to be taken of this factor.

We are all agreed on the necessity for a reduction in the present large number of tariffs in this country, domestic tariffs demanding the first attention. Now that all undertakings are purchasing in bulk from the Central Electricity Board with a unit charge of less than $\frac{1}{4}$d. and with the variable or copper losses at less than 10 per cent, there is no reason why power for domestic and other purposes should not be available to all consumers, urban or rural, at a running charge on a 2-part tariff of between 0·3d. and 0·5d. per unit. The standing charge should be variable, as truly as possible in proportion to the actual fixed charges in which the particular type of consumer involves the undertaking, but should not be assessed according to floor area, rateable value, or number of rooms. The latter are not true guides to the above costs, and give results which are unfair to some users and too generous to others.

Whatever the domestic tariff devised to supersede all

others, it should aim at reducing the large amount of metering required at present, and only one meter should be necessary per consumer. In addition to this, it should be capable of taking the place of the pure flat-rate tariff in an entirely voluntary manner, should include meter rent, and should possess the ability to reduce the running charge to 0·5d., or less, above a certain consumption a little greater than that required for lighting and wireless purposes. Further, the fixed charge should not be dependent on a measured maximum demand, for this worries the consumer, hinders development, and is not a criterion of the real cost to the undertaking.

Mr. W. J. Heaton: I am particularly interested in the paper from the point of view of its application to plastic brickmaking. The chief problem of the industry is drying, for which exhaust steam is used. A standard brick contains about $1\frac{1}{4}$ lb. of moisture, which must be evaporated before burning can take place. This means that over 2 tons of moisture must be evaporated from drying bricks per day, about 30 million cub. ft. of air being required to carry this moisture away.

I find that most of the steam engines in use to-day are over 30 years old and consume from 40 to 45 lb. of steam per h.p. per hour. Quite a number of brickmakers are now instilling fan equipments to transfer the waste hot air from the kilns to the drying sheds, which means that the amount of steam required is becoming less. I think the time has come for the brickmaker to consider the economics of turbo-electric or direct turbine drive, or a combination of both. The pass-out turbine is going to prove the most useful, and it will be the greatest rival to the grid unless some alternative in the way of steam or very cheap electric heating can be provided.

I should like here to draw a comparison between the costs of steam and electric drives. The costs per month are as follows: for steam, day cost £140, night cost £87, total £227; for electricity, day cost £182, night cost £22, total £204. The steam cost (fuel and boilerman's wages) for the day-time is the cost of power and drying steam, and for the night time of live steam blown under the drying sheds purely for drying purposes. The electricity cost for the day-time is an estimated one (based on the indicated horse-power of the engines), and for the night time is the cost of fanning hot air.

The difference is really very small, and it is most probable that in the case of electricity more drying heat will have to be provided.

The tariffs in operation in one district of the S.W. Lancashire brickyards are a barrier to the use of electricity. There is a maximum-demand charge of £4 per kW per annum, which brings the cost per unit to nearly $\frac{3}{4}$d. At this price the public supply cannot possibly compete with modern small generating plants.

Mr. L. C. Grant: It is not clear to me what size of plant the author is dealing with in the paper. Table 4 deals with plant of an average size of 10 kW, Table 2 with 1 400 kW, and the Summary mentions 400 kW. Most of the information I have here deals with plant of 400–1 500 kW.

My own experience is that the fuel used and the method of generating power are not for medium-size plant nearly so important as the load factor, and from the public supply authorities' point of view the most important thing is the 2-part tariff.

I have a few figures which may be of some interest. Taking first the case of a plant required to deal with an average load of 700 kW and a peak load of 1 200 kW, worked with the high load factor of 60 per cent, the average cost of 6 million units per annum at 60 per cent load factor is 0·320d. For a completely duplicated generating plant the cost is 0·402d. per unit. A very favourable public supply tariff which was offered for this work averaged just under 0·320d. per unit.

Another plant which has an average load of 1 200 kW at a load factor of 22 per cent, a power factor between 0·6 and 0·7, and an annual consumption of 3 million units, supplied from a generating plant with approximately 50 per cent spare plant, shows an average figure of 0·505d. per unit. It is questionable whether a 2-part tariff of the usual order could compete with this figure, and with the charges available in North Wales, where the works is situated, it would be even more doubtful. I am anxious to make the point that it is relatively unimportant whether steam turbines or small Diesel engines be used, until the load factor and its bearing on the charges have been investigated.

The author appears to regard pass-out steam turbines as being in most or all cases a proved case for a self-contained generating plant. With turbines of the 500–1 000-kW range it is very difficult indeed to make any appreciable economy as compared with a full condensing turbine until the pass-out steam reaches one-third or more of the full-load steam. Below that amount there is no case for a self-contained plant.

The author seems to consider that public-supply electricity charges are at too low a level. I am afraid I do not agree. Most charges in industry are cut very fine nowadays, and where the relative merits of a generating plant and a public supply are being compared the latter is usually cheaper by only a small margin, and often the decision is not made on the basis of economics at all.

The Author's Reply to the London, Manchester, Newcastle, and Liverpool Discussions.

Mr. J. A. Sumner (*in reply*): I should like to thank the various speakers for the useful criticisms which they have put forward and for the suggestions which have been made. At the same time I would take this opportunity of setting out a little more clearly the reasons for introducing the topics of tariff standardization and the merging of supply areas. An analysis of the discussions shows that interest has centred more upon the question of tariffs and the comparisons of different types of areas than upon the demonstrations which are made as to the cost of running private plants.

The paper was put forward with a two-fold purpose, which is aptly summarized by Mr. Carne. It was felt that as the cost of running private plants, particularly Diesel engines, had never, to my knowledge, been set out on a basis of annual load factor, it was important that the costs be demonstrated upon this basis before they could be compared with the cost of public supply,

which is usually charged upon a two-part tariff, using a fixed charge per annum for demand and a separate unit charge. Considerable data were in my possession as to the costs of running private plants, and it was considered that it should be possible to derive generalized two-part costs from these data so as to obtain the tariff or tariffs at which the public supply would be competitive.

One or two speakers have intimated that Part 2 of the paper, which briefly describes a particular semi-rural distribution area, should have been omitted. I think it would have been insufficient to have derived the generalized two-part cost for private plants and then to have merely stated that this derived figure was the competitive price of the public supply. It seemed necessary to show clearly, by demonstrating the full series of steps, whether a public supply undertaking could indeed afford to give supply at the above figure. To give this analysis in any detail meant that the whole of the costs of a particular undertaking must be obtained, and as I had available the costs for the particular undertaking which served as the illustration, it was obvious that these were the required facts. From this point several difficulties of construction arose. It was considered that to present Table 7 without indicating the main geographical and technical outlines of the undertaking would involve justifiable criticism, to the effect that discussion of the public-supply case had been stultified because of the provision of insufficient data. It was also decided to ensure that the somewhat high cost of supply from this particular rural undertaking should not be quoted apart from its context as the general cost for public supply, and the analysis of public-supply costs was therefore extended to include urban undertakings for which certain figures were available.

I should have to agree that the result of the study of the various classes of public supply undertakings has led me to introduce certain features into the paper which are more properly complete topics for separate papers and discussions, but the main object in writing the paper was to encourage the wider use of electricity. The extension of the study of public-supply costs revivified certain long-standing views which I held that this increase in sales is only to be obtained by some standardizing of tariffs and the merging of supply areas.

In each of the discussions interest has been focused upon certain common parts of the paper, and I propose to reply under main headings to these so as to avoid repetition. I hope that all the points raised by the various speakers will be adequately covered by the adoption of this method and that they will appreciate why only the detailed criticisms and inquiries are dealt with in the later and more brief replies to individual speakers.

Table 1, *Fig.* 1, *and Tables* 8 *and* 9.

I would reply to a number of speakers that Fig. 1 was not intended to be quantitative, but merely indicative of the general conclusions which can be derived from Table 1. This Table has been of great interest in showing the widely divergent conclusions which various speakers have drawn from it. I must still adhere to the conclusions which I have drawn in the paper from Fig. 1, i.e. that a rural undertaking is not more expensive to develop than an urban undertaking, up to a figure of approximately 70 per cent of full development of the possible consumers in the rural area. Up to this point it is contended that the costs of giving supply are approximately equal, but once the point is reached, there is an increase in the cost *for both urban and rural areas*. The rate of increase is, however, greater for rural than for urban areas, although it is not correct to say, as several speakers have suggested, that in rural areas 70 per cent represents the economic limit of development, or that the other 30 per cent of consumers can never receive a supply.

This similarity in cost of development between rural and urban areas appears to have been accepted by most speakers, but the proposal which I have made that rural and urban distribution areas should be merged so as to facilitate tariff standardization has been criticized on the ground that it would involve subsidizing the rural areas at the expense of the urban areas. But, obviously, such subsidizing would only be necessary for rural areas which had more than 70 per cent development, and then only to a limited degree at the expense of the urban areas which were still more highly developed. I do not think that this contingency is likely to give anxiety at the present moment, if we consider the relatively low stage of development in rural as compared with urban areas.

Mr. Hetherington states that he still shares the general opinion that rural electrification does cost more per consumer than urban electrification; he also raises the rather important point as to the relative yield from each of the two classes of consumers. As regards the first point, I must agree that it is a general opinion that rural electrification is the more expensive, but must disagree that it is a correct opinion; I am inclined to think that it is held because the matter has never received any exact quantitative investigation. I have recently obtained certain additional statistics which strengthen the surmises made in the paper as an outcome of Table 1; and these data show that it is possible to electrify rural areas at a cost varying from £22 to £17 per consumer on, or near, the route of mains.

I suggest that it is possible to electrify rural areas at a cost in the region of £20 per consumer on the route of mains, and that the cost is probably higher in urban electrification. The point is obviously of great importance and worthy of further investigation; if the view can be justified which I express, that rural electrification is as cheap, or cheaper, than urban electrification, the greatest objection to the merging of areas will have been removed.

Mr. Hetherington then raises the important question of the relative yield per consumer, in rural and urban areas respectively. Very few of the other speakers appear to have related this matter with the question of relative capital expenditure, whereas the two matters must essentially be considered together before any conclusion can be arrived at as to the financial relation between rural and urban electrification.

The comparison between a rural and an urban area requires us to consider the following factors, all of which are considered in the paper.

(*a*) The relative costs of electrification (Table 1).

(*b*) The relative stages of development (Table 8 and Figs. 7 and 8).
(*c*) The relative yield at, or near, full development (Table 9).

As regards the first factor, I have already indicated that, in my view, the rural area may be developed as cheaply as the urban area, to a fairly advanced stage of development. The second factor is only of temporary importance if we consider a unified, national, or even regional, distribution area; the standardizing of tariff bases and prices would certainly tend to equalize development. The third factor, the relative yield from an area, is of greater importance, but unfortunately very few data are yet available to us. The yield from any area will have some relation to the tariffs which are in force, but I am inclined to the view that the domestic yield will not differ materially between two areas, one rural and the other urban, which have similar tariffs. This leaves the questions as to whether there is a larger load, other than domestic, available in urban areas. Here I can only refer readers to the particular semi-rural area described in the paper, and suggest that the industrial power load available in rural and semi-rural areas is considerably greater than might have been anticipated. Table 8 of the paper does seem to provide a strong suggestion that the fully developed yield from a rural area can compare favourably with the yield from an urban area.

The whole trend of the discussion has been to suggest that rural electrification is fraught with grave dangers and is likely to be a burden upon the future. I do not think that this view can be substantiated, and I imagine that accurate data may shortly be available which will disprove it. I am quite convinced that the phenomenal growth of demands for power, etc., which so often occur in the most unexpected places, would justify much bolder steps being taken on a national scale, as regards both capital extension and reductions in prices. One purpose of the paper has been to show that the power and non-domestic load in a rural area is much larger in its potentialities than is generally realized, and whilst it may be true, as Mr. Hetherington remarks, that the yield from a rural domestic consumer is lower than for the urban consumer, I hold the view that the rural domestic load is only a small portion of the total rural load which is potentially available.

Standardization of Bases of Public Supply Tariffs.

It will be found that the somewhat qualified approval which is given to the suggestion made strongly in the paper, that public supply tariffs must be standardized to obtain an increased rate of sales, as well as the definite objections which are raised against the suggestion, are made by engineers in charge of public supply undertakings. It is of still greater significance that full approval comes from those engineers who represent the consumers of electricity, and I must adhere strongly to the opinion which I have formed as a result of many inquiries and of close contact with many actual and potential consumers, that a single national tariff basis is essential to obtain an increased rate of sales and to the fuller popularization of electricity.

Whilst there is much sympathy with the argument of several supply engineers that the tariffs to which they have devoted much thought and labour would not be suitable if applied outside the area of supply for which they were designed, I do seriously ask those engineers to study carefully the remarks of Messrs. Forbes-Jackson, Frith, and others, and then to decide whether the extraordinarily large number of tariffs, when compared with the short range in prices which is covered, does not justify a complete abolition of parochialism in tariff-making. It should be observed that the only definite suggestion which has been made on this subject in the paper is that contained in the last paragraph, in which a standardized national tariff *basis* is suggested. In my view, the imposition of a standardized price per unit common to all areas is impracticable and unwise as an immediate step. If, however, a standard national tariff *basis* were at once imposed a large increase in sales would soon follow; this increase in sales would have the effect of increasing "distribution efficiency," so that the cost per unit sold would reach a figure which would be approximately the same for all undertakings. Subject only to their different stages of development, the standardization of charges would then be naturally achieved for all except the special demands and "off-peak" loads. Tables 8 and 9 show clearly the closeness of costs which exist between rural and urban undertakings which are at a reasonably advanced stage of development, and it was desired to suggest that if all the undertakings were energetically developed, the cost per unit sold would become practically equal, independent of the type of area; standardization of prices would then be achieved satisfactorily. Confirmation of this matter has recently been published in a paper written for quarry owners.* In this paper, the author states that "if it is the aim of the Central Electricity Board to increase further the demands for bulk supplies, the first step to take is to inaugurate a *standard tariff* throughout the country . . . as it cannot be disputed that one of the most serious drawbacks to the further extension and use of bulk supplies is the extraordinary variation in the tariff rates offered in different parts of the country. . . ."

Merging of Supply Areas.

This suggestion was developed by means of Table 1 and Fig. 1, where the capital cost involved in giving supply to certain rural and urban undertakings is shown to be very approximately equal, and later, in Fig. 11 and Table 8, where it is suggested that the cost per unit supplied is approximately the same for rural as for urban areas, where there are equal stages of development and the power load has been developed. I do not think that the significance has been fully realized in the discussion, of the similarities in capital costs and cost per unit sold which are shown in the paper for *equally developed* rural and urban areas. If the similarity shown for the particular cases of Tables 1 and 8 are at all general, it would dispose of the criticisms of several speakers that merging areas would mean subsidizing the rural at the expense of the urban areas.

It is agreed, however, that equality in the costs of developing areas is not a reason for suggesting that such

* S. Millington: "Electricity in Quarries," *Quarry Managers' Journal*, July, 1935.

areas should be merged. The merging of areas is called for, in my opinion, because of the impossibility of achieving even a reasonable national standardization of tariff bases or prices unless and until this merging takes place. The discussions have shown that the difference in prices between undertakings is due to two main causes:—

(1) The differing stages of development.
(2) The need to suit tariff bases and prices to meet local conditions of diversity and load factor.

As regards the first point, a newly developed undertaking connects consumers slowly even in urban and other paying areas, and is almost certain to incur a deficit for the first 2 or 3 years of operation. As Mr. Fennell points out, it is bound to charge high prices in the early stages of few consumers, and thereby obtain profits as soon as possible, or forfeit the confidence of financiers. These prices are gradually reduced as "distribution efficiency" increases, but since each undertaking is unique in its stage of development so we are bound to have differing prices for each. With respect to the second point, several speakers have pointed out that merging of areas would result in a better diversity for the whole than for any of its components. It is therefore only in respect of the differing development that merging could possibly result in any subsidizing of a newly developed area at the expense of the more highly developed one, and it is very probable that the improved diversity of the merged scheme would neutralize even this amount of subsidizing.

Mr. Fennell, in common with several other speakers, agrees with me in deploring a policy of high prices and consequent small sales, and he suggests that the rural area of a particular undertaking can be developed with financial stability only if it is developed and merged with the already developed and paying urban area. This is, of course, the whole of my thesis, except that I am considering all the urban areas in this country as one unit and all the rural areas as the complement. I do not see how it is possible to avoid the undesirable policy of high prices and small sales (with its corollary of an increasing number of different prices for electricity between undertakings) until a central monetary fund is formed from which to finance newly developed undertakings. This suggestion on my part has been criticized at several centres, in common with the one which I have made that undertakings should be merged and prices averaged, on the grounds that it would lift the price of electricity to fully developed undertakings. I fail to see that this would be so, although I should agree that the effect would be temporarily to arrest the reduction in prices to consumers in fully developed undertakings. On the other hand, the newer undertakings would confidently commence operations with low prices, as they would be guaranteed against their initial deficit and would consequently reach a condition of financial stability more quickly because of the increased sales. It is here that the contentions made in the paper in Fig. 1 are of importance, since, if they are correct, we can say that up to a point of approximately 70 per cent of national development, the merging would only cause an arrested reduction in price in the urban areas but would give an accelerated reduction in price in the rural areas; and the prices in most of the fully developed urban areas are sufficiently low that a temporary stabilization of those prices would not arrest normal development.

Private Plants.

The discussion on this part of the paper has been confined chiefly to general statements which may be resolved under the headings given below. The statement made in the paper that private plant owners do not usually know their costs has, I think, been generally agreed, even by the private-plant protagonists. I should agree with the speaker who states that these costs are fairly accurately known where large private plants above 500 kW are in use; but such plants are not dealt with in the paper. It is very usual to find private-plant owners who state, firstly, that the public supply is too expensive to permit them to change over, and then inquire what the tariff will be. This hereditary view that electricity is expensive is one of the big obstacles to increased sales; and the present multiplicity of tariffs is tending to perpetuate the view.

Table 2 and Weighting Ratio.

Table 2 was included so as to indicate the component costs of typical works, in the hope that the Table would be useful to other engineers who are dealing with change-over problems. It has only an incidental connection with the various factories which are in the area of supply, and no conclusions are deduced in the paper from the Table except the weighting ratio, which is almost exactly the same for other works in the area which are not included in the Table.

The weighting ratio which I have used to weight the actual derived costs of private plant has received no direct criticism, despite the fact that it is the most important factor in the final comparison between private plants and public supply costs. Mr. Mallinson refers to "the weighting of private plant figures by provision of spare plant," but the weighting ratio used does not provide for spare plant, although this point is not made sufficiently clear in the paper. Mr. Swales appreciates the meaning more clearly, but I do not think that even he has realized the full significance of the matter. The real definition of the weighting ratio is illustrated in Fig. 3, and can be summarized by stating that if a manufacturer who has an averaged demand of $60 \cdot 6$ kW on a public supply undertaking (whose kW charge is based upon the average of all the demands in any given 30 minutes) were to change over to private plant, he would require to install an engine with a capacity of $1 \cdot 65 \times 60 \cdot 6$ kW, i.e. 100 kW, and this engine would only meet the ordinary series of peak loads on his works. He would then have no spare plant available, and if adequate spare plant of 100 per cent were installed, as it ought to be, with small engine capacities, the correct weighting ratio would be $3 \cdot 3$. I should agree that there is not the need to install 100 per cent spare plant where larger factories are concerned, owing to the fact that the running private plant is split up into a number of units; and, for the same reason, a greater advantage could be taken of the diversity of load demand. For small plant,

however, and particularly where only a single engine is available to meet the whole demand, the weighting ratio would stand at 1·65 minimum, and any spare plant which was installed would increase this figure.

Fig. 2 *and Table* 3.

The capital costs of Diesel electric plant, which are given in Fig. 2 and Table 3, have been strongly criticized by Mr. Mallinson and several other speakers as being too high. In reply, I must point out that Table 3 is an actual abstract from a Diesel engine maker's (1934) catalogue and that the capital costs for the larger plant in Fig. 2 are the actual manufacturer's tender prices (1933) for plant quoted for in competition with a public supply from the particular undertaking instanced in the paper.

It does not appear to have been sufficiently clearly shown in the paper that the generalized costs (page 316) of £4·95 per kW + 0·4d. per unit for large plant is derived entirely from Fig. 2, and the cost of £6·6 per kW + 0·53d. for small plant is derived solely from Table 3. It should be realized that the two sets of figures are derived quite independently of each other, and are then given as the upper and lower generalized limits of cost. There is not a sudden increase in cost at, say, 46 kW, and the small-plant analysis ceases at this figure simply because the particular manufacturer's list ceases at 45 kW. In general, there has been no detailed statement or criticism in the discussions which causes me to alter my opinion that these finally derived figures are correct. It is of interest to refer again to Mr. Millington's paper* as he speaks with considerable actual knowledge of the cost of operating Diesel engines and as an unbiased observer. He finds that "the costs of operating a small Diesel plant may be summarized as £5·5 per kW of maximum demand + 0·55d. per unit. . . ."

The above figures assumed 50 per cent greater installed plant capacity than the normal highest maximum demand on the quarry.

Example of Steam Plant for combined Power and Heating.

This example appears to be one of the most criticized and vulnerable portions of Part 1 of the paper; the most pertinent criticism is that made in the very well-considered and restrained remarks of Mr. Webber. The example was included in the paper because I desired some discussion which might have the effect of stripping this type of plant of the somewhat magical properties which are so often attributed to it. My experience has been that when an increase in load which necessitates additional plant occurs in an institution or factory where this type of plant is installed, the decision as to whether further private plant or the public supply shall be installed is nearly always made in favour of the private plant; and in two cases which came to my knowledge, the relative costs were not even ascertained, owing to a considered report on the matter stating that "even a super-power station cannot compete with a pass-out steam plant."

The discussion has confirmed the statement which I made in the paper that "where the heating and power demands coincide both in points of time and quantity" the pass-out cycle is a very efficient one, and is a formidable competitor to the public supply. But where an increase of power load occurs in a small institution or factory, the original highly efficient pass-out plant may lose its efficiency unless an equivalent additional heating load can be economically created; it is often the case that it may then be economic to scrap the pass-out plant and take a public supply.

Small Private-Residence Plants.

The statements which were made by practically all the speakers on this matter show full agreement that the public supply is more suitable and cheaper than private plants in small residences. I have not always found that the owners of these plants are willing to agree to this, when they are approached to take a public supply, and it was hoped that the publication in the paper of the data given in Table 9 may be of use to other engineers who have to deal with this problem.

London Discussion.

I would thank Mr. Hetherington for his support of my statement regarding the lack of knowledge by private-plant owners of their costs of running, and of the need to insist upon a knowledge of load factor before comparisons of private-plant costs can be made with a public supply based upon a 2-part tariff. Mr. Hetherington, in common with several other speakers, deprecates my suggestion that public supply tariffs should be based purely upon the competitive cost of operating private plants, but this course is common practice in many undertakings and is likely to remain so in the absence of some standardizing of public supply prices and greater knowledge of private-plant costs. I also deprecate the course, as I maintain that in many cases a mild form of blackmail is being levied on supply undertakers when a threat is made to replace the public supply by private plant unless lower prices are quoted for the former supply. In many cases, incorrectly low figures are quoted of the operating costs of private plant, and with the hundreds of different prices charged by undertakers for electricity (most of them being confidential) it is not a difficult matter for a consumer to quote hypothetical figures and demand to be supplied at that lower price. As regards 2-part costing, the method adopted in Table 7 may not be orthodox, but I would mention that when I made the analysis by far more complicated methods, using orthodox methods, there was very little difference in results; this experience is borne out by Mr. Thwaites. My method has, at least, the merit of being simple, and if it leads to results which are very close to those achieved by elaborate costing methods, I would suggest that it meets necessary requirements.

Surely the position is, that in costing electricity we must either carry the process to its logical end or we must be arbitrary and stop at some point in the process. If we complete the process, then no two consumers, even in the same undertaking, will obtain electricity at the same price; and if we become arbitrary we have not a standardized definition of arbitrariness. The remaining remarks of Mr. Hetherington are dealt with earlier in my reply under the various headings.

The first portion of Mr. Thwaites's remarks I have

* *Loc. cit.*

already dealt with in the preface to the reply, and I am very glad to see that he arrives at similar tariffs to those derived in Table 7 although he adopts a different basis of calculation. As regards the £10 000 proportion of the £16 900 which Mr. Thwaites assumes is attributable to high-tension consumers, I would point out that the actual figure is very much lower than this, since the £16 900 is merely the total of the revenue charges, exclusive of cost of current.

In reply to Mr. Sparks, I think the fact is quite clearly shown that the paper does not deal with plants exceeding 500 kW. On the question of plant life, I would stress that in the particular cases which he quotes, obsolescence, or economic life, is chiefly considered, as this factor is likely to be far more important than mechanical life: this is due to the abnormal growth of electricity supply and the consequent reductions in cost. I do not agree that, because of the grid system, the urban areas subsidize the rural areas, although I should agree that it is quite true that the tendency to equalize the wholesale prices in rural and urban areas is largely due to the advent of the grid, it being one of the principal objects of the 1926 Act to achieve that end. It is, however, misleading to suggest that the rural areas are subsidized by the urban areas, because that would imply that the price in the latter was increased in order to reduce that in the former area. Since the 1926 Act contains specific protective provisions, such as Section 13, which prevent any such increases, this is clearly impossible. The operations of the grid, in fact, give rise to savings which can be, and are, applied in lowering the price in rural areas without increasing the price in urban areas; and I maintain that the formation of an Electricity Distribution Board, which could finance distribution extensions from a central monetary fund obtained from the savings which the Board could effect and in part from the large annual surplus available, would have a similar effect.

I am not quite clear why the conclusions on page 332 of the paper are "sweeping statements" because they are based on 65 per cent average load on the engine. As a general case, there are very few plants of the type mentioned in the paper which have an average load of 65 per cent on the running plant or even 32½ per cent of the installed capacity, in the somewhat rare cases where complete spare plant is installed. Finally, I think Mr. Sparks has probably overlooked the former part of paragraph 2, page 332, which refers specifically to the larger Diesel engines before dealing with the six engines in Table 3.

I would thank Mr. Bolton for drawing my attention to the figure of £3·25 in Table 7; this should be £3·50 and has been corrected for the *Journal*. The diversity of 1·55 for h.t. supplies is quite correct, as it is a figure which is capable of fairly exact measurement by means of kVA meters at the h.t. consumers' terminals and of kVA at the one large outgoing main substation from which the h.t. supplies emanate. For a system that is, as yet, largely undeveloped on the l.t. side but which has a highly developed h.t. industrial load, it is quite possible that the diversity on the l.t. side may be lower.

I appreciate very much the carefully considered remarks of Mr. Webber and also those of Mr. Crocker, although I cannot agree that the reference in the paper to the pass-out steam case was pointless. It serves, at least, to illustrate my contention that many private-plant owners do not know their real costs for the reason that the owners of the pass-out plant which is used as an example demanded a public supply at "not more than ½d. per unit, as otherwise their own plant would be cheaper to run."

Mr. Forbes-Jackson's remarks illustrate admirably my comments that the large number of prices for electricity is causing confusion, and worse, in consumers' minds. His remarks are all the more pointed in that they represent the view of a trained electrical engineer who is in the position of a consumer of large amounts of electricity.

Mr. Molle's contribution is valuable not only because of his confirmation from experience of the various data given in the paper, but chiefly for his summary of a wide experience in consulting work. I should imagine that he is including cases which cover experience with private plants installed outside England, judging from his remarks regarding the difficulty of obtaining skilled attention to plant, and I should fully agree that in such cases the pure economic (cost) grounds are insufficient to determine the right type of plant to be used. In any case, either for England or abroad, what have been termed "the invisible exports," i.e. the non-financial items which affect the comparison, cannot be left out of consideration and may be the determining factor in the decision as to which type of supply to adopt. Generally, I am inclined to think that they will favour the public supply.

Manchester Discussion.

Mr. Mallinson and I are in perfect agreement except on the question of private plants and public supply costs. I do feel, however, somewhat disappointed in the educative value of the paper when he suggests that private plant owners have no need to know their "cost per unit." Even if it were correct that the total annual cost of power from the public supply is not known until the end of the year, I fail to see how the total annual cost of running the private plant can be ascertained earlier. As regards the future trend in crude-oil costs, I think that the 1935 Budget has already indicated that political needs can prevail over economic laws. The substance of Mr. Mallinson's remarks on private plants is dealt with under the prefatory general headings to the reply.

Finally, it would appear that Mr. Mallinson does not disagree with my conclusions as to capital cost if the plant-costs which I give can be shown to be correct. In view of my earlier statement giving the source of my information on this point, I feel that Mr. Mallinson's only reservation can quite properly be eliminated.

I must thank Messrs. Swales and Allcock for their support, all the more valuable because it is based on a very wide and detailed experience. The instance that Mr. Allcock quotes in support is of importance because he gives all the essential component figures of cost which are required before the usefulness of the instance can be judged. In reply to Mr. Swales's inquiry regarding Table 2, the figures in col. 5 are the actual demands of the particular consumers, measured on the h.t. side. In order to include all the losses including transformer

losses, measurement must be on the h.t. side. It is agreed that the load factors in col. 5 would be higher if the various losses were excluded, but it would not present the true facts of public supply, which would then have been favoured unduly. It was not considered that the load factors in col. 7 were unduly high, considering the type of factory or works concerned, but in any case, the figures in cols. 3 and 5 are taken from the meter cards of the various consumers.

I think Mr. Fennell has confused the case which I have given for the particular undertaking (Figs. 7 and 8 and Table 8) with that given in Fig. 11 for a much larger number of quite separate undertakings. He will doubtless have noticed that, for the particular undertaking, I have specifically mentioned and demonstrated that an increased power load does reduce the total and domestic charges, as would be expected if the hypothesis is true that increased sales improve distribution efficiency and thereby reduce total costs. Both at the beginning and end of the paper I accept this hypothesis as being correct, and it is a part of my thesis that the lack of reduction in domestic costs as shown in Fig. 11, despite increased power sales, indicates that tariffs are based upon expediency and should be co-ordinated on a national basis.

Mr. Frith's remarks regarding the need for tariff unification are valuable, as he has had many years' experience on behalf of consumers. I, also, see no reason why supply engineers should not give lighting at power rates; if the tariff in force is a 2-part tariff based on load factor, it does not seem to me that there is any need to discriminate between lighting and power, or any other class of demand.

Newcastle Discussion.

I gather from the very logical opening remarks of Mr. Carne that he is in full agreement with the two-fold objects of the paper, and I particularly appreciate his endorsement of the method of cost analysis which I have used. I agree that the method does not attempt to achieve highly accurate costing and that complications would ensue if it were used for detailed analysis. But, with all deference to Mr. Carne, I must submit that highly accurate and detailed costing must become far more catholic in its application, unless it is to cause a still greater parochialism in tariff-making.

I appreciate also his suggestions as regards Fig. 11, but I still do not find the conclusions that I had hoped for after drawing the families of curves which accrue from the re-drafting. Is it not true that certain undertakings boast that "their load is made up of the lighting load," and others that "theirs is based upon the power load," and so on, each undertaking having given special facilities to encourage a particular type of load? It is for these reasons that we have uneconomic flat rates of 2d. and 3d., and probably we have here the whole reason for the immense number of tariffs which are in vogue throughout the country. But the ideal is, surely, to give a "cheap and abundant supply" without any favouring (and often subsidizing) of a particular class of load at the expense of other classes. Broadly speaking, it is true that the smaller the undertaking, the greater is the need to encourage certain classes of loads in order to retain a good system load factor, and this is the strongest argument in favour of merging small into large undertakings.

I do not agree with Mr. Clement's argument which I understand to be that if the domestic tariff in an industrial town is lower than the domestic tariff in a residential town, the average price will be higher in the industrial town if less units are sold. Further, because power consumers are shown in Fig. 11 to be paying less than 1d. per unit, it does not follow that it would not be economical for them to pay 1d. per unit. It would obviously be dearer, but, as I maintain, and as is demonstrated in the paper, if the only alternative source of power costs a minimum of 1d. per unit for general cases, then it is still economical to pay 1d. per unit for the public supply. I do, however, agree most fervently with Mr. Clements that electrical energy is a raw material and an economic necessity to the country, and the cheaper it can be sold the better for our well-being. The demonstration of methods as to how this desideratum can be achieved is the sole object of the various suggestions in the paper.

The sale of electrical energy to modern collieries is, as Mr. Burns suggests, not easily effected, and may be classified as a special case which the paper hardly purports to cover. I must agree with him that this is a subject which requires special attention, but I see no reason why such a special case is not subject to the beneficent effects of standardizing tariff bases, or even prices.

In reply to Mr. Cross, the annual standing charges per kW (Table 4) vary from £12 to £5 per kW, and, as was stated in the paper, it was necessary in many cases to accept the private-plant owner's allocations as to fixed and running charges. I should, of course, agree that if the owner could use the plant for other purposes than lighting, he would reduce his costs per unit owing to the increased annual load factor. The figure of £150 per kW is the figure given in the plant-manufacturer's list, and is for the plant supplied and delivered but not fixed. I suggest on page 312 that "high prices are almost inevitable in the initial stages of an undertaking so long as the financial results of each undertaking are kept separate." I do not, however, agree that high prices are wise in the initial stages, as they hinder sales. The merging of undertakings is recommended in the paper in order to bring about the result that we both desire.

In reply to Mr. Phelps, I regret that the type of plant was not stated for each case in Table 4, but his conclusions are correct except for col. 1, which refers to a petrol-paraffin plant. The costs of the plant in col. 7 were extracted from a rather biased local authority, and this may account for the somewhat low costs. I very cordially agree with Mr. Phelps as to the difficulties of obtaining costs of this nature. I am pleased to have his approval of the use of 3-phase as compared with single-phase lines, and I regret that the nature of the paper precluded the addition of detailed distribution costs, as well as certain other technical data which I should have preferred to include. The costs which Mr. Phelps requires are given in Table B, and are in no way special costs; they refer to recent l.t. distribution

carried out in rural villages, and to the 11-kV lines associated with the supply to those villages. No claim is made that the costs are unduly low, but it is claimed that they are accurate owing to the special steps which are taken to keep a careful and current check upon the cost of all capital work. All the cost components are included, these being materials (exclusive of switchgear), labour, tree-cutting, compensation paid for trees, etc., with an overall charge of 10 per cent to cover supervision and administrative charges.

TABLE B.

Location	Total length	Component costs			Total cost	Cost per mile
		Material	Labour	Tree-cutting, compensation, and other charges		
11 kV, 3-phase, 3-wire, 0·025 sq. in. steel-cored copper.						
	miles	£	£	£	£	£
M.169	1·0	151	70	28	250	251
M.191	1·75	247	80	29	356	205
M.200	0·41	53	27	10	91	225
M.150	19·15	3 068	1 340	947	5 356	279
M.154	5·15	787	393	281	1 462	284
M.158	1·19	198	108	113	405	340
Total	28·65	4 504	2 018	1 408	7 920	276*
400/230 volt, 3-phase, 4-wire, 0·05 sq. in. copper lines.						
M.148	0·32	61	17	14	93	292
M.150A	6·82	1 192	557	434	2 184	320
M.154A	2·27	337	173	215	727	320
M.156	0·06	15	8	5	29	457
M.161	0·11	63	15	9	88	697
M.171	0·35	56	28	16	101	288
M.176	0·5	44	26	15	85	172
Total	10·43	1 768	824	708	3 307	316*

* Average.

It is rather refreshing to be accused by Mr. Jackson of having unduly favoured the crude-oil engine manufacturers, but I should agree with him that the annual load factor of the plant in most factories working an 8-hour day is in the region of 17 per cent.

The support from Mr. Smillie is also very acceptable, as it is based upon a wide experience of the subject. I am glad that Mr. Smillie stresses that no two cases of power-sales negotiations are identical in actual practice, as I was precluded from dealing with this point owing to the general nature of the paper.

LIVERPOOL DISCUSSION.

In reply to Mr. Hogg as regards Table 1, I would rather put it that up to an approximate figure of 70 per cent of electrified dwellings in a rural area, the capital cost is the same as for electrifying 70 per cent of the dwellings in the urban area. The economic limit in the rural area is not necessarily reached then, but it would be more expensive to electrify the remaining 30 per cent in the rural area than the 30 per cent remaining in the urban area; I think that this is the view which Mr. Nelson takes upon this particular matter.

The increase for Diesel plant from £3 to £4·95 per kW is not on account of spare Diesel plant but owing to the fact of the maximum demand charged for by the public supply being the averaged demand over 30 minutes, whereas Diesel engines must be installed of such a size as to meet the highest momentary peak of which that averaged demand is composed. This relation is illustrated in Fig. 3. The " long-pin " insulators in question consist of insulators rather larger than usual insulators (3-shed type) but the large clearance from conductor to the earthed cross-arm is provided chiefly by having extra long pins; the actual distance between conductor and cross-arm is $10\frac{1}{2}$ in. The earth resistance at each pole on the system is measured periodically for all lines except the very small rural lines, and very variable results are obtained. With very few exceptions, however, the single earth-plate has been of sufficiently low resistance to permit of the fault current being at least equal to that required to operate the tripping relays. Trouble due to lightning has been experienced, and lightning arrestors of the zinc-oxide pellet type have been—and are being—installed. It is difficult to measure the effectiveness of these arrestors, but lightning trouble has been almost completely eliminated on those lines where the arrestors are installed.

I am very gratified that Mr. Nelson agrees with me as to the practicability of standardizing tariff bases, and particularly that he feels that it should soon be possible to obtain some measure of equality in the selling prices of electricity.

In reply to Mr. Harrison-Watson, I would state that Table 4 and Fig. 12 refer to particular cases of small private-residence plant. Figs. 4A and 4B refer to the general case for small automatic petrol plant, and the data for the curves are obtained from a manufacturer's catalogue. Interest and depreciation are taken at 12 per cent. Whilst it was not my intention in the paper to discuss the best form of public supply tariff, I think that a sliding-scale rebate system would suggest itself, but it is very probable that the unit charge would need to be coupled to an annual fixed charge and thereby permit of adherence to a two-part tariff. It seems very probable that the difficulties which lie in the way of determining the correct basis for the fixed charge will ultimately lead to a common service charge derived as in the lower two columns of Table 9. These remarks apply to a great extent to the points raised by Mr. Hodge.

Mr. Hatch postulates a case of supply which cannot tolerate an interruption equal to 1/10 000th part of the time that supply is required. I should agree that this condition can probably be achieved by the use of private plant, but only at a much greater cost than is visualized for the private plant dealt with in the paper. I should also state that equal continuity can be obtained from the public supply if the consumer is willing to pay his proportion of the additional cost of ring mains and duplicate feeds. The instance of the failure of supply from the grid should be considered over a much longer period than Mr. Hatch has taken in his example.

Mr. Howard's remarks amplify in very considerable detail a number of points which I should have liked to deal with at greater length in the paper. This is particularly so where he stresses that diversity is an argument in favour of large distribution networks, and, later, the need for reducing metering to the minimum.

The contribution of Mr. Heaton is of value as it deals in a most constructive manner with the very special case of brick-making. But he must bear in mind that the average brickworks is so small, and has such spasmodic working, as not to justify the installation of pass-out steam plant. In the case of the larger brickworks the annual charges on the heavy capital expenditure required to deal with the rather elaborate fan equipments and pass-out turbines may become a serious item for consideration.

A STUDY OF THE INDUCTION WATT-HOUR METER, WITH SPECIAL REFERENCE TO THE CAUSE OF ERRORS ON VERY LOW LOADS.

By T. Havekin, B.Sc., Ph.D., Associate Member.

(*Paper first received* 20*th February, and in amended form* 16*th June*, 1934; *read before the* Meter and Instrument Section 1*st February*, 1935.)

Summary.

During the past 10 years meter designers have met with considerable success in extending the working range of the induction meter in the overload direction, and this paper opens with a brief review of some of the more important considerations involved in such improvement. It is shown that this subject of curve extension ultimately compels some consideration of the conditions which govern the registration on low loads, and it is suggested that the meter curve might possibly be further extended by a better understanding of the factors which control low-load registration. In this connection a mathematical theory is developed, the experimental determination of the various torques is described, and the low-load curve of the meter is derived from calculations. The theoretical curve is compared with the curve obtained by an actual test on the meter. There is sufficiently close agreement between the curves to leave little or no doubt as to the validity of the mathematical treatment, and this in turn shows clearly that the anti-creep device is the cause of the large registration errors on very low loads.

(1) Introduction.

The calibration curve of the single-phase watt-hour meter is frequently referred to simply as the "curve" of the meter, and this expression will be used throughout the paper where there is no likelihood of ambiguity.

Though B.S.I. recommendations* concerning limits of error in alternating-current meters make no reference to loads outside the range $\frac{1}{20}$ to $1\frac{1}{4}$ times full load, yet there has been a marked tendency during recent years to extend the working range of induction meters in the overload direction.* It is not unusual now for manufacturers to claim sustained accuracy up to twice full load, whilst curves have been published indicating quite small errors in registration up to, and even beyond, three times full load. The curves in Figs. 1, 2, 3, and 4, show clearly the considerable and progressive improvement which has been effected in the accuracy of registration on overload by one British manufacturer since the year 1921, and these curves may be regarded as indicating in a general way the trend of modern meter design, not only in England but also in Canada and the United States.

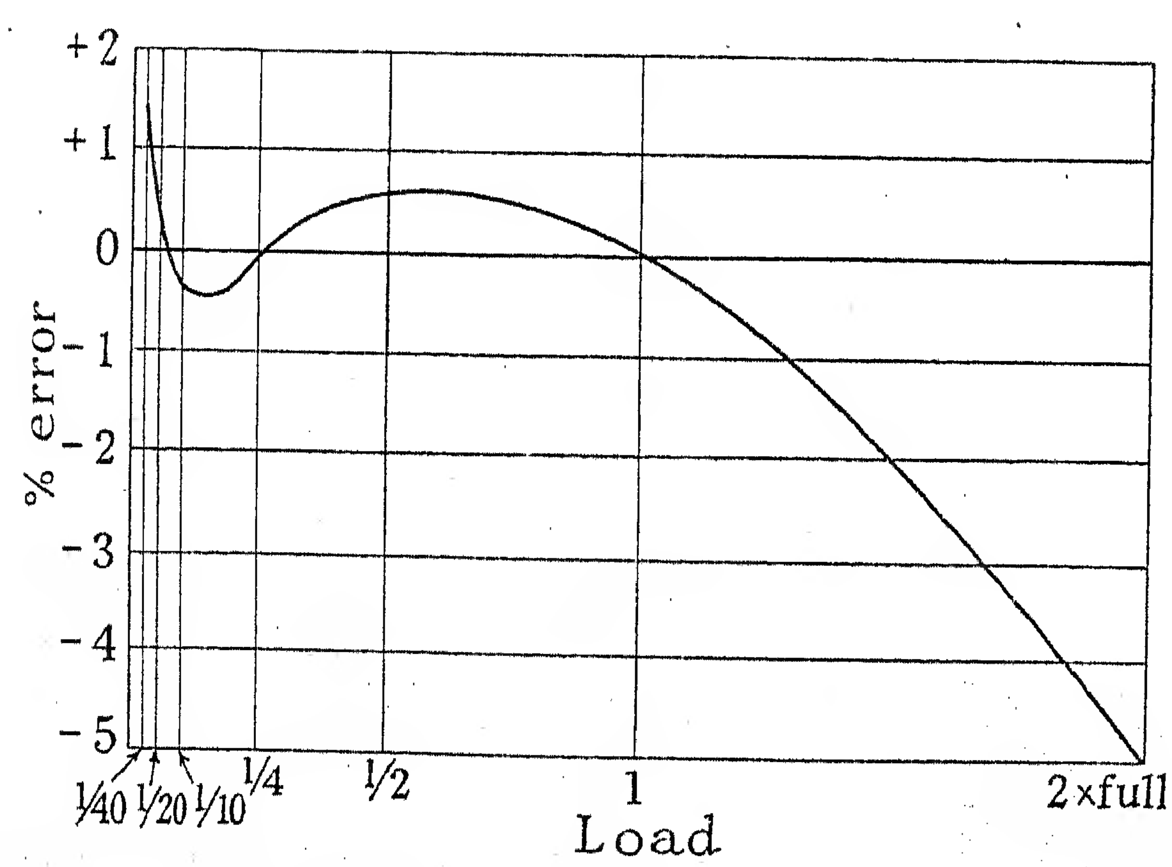


Fig. 1.—Accuracy of registration, 1921.

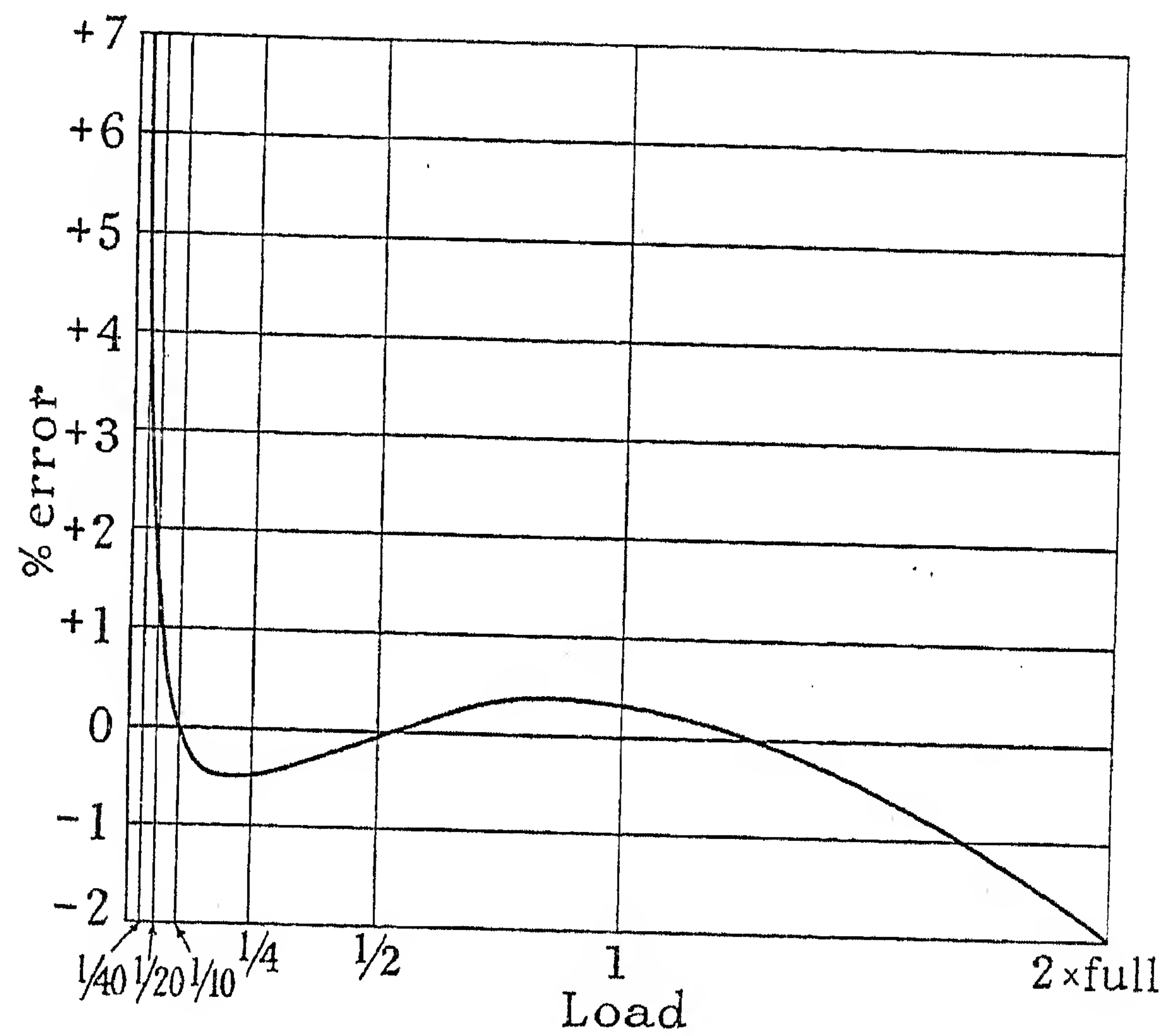


Fig. 2.—Accuracy of registration, 1925.

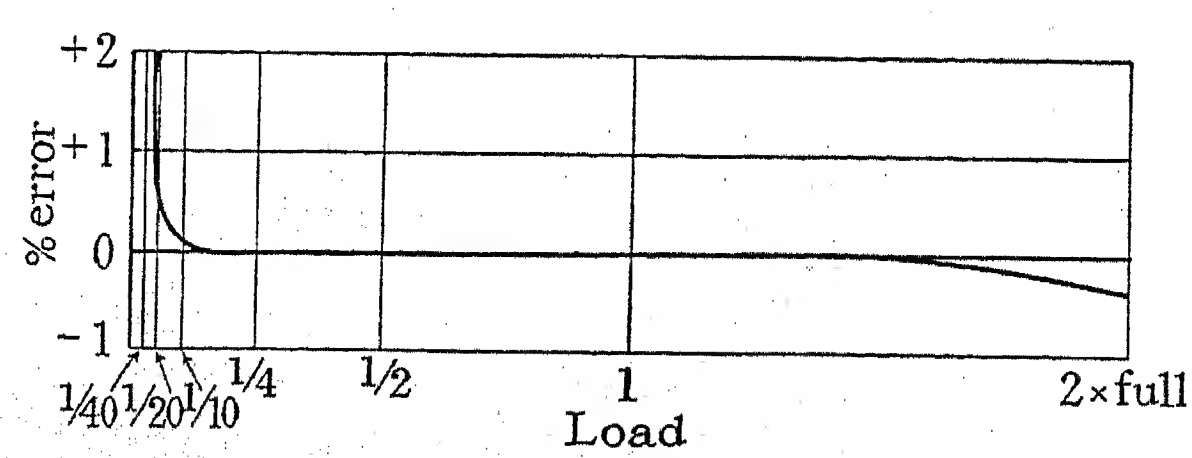


Fig. 3.—Accuracy of registration, 1928.

We can conveniently refer to that part of the curve corresponding to loads above full load as the overload

* B.S.S. No. 37—1930, p. 15.

* J. L. Carr: *Journal I.E.E.*, 1929, vol. 67, p. 865; E. Fawssett: *ibid.*, 1931, vol. 69, p. 546.

characteristic; in view of the accomplished accuracy in registration in this region of the curve, it is obvious that the factors which influence the overload characteristic are well understood. Nevertheless, a brief review will render this study more complete, and should place in better perspective the main subject of this paper.

In order to avoid cumbersome descriptions the flux provided by the voltage coil of the meter will be referred to as the shunt flux, and that provided by the main current coil will be called the series flux.

Though there are several factors which tend to deform the overload characteristic, it is well known that the most potent cause of negative errors on overload is the braking action of the series flux. If I is the current, and $d\theta/dt$ is the angular velocity of the rotor, then the brake torque due to the series flux is proportional to $I^2 d\theta/dt$; but since the angular velocity of the rotor is approximately proportional to the current, assuming constant voltage, the series brake torque may be said to be approximately proportional to $(d\theta/dt)^3$. It is obvious, therefore, that a reduction in the normal working speed will lead to a decrease in the magnitude of the unwanted retarding torque.

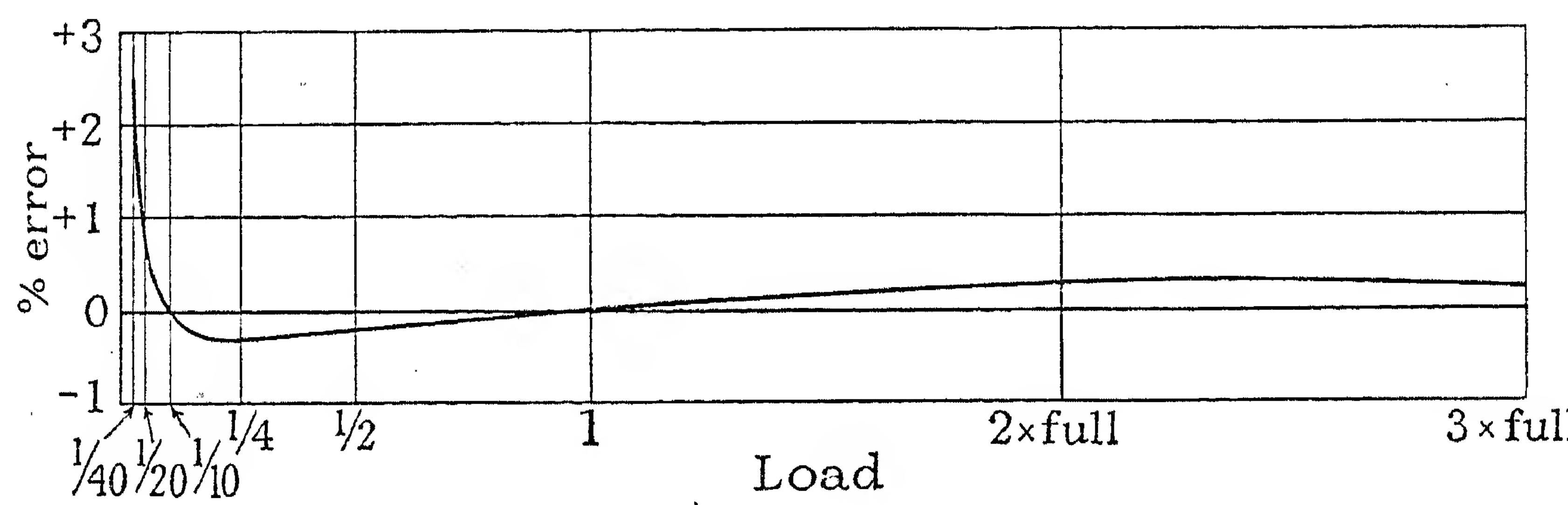


FIG. 4.—Accuracy of registration, 1934.

During the past 10 years, marked improvement in the overload characteristic has been achieved, either (*a*) by employing some form of compensation or (*b*) by reducing the full-load speed of the rotor.

In general, compensating devices are not desirable. To mention only one defect, their correcting action is usually dependent on the magnitude of the main current, and so the compensating effect varies with the power factor of the load.

A reduction in the full-load speed of the rotor in order to secure the desired improvement is obtained either (i) by the use of a more effective braking system, or (ii) by reducing the normal full-load torque. Obviously, a combination of (i) and (ii) will serve the same purpose.

A better braking system may necessitate the introduction of a larger-diameter rotor disc. The brake magnet can then be set to act at a greater radius, and thus produce an increased retarding torque; but the larger disc leads to an undesirable increase in the overall dimensions of the meter, and may also result in a heavier rotor. If this method is rejected we may arrange for greater retarding torque by increasing the permanent-magnet flux. This demands the employment of a larger brake magnet, or the use of more costly material such as cobalt steel, instead of tungsten steel as is usually used. In either case, the improvement is accompanied by increased cost, and this would be regarded by most manufacturers as a serious disadvantage. It may be remarked here that the brake magnets used in many meters are probably less efficient than they should be, in proportion to the weight of steel used. The braking effect of a magnet, on the type of disc common to most induction meters, is influenced very much by the shape of the magnet pole-face, but no simple law appears to have been established hitherto which defines the relationship between braking effect and pole-face shape. This is the more remarkable because the permanent-magnet system, whether consisting of a single magnet or of a pair of magnets as is sometimes used, is one of the most costly of the meter components. Any theoretical development or empirical rule which would lead to a more economical use of magnet steel in this connection would be of great value.

Dealing now with (ii), the full-load torque may be reduced by decreasing the number of turns of the main current coil. At a given current, the fewer ampere-turns result not only in a lower rotor speed but also provide less flux, and the combined reduction of both speed and flux leads to a marked diminution of the retarding effect of the series flux.

It is probable, however, that the lower full-load speed, which is to be noted in many of the most modern induction meters, has been obtained by having recourse to a combination of methods (i) and (ii). To rely on a reduction of the series ampere-turns alone has not, in some cases, been advisable, because the accompanying fall in the value of the main driving torque has undesirable consequences in the low-load region of the curve. Though the driving torque due to the independent action of the shunt flux, as well as the retarding torque due to friction, may be of negligible magnitude compared with the full-load torque, yet both small torques assume much greater significance when compared with the torque corresponding to $\frac{1}{20}$ full load. Hence a prudent correction of overload errors, by means of a reduction of the main driving torque, will be marked by a careful regard for the possible deleterious effects in the low-load region of the meter curve.

We see, therefore, that although overload improvement may be the immediate objective, the designer is compelled ultimately to pay some attention to the low-load end of the curve, and a little reflection in this direction soon reveals a rather different aspect of the subject of curve extension.

As an example, if a given meter registers accurately

only between $\frac{1}{20}$ and full load, it can be converted into a double-load meter if the curve between $\frac{1}{40}$ and $\frac{1}{20}$ load is brought within acceptable limits of error. Whether an extension of the curve in the low-load direction is either possible or desirable certainly calls for some inquiry, though, apart from this, a clear understanding of the conditions which govern the low-load performance of the meter is highly desirable. It is to this phase of the subject that the remainder of this paper is devoted. We shall consider in detail all the factors which influence the motion of the rotor over a load range from starting current up to $\frac{1}{20}$ full-load current.

(2) Determination of the Low-Load Curve of the Induction Watt-Hour Meter.

Two meters (A and B) made by different manufacturers were tested in order to emphasize the rather remarkable

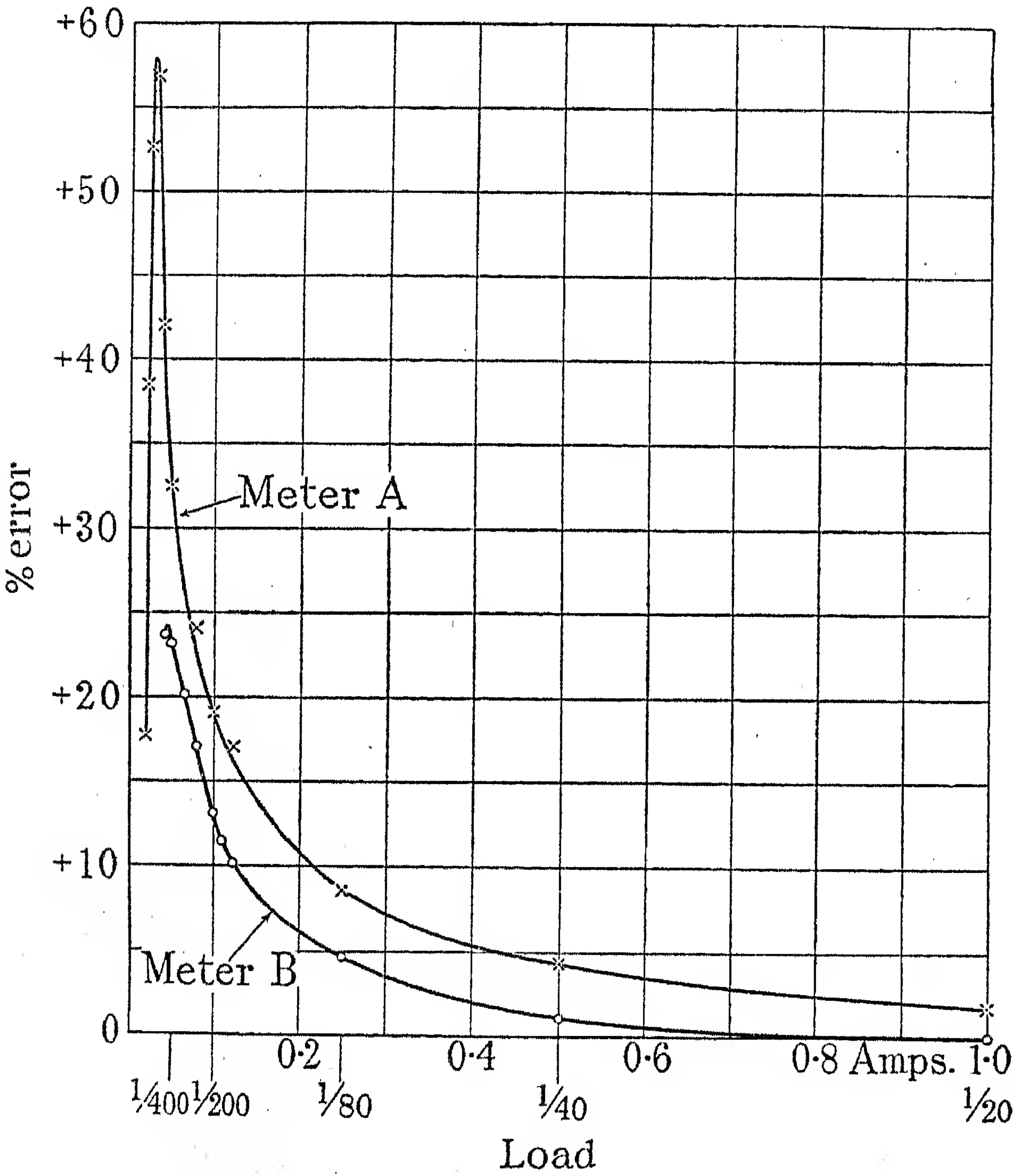


Fig. 5.

character of the low-load curve, and to show that the general form of the curve is not peculiar to the products of one manufacturer. Most modern induction watt-hour meters yield corresponding curves which bear a general resemblance to those found by test.

The full load at unity power factor and 50 cycles per sec. for each of the two meters is 200/250 volts, 20 amperes. The tests were conducted at 200 volts and unity power factor. The curves are shown in Fig. 5.

The standard wattmeter used for the tests had four current ranges, and was first checked against a potentiometer.

Though the errors shown by curve A are greater than those indicated by curve B, yet the curves exhibit a general resemblance in form, and this, in conjunction with the fact that the errors in both cases are all positive, suggests similar disturbing influences in both meters. It is quite clear that large errors are to be expected if we attempt to extend the working range of the meter into the low-load region without first eliminating the vitiating influences, and a desire to accomplish this would provide an adequate motive for our investigation. Quite apart from such a consideration, however, it is suggested that the remarkable shape of the curve is sufficient to stimulate inquiry into the cause of errors of such magnitude as those revealed.

Our problem, then, is to study in detail all the forces acting on the meter rotor under low-load conditions, to deduce therefrom the equation of motion of the rotor, and to derive from the solution a theoretical low-load curve. The degree of agreement between the theoretical and experimental curves will enable us to judge with what confidence we may expect the analysis to expose the cause of the low-load errors. Meter B was chosen for the research.

(3) Theory of the Single-Phase Induction Meter when measuring Very Low Loads at Unity Power Factor.

Torques Acting on Rotor.

We will consider one by one all the torques acting on the rotor under working conditions, and the expressions used to represent the torques will be adapted to low-load conditions. The stated expressions for the torques will be amply justified later, when we deal with the individual measurements.

(i) *Main driving torque.*—If the voltage and current are represented by E and I respectively, and the power factor is unity, this torque is proportional to the product EI. As the voltage is constant,

$$\text{Torque} = \mathbf{T}_1 = K_1 I \text{ dyne-cm, where } K_1 \text{ is a constant.}$$

(ii) *Shunt torque.*—The shunt flux exerts an independent torque through the medium of the low-load adjustment. It is true that this torque serves to compensate for friction, but it is by no means of constant magnitude throughout one revolution of the rotor. The shunt torque may be regarded as being, in general, greater then the friction torque, but continuous rotation, when the voltage circuit alone is excited, must be prevented; for besides constituting an obvious violation of ethical principles, uninterrupted registration on voltage alone would prove a fruitful source of complaint by vigilant consumers. To provide for this contingency we have some form of anti-creep device such as a hole in the disc. It is well known that the effect of the hole, especially when in close proximity to the shunt stator, is to distort the eddy-current path, with the result that the shunt torque varies with the position of the hole. We see, therefore, that the torque is a function of θ, the angle of rotation of the rotor, and we shall express this provisionally as

$$\mathbf{T}_2 = f(\theta) \text{ dyne-cm.}$$

(iii) *Driving torque due to series flux.*—Unless the series stator is fixed in a given position of symmetry with respect to the rotor, there will be an independent driving torque due to the series flux, but as careful adjustment is always made by the manufacturer with the object of reducing this torque to zero we have

$$\mathbf{T}_3 = 0$$

This relation was verified experimentally in connection with the meter with which we are concerned, and the analysis is developed with due regard for this condition.

(iv) *Torque due to permanent magnet.*—The permanent-magnet retarding torque is proportional to B^2, where B is the flux density in the magnet gap, and to the rotor speed $d\theta/dt$. Hence $\mathbf{T}_4$ is proportional to $B^2 d\theta/dt$, but, since B is constant for a given magnet,

$$\mathbf{T}_4 = K_4 d\theta/dt \text{ dyne-cm}$$

where K_4 is a constant.

(v) *Retarding torque due to shunt flux.*—Owing to the motion of the rotor disc through the shunt flux, eddy currents are created in the disc; these are proportional to the voltage E and the rotor speed $d\theta/dt$. The interaction of the flux and eddy currents results in a retarding torque $\mathbf{T}_5$ which is proportional to $E^2 d\theta/dt$. As the voltage is constant during the test, we have

$$\mathbf{T}_5 = K_5 d\theta/dt \text{ dyne-cm}$$

where K_5 is a constant.

(vi) *Retarding torque due to series flux.*—It follows from a consideration of (v) above that the series flux will also give rise to an independent retarding torque which can be expressed as

$$\mathbf{T}_6 = K_6 I^2 d\theta/dt$$

where K_6 is a constant and I is the series current.

Both speed and current are so low within the range of our tests that $\mathbf{T}_6$ has no appreciable magnitude (see Fig. 11). Hence

$$\mathbf{T}_6 = 0$$

(vii) *Bearing-friction torque.*—The friction torque due to the registering mechanism and rotor bearings may be assumed to be constant. Therefore,

$$\mathbf{T}_7 = F \text{ dyne-cm}$$

(viii) *Air-friction torque.*—Air friction is negligible at the low speeds corresponding to the very low loads under consideration, and therefore

$$\mathbf{T}_8 = 0$$

(This will be referred to in Section 4*a*.)

Equation of Motion.

If $\mathbf{I}$ represents the polar moment of inertia of the rotor, the equation of motion is

$$\begin{aligned}\mathbf{I}d^2\theta/dt^2 &= \mathbf{T}_1 + \mathbf{T}_2 - \mathbf{T}_4 - \mathbf{T}_5 - \mathbf{T}_7 \\ &= K_1 I + f(\theta) - K_4 d\theta/dt - K_5 d\theta/dt - F \\ &= K_1 I + f(\theta) - N d\theta/dt - F\end{aligned}$$

where $N = K_4 + K_5$.

Let us consider the motion for some given value of the main current (I amperes), and let us write $K_1 I = U$ dyne-cm. We have then

$$\mathbf{I}d^2\theta/dt^2 + N d\theta/dt = U - F + f(\theta) \quad . \quad . \quad (1)$$

Before any useful attempt can be made to solve equation (1) we must have some definite knowledge of the form of $f(\theta)$. The constants of the equation, as well as a curve showing the relation between $\mathbf{T}_2$ and θ, were determined experimentally, and Section (4) is devoted to the details of the measurements.

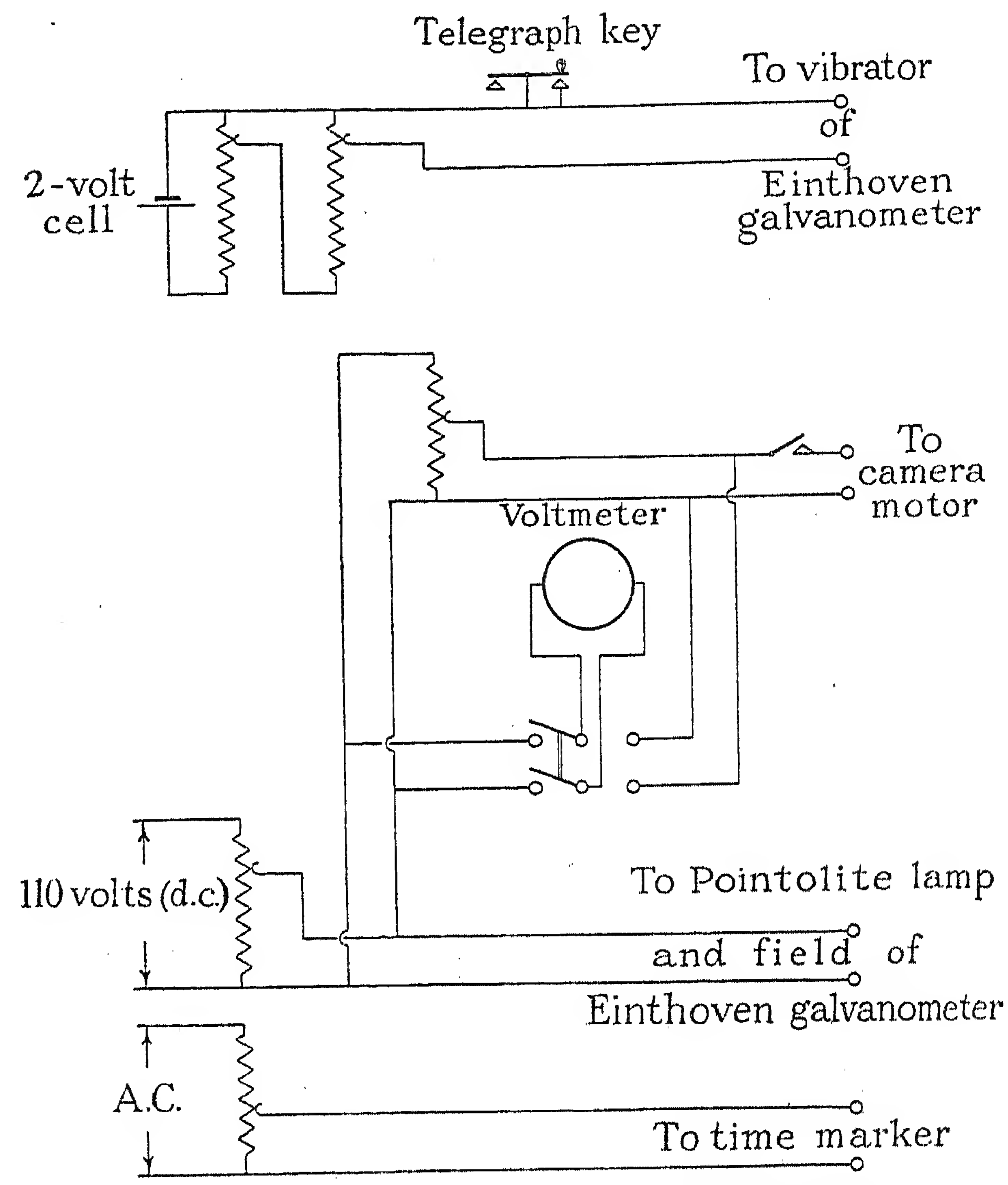


FIG. 6.—Arrangement of apparatus for friction measurements.

(4) DETERMINATION OF THE CONSTANTS OF THE EQUATION OF MOTION.

(*a*) *Measurement of the Torque due to the Combined Friction of the Registering Mechanism and Rotor Bearings.*

For this measurement the brake magnet was removed, and the rotor was set in motion by hand. The only torques opposing the rotation are those due to the friction of the air, bearings, and registering mechanism. The value of the air friction can be rendered negligibly small by choosing a low speed of rotation. The rotor is then gradually brought to rest by the friction torque of the bearings and registering mechanism. The torque is assumed to be constant, and its value is given by the expression

$$F = \mathbf{I}d^2\theta/dt^2 \, *$$

If we can construct a space/time curve giving the relation $\theta = f(t)$ for the motion of the rotor under the

* H. LAMB: "Dynamics," p. 156; *Bulletin of the Bureau of Standards*, 1913, vol. 10, p. 177.

action of a constant torque F, we can then, by successive differentiation, arrive at a value for $d^2\theta/dt^2$.

The Einthoven galvanometer was found to be admirably suitable for the production of chronograph records. The arrangement of the apparatus is shown in Fig. 6. The Pointolite lamp circuit required 80 volts, and the voltage of the camera motor was adjusted to give a suitable photographic film speed.

The time marker is a small synchronous motor which causes the light passing to the photographic film to be interrupted 5 times per second, when the frequency of supply to the motor is 25 cycles per sec., thus giving time marks on the film at intervals of 0·2 sec.

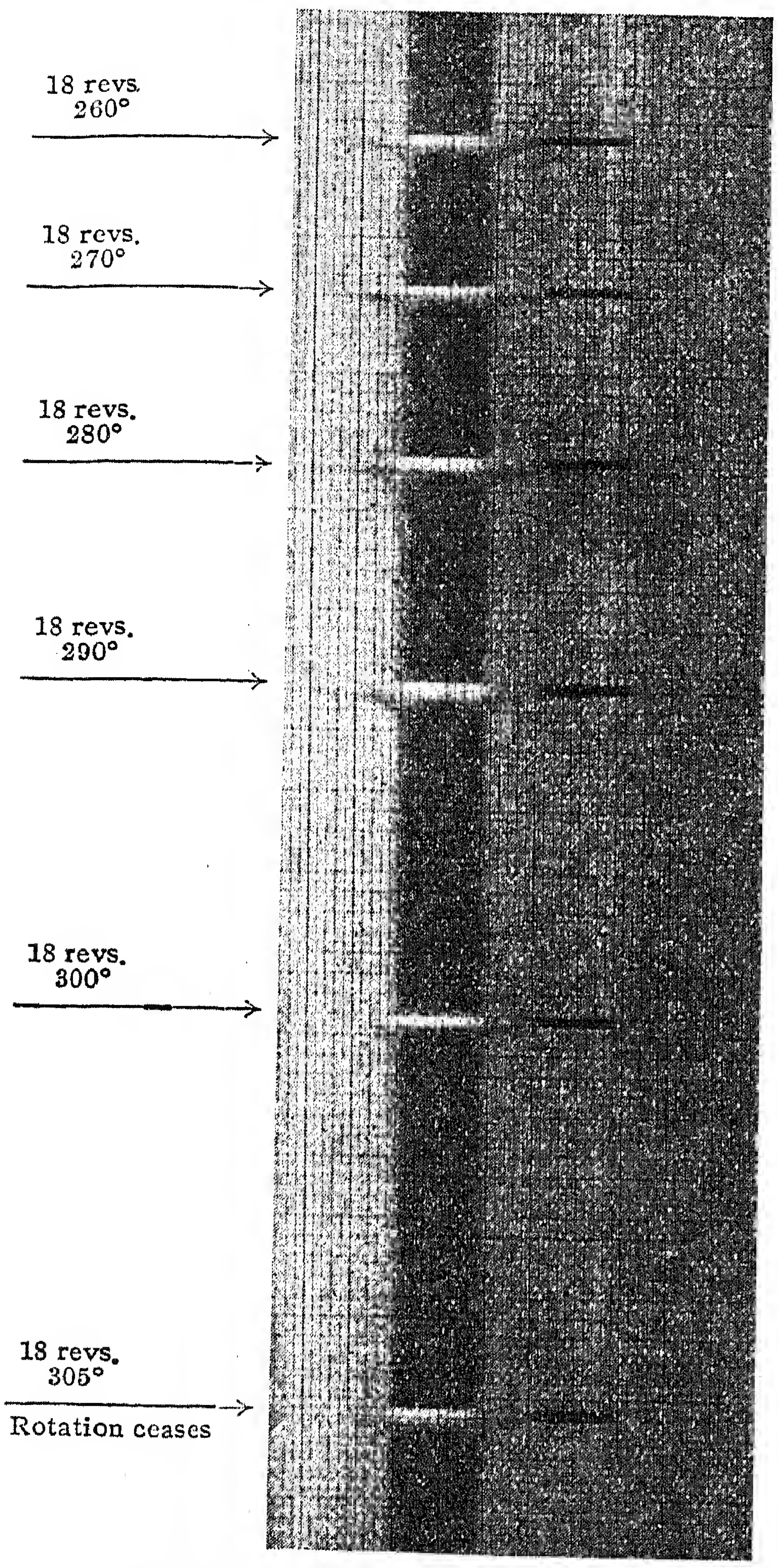


FIG. 7.—Portion of chronograph record showing the last 45° of rotation.

Two slide-wire resistances and a 2-volt cell were connected as shown, and the resistance slides were carefully adjusted so that on depressing the telegraph key a well-defined deflection of the galvanometer vibrator was obtained.

Before the tests were made to obtain curve B, Fig. 5, fine radial lines had been marked round the outer surface of the meter disc at intervals of 1°, longer lines marking intervals of 10°. As the disc rotated, the telegraph key was smartly operated by hand to record the passing of selected marks on the disc under a fixed pointer specially fitted for the purpose.

For the friction tests the rotor was set in motion by hand, and the meter-case front was placed in position. The rotation could be observed through the window, and when the speed was considered to have decreased to a sufficiently low value a chronograph record began, and continued until the rotor came to rest. A print of a portion of the chronograph record is shown in Fig. 7; the total length of the record is about 6 ft. The broad dark line, which is broken at intervals, denotes the position of rest of the galvanometer vibrator. A discontinuity in the line occurs at the instant the telegraph key is depressed, so that the interval between two successive breaks corresponds to an observed angle of rotation. It will be noticed that the beginning of each interruption in the line is sharp and well defined, and that there is no difficulty in counting the number of fine horizontal lines between successive interruptions. Since the horizontal lines are produced by the time marker, the time-interval for each observed angle of rotation is easily determined. The instant at which

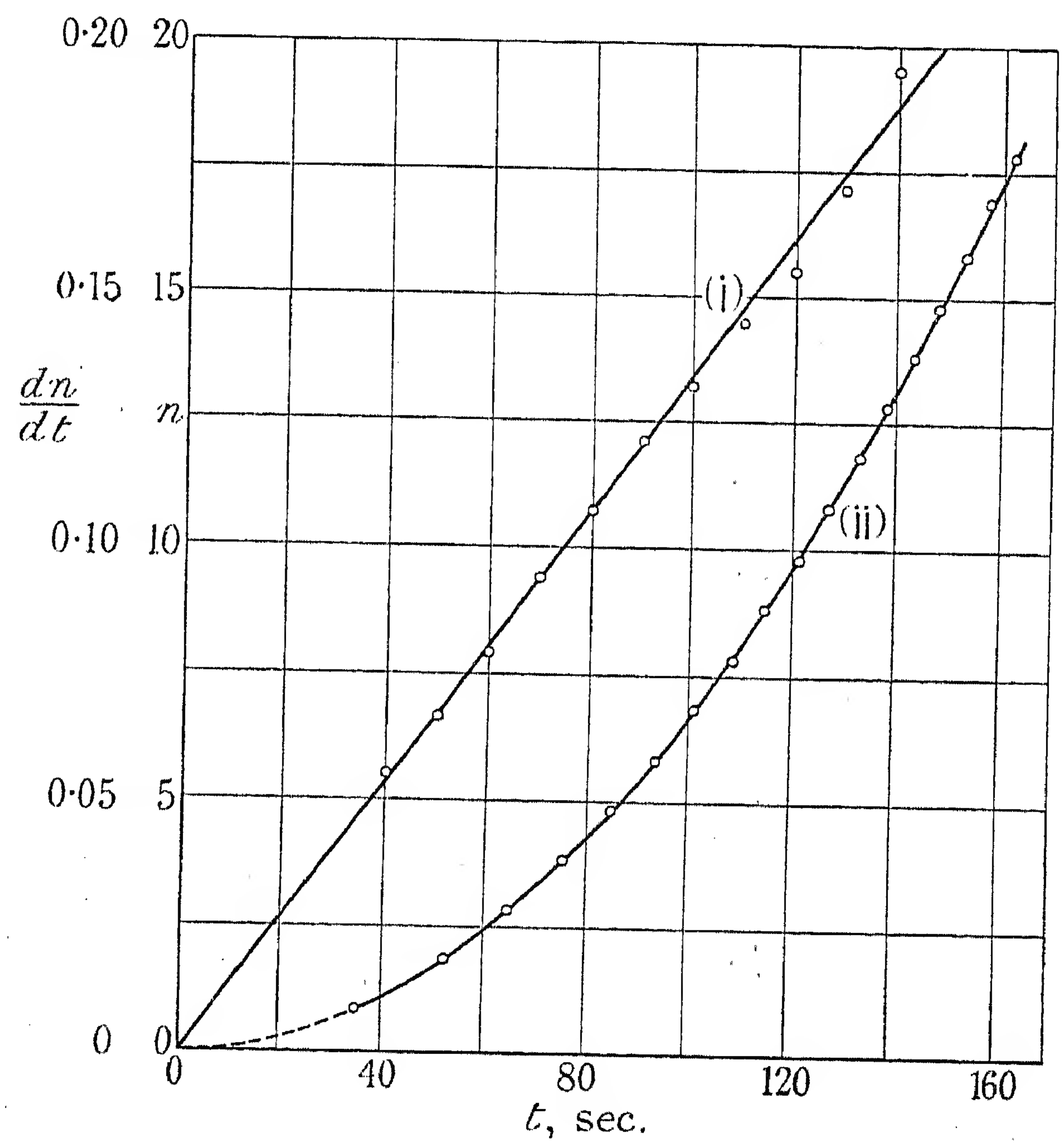


FIG. 8.

(i) Velocity/time curve from graphical differentiation of space/time curve.
(ii) Space/time curve from chronograph record.

rotation ceased was regarded as the zero of time, so that the information provided by the chronograph record could be arranged in such a way that a curve could be constructed to represent the accelerated motion of the rotor under the influence of a constant torque equal to the required friction torque.

The space/time curve is shown in Fig. 8, together with the velocity/time curve, which is obtained by graphical differentiation. It is obvious on examining the velocity/time curve that air friction has no appreciable effect for rotor speeds below 0·15 rev. per sec.; and

since the maximum rotor speed during the determination of the low-load curve B, Fig. 5, was about 0·03 rev. per sec., it is clear that we are justified in writing $\mathbf{T}_8 = 0$ in our mathematical analysis.

From the slope of the velocity/time curve, $d^2n/dt^2 = 0{\cdot}2/148{\cdot}8$, where n is the number of revs. per sec. and t is the time in seconds.

Thus $d^2\theta/dt^2 = 2\pi \times 0{\cdot}2/148{\cdot}8$, where θ is the angular displacement in radians. The moment of inertia of the

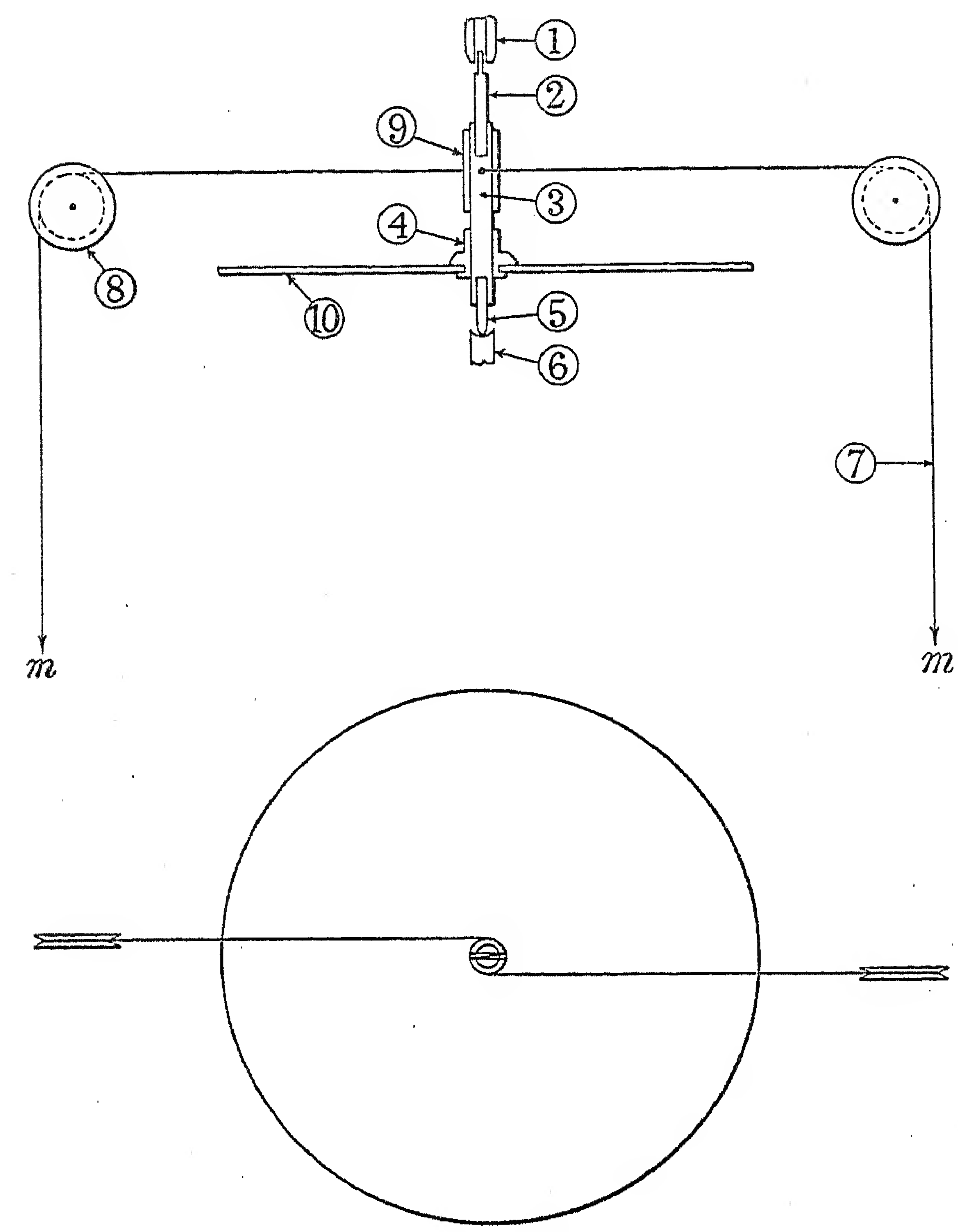


FIG. 9.—Method of measuring retarding torques.

1. Upper bearing.
2. Steel pivot.
3. Aluminium spindle.
4. Brass collet (plan of collet omitted).
5. Steel pivot.
6. Lower bearing.
7. Fine cotton thread.
8. Light pulley with steel pivots resting on sapphire bearings.
9. Brass sleeve, to increase effective diameter of spindle.
10. Aluminium disc.

rotor is shown in Section 4(d) to be 246·2 g-cm². Hence for the friction torque we get

$$F = \mathbf{I}d^2\theta/dt^2$$
$$= 246{\cdot}2 \times 2\pi \times 0{\cdot}2/148{\cdot}8$$
$$= 2{\cdot}1 \text{ dyne-cm}$$

(b) *Measurement of the Retarding Torques due to the Brake Magnet* ($\mathbf{T}_4$), *the Shunt Flux* ($\mathbf{T}_5$), *and the Series Flux* ($\mathbf{T}_6$).

For this set of tests the brake magnet was returned to its initial effective position. The meter was complete except for the registering mechanism, which was omitted for convenience. The total friction torque is so small that it can be ignored in comparison with the torques to be measured.

The method of measuring the torques is illustrated in Fig. 9. A brass sleeve is carefully driven over the rotor spindle in order to increase the effective diameter. A fine cotton thread passes through a hole which is drilled diametrically through the sleeve and spindle, and the thread is held in tension by the weights m. The rotor is turned—either by hand or electrically—in a clockwise direction in order to wind the thread in a layer round the spindle, and it is arranged that the layer forms in opposite directions simultaneously away from the hole. The length of the layer is small compared with the distance between either pulley and the spindle, so that the two lengths of thread on either side of the spindle can be considered to be always parallel. The spindle is therefore acted upon by a couple tending to turn the rotor in an anticlockwise direction, this being the direction of rotation under working conditions. The effective diameter of the spindle, allowing for the thickness of the thread, is $\frac{1}{4}$ in. Hence the torque is $m \times 2{\cdot}54/4$ g-cm, where m is measured in grammes.

The brake torque due to the permanent magnet was first measured. The rotor was turned at constant speed by weights each equal to m grammes against the retarding action of the brake magnet, and the time taken

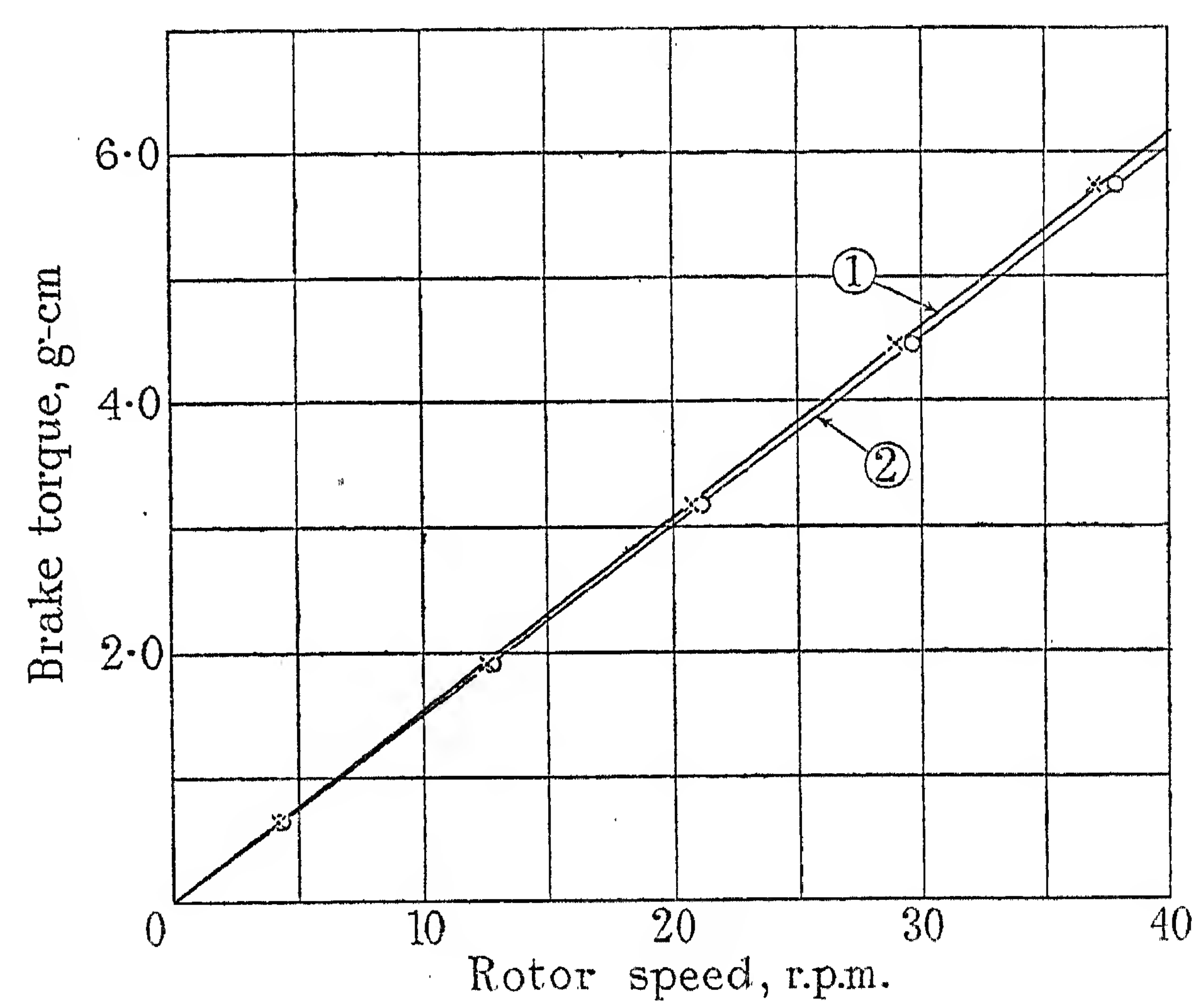


FIG. 10.

1. Combined action of brake magnet and shunt flux (200 volts).
2. Brake magnet alone.

to complete a given number of revolutions was found by stop-watch.

The second measurement was to find the brake torque due to the combined action of the permanent magnet and the shunt flux. A 200-volt 50-cycle supply was applied to the voltage circuit of the meter, and the speed and torque were determined as before. The results of these tests are plotted in Fig. 10.

The object of the third test is to obtain values for the combined braking effect of the permanent magnet and the series flux, but we must now bear in mind that both the current and the speed are variables. A given current was therefore established in the main current circuit, and the corresponding speed was obtained by adjusting the weights m. If the meter registered

without error the speed and current would bear a constant ratio, but by actual test the corresponding current and speed values under normal working conditions were found to be: 20 amperes, 39·5 r.p.m.; 18 amperes, 35·8 r.p.m.; 16 amperes, 32·1 r.p.m.; 14 amperes, 28·0 r.p.m.; 12 amperes, 24·0 r.p.m.; 10 amperes, 20·0 r.p.m.

These values were used in determining the combined brake torque, and the results of the measurements provide the curves in Fig. 11.

It will be seen that the braking effect of the series flux becomes insignificant for speeds below 20 r.p.m., and since the maximum speed in our low-load range is about 2 r.p.m. the omission of the torque $\mathbf{T}_6$ from our analysis is clearly vindicated. From the graph of the

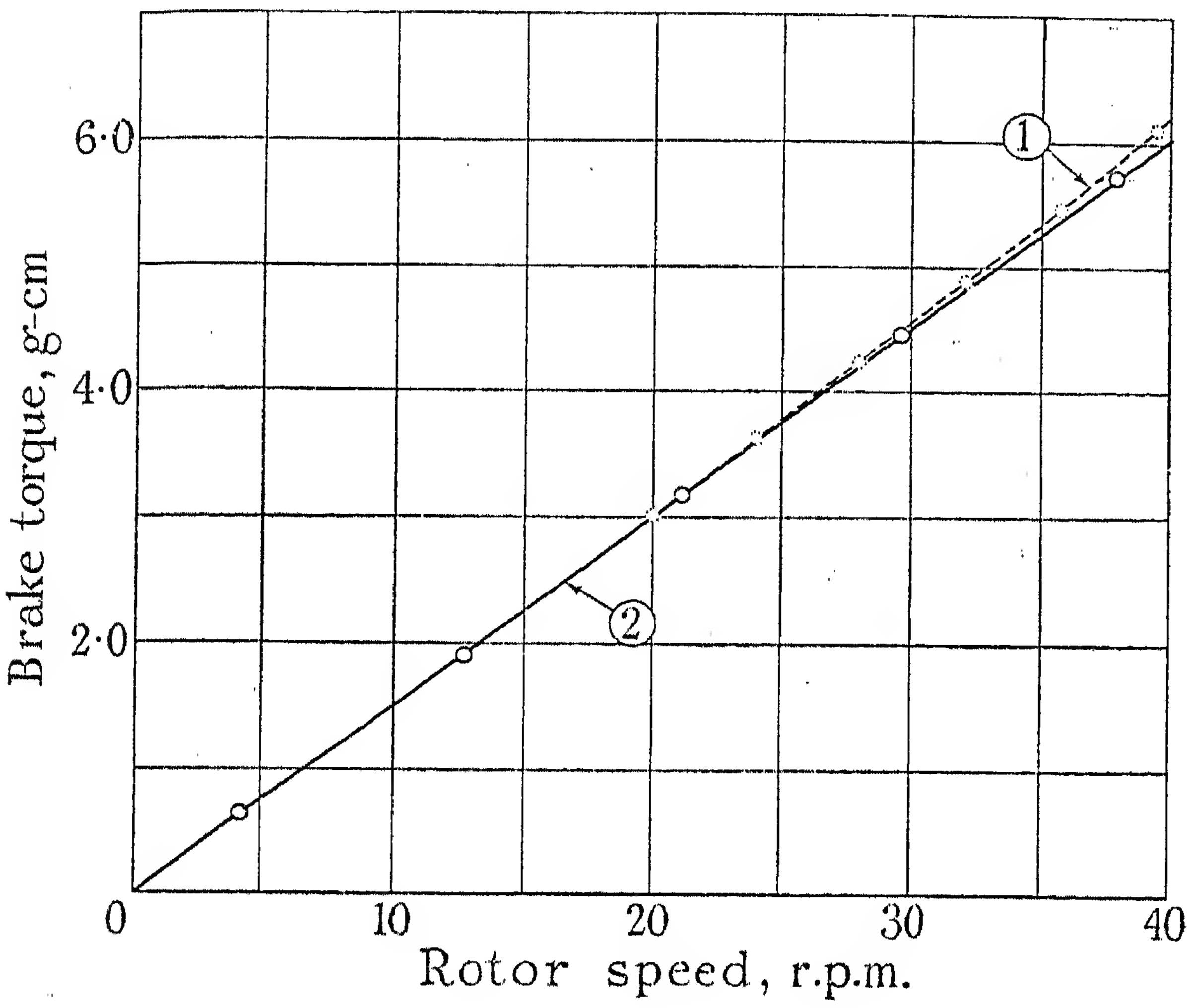


Fig. 11.

1. Combined action of brake magnet and series flux.
2. Brake magnet alone.

combined braking effect of the permanent-magnet and shunt flux (Fig. 10) we find that the torque at 40 r.p.m. is equal to 6·14 g-cm. Hence

$$N = K_4 + K_5 = 1\,438 \text{ dyne-cm at 1 radian per sec.}$$

(c) *Measurement of Main Driving Torque by Suspended-Weight Method.*

The brass sleeve which was used for the previous test was removed from the spindle, and the permanent magnet was detached from the meter. Referring to Fig. 12, a weight m grammes hangs suspended from one end of a fine cotton thread. The other end of the thread is fixed, and at a point l cm below the fixed end a second thread is attached which extends horizontally to a small vertical pin secured at the edge of the rotor disc. The horizontal thread is held in this position under the action of two opposing forces, one due to the torque of the rotor corresponding to a given constant electrical load, and the other due to the horizontal component of the displaced weight m. The weight m or the distance x is adjusted until the horizontal thread is tangential to the rotor disc and at the same time the system is in equilibrium. The load torque is thus given by $mr_a x/\sqrt{(l^2 - x^2)}$ g-cm, the meaning of the symbols being indicated in Fig. 12. The readings for these tests are plotted in Fig. 13, from which we obtain the relation between current and torque at constant voltage. Deriving a mean value from Fig. 13, we get

$$\text{Torque per ampere} = K_1 = 3 \cdot 04/10 \text{ g-cm}$$
$$= 298 \text{ dyne-cm.}$$

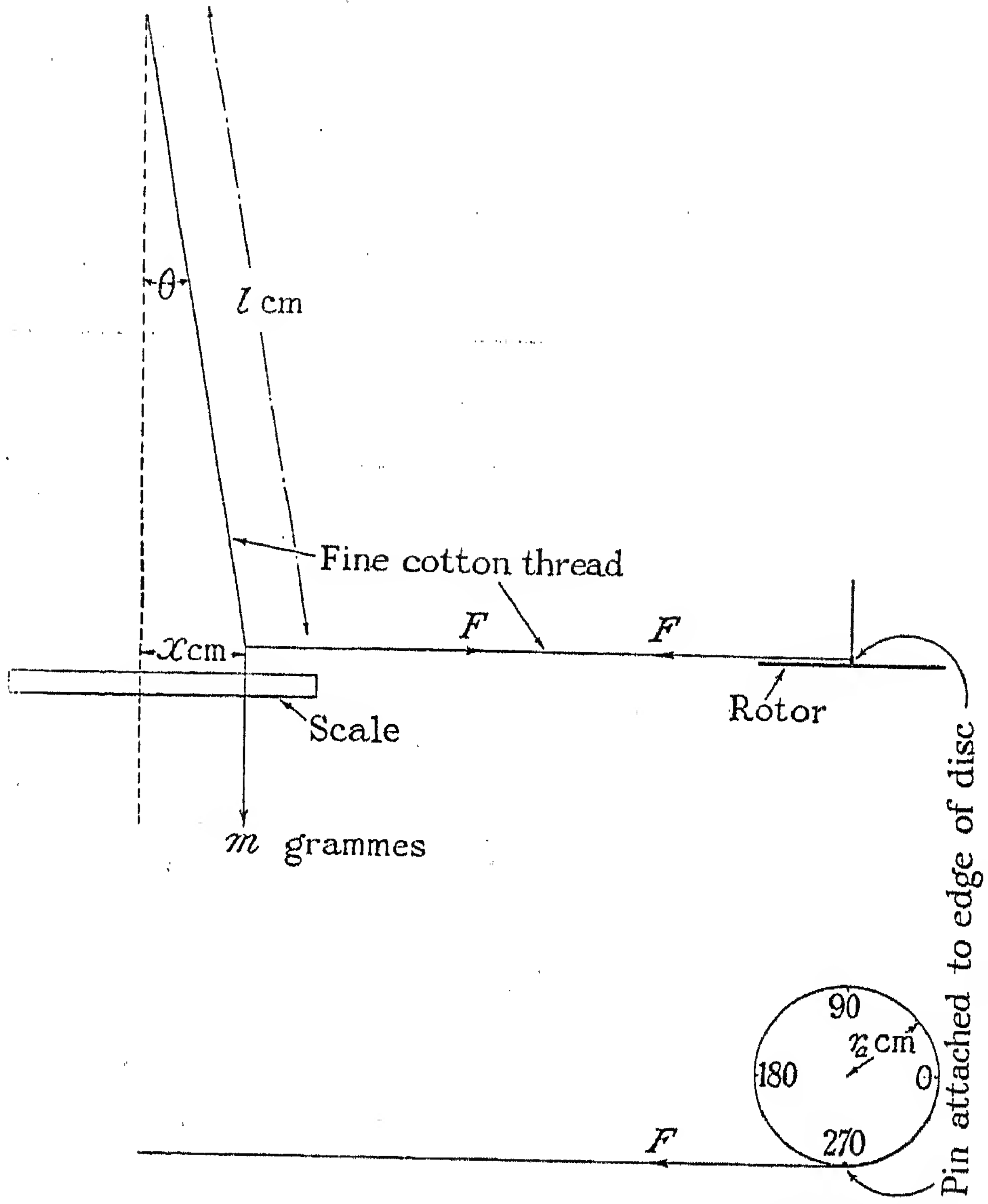


Fig. 12.—Measurement of load torques by suspended-weight method.

(d) *Description of Suspended Rotor, and Measurement of* (i) *Moment of Inertia of Rotor and* (ii) *Torsion Constant of Suspension.*

It is necessary to turn our attention to the devising of some means of measuring the small driving torque produced by the shunt flux. The primary function of this torque is to compensate for solid friction, and from this consideration alone it follows that the order of magnitude will be a few dyne-cm. Moreover, the torque is variable under the influence of the anti-creep device, so that the method adopted must enable us not only to measure very small torques but also to make the measurements for different positions of the rotor throughout the range of one complete revolution.

The rotor was suspended by means of a phosphor-bronze strip from a torsion head. By using levelling screws under the meter stand, and an adjustment at the torsion head for raising or lowering the suspension vertically, the rotor could be made to occupy its normal working position in the meter. The jewel bearing was lowered slightly so as to leave the rotor hanging freely, but it was retained in the meter because it provided a centre point over which the rotor pivot could be observed

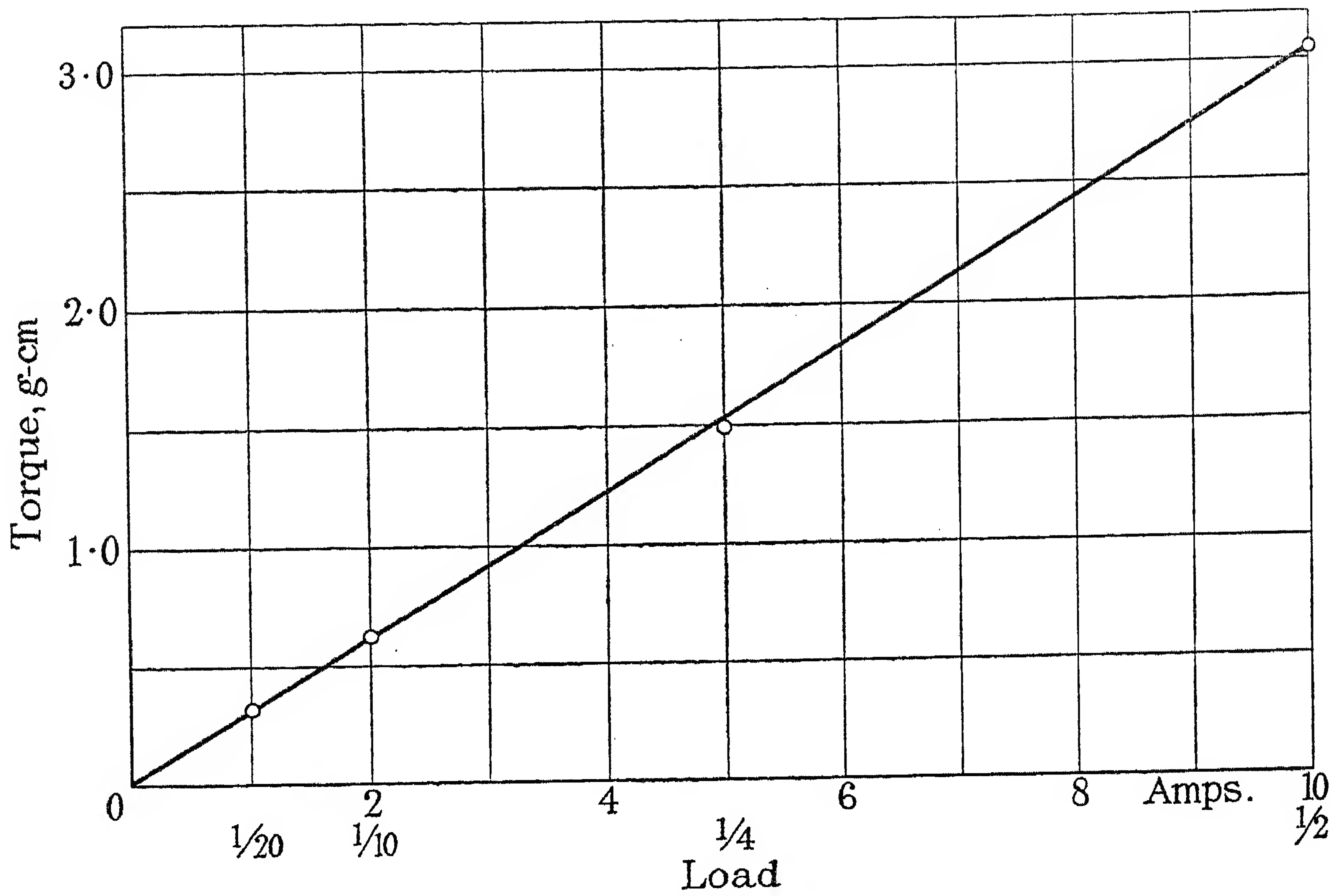


FIG. 13.

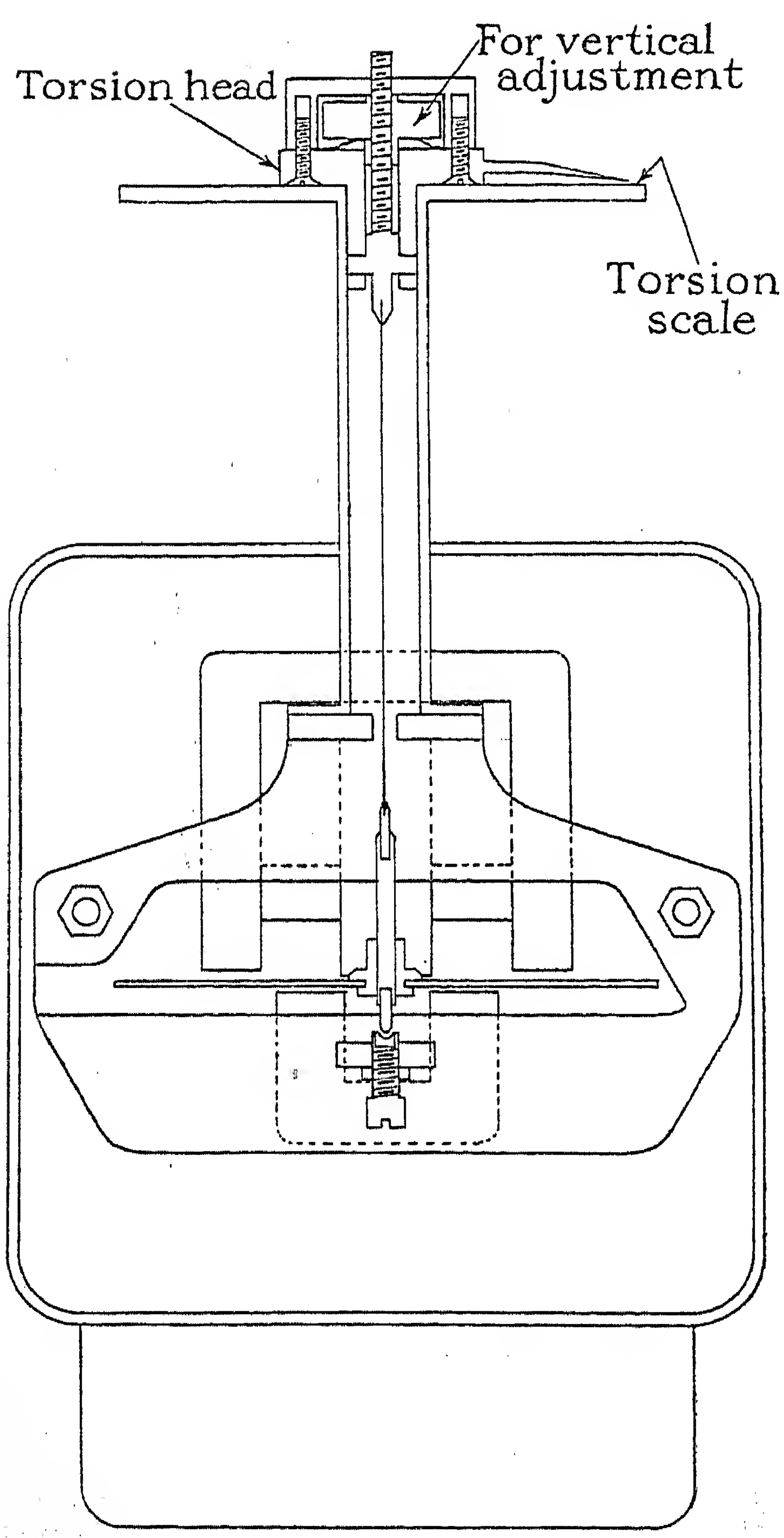


FIG. 14.

to hang truly. The torsion pointer turned smoothly over a circular scale marked in degrees, and during the rotation the rotor continued to hang without perceptible deviation from the one axis of rotation. A drawing of the suspended rotor is reproduced in Fig. 14.

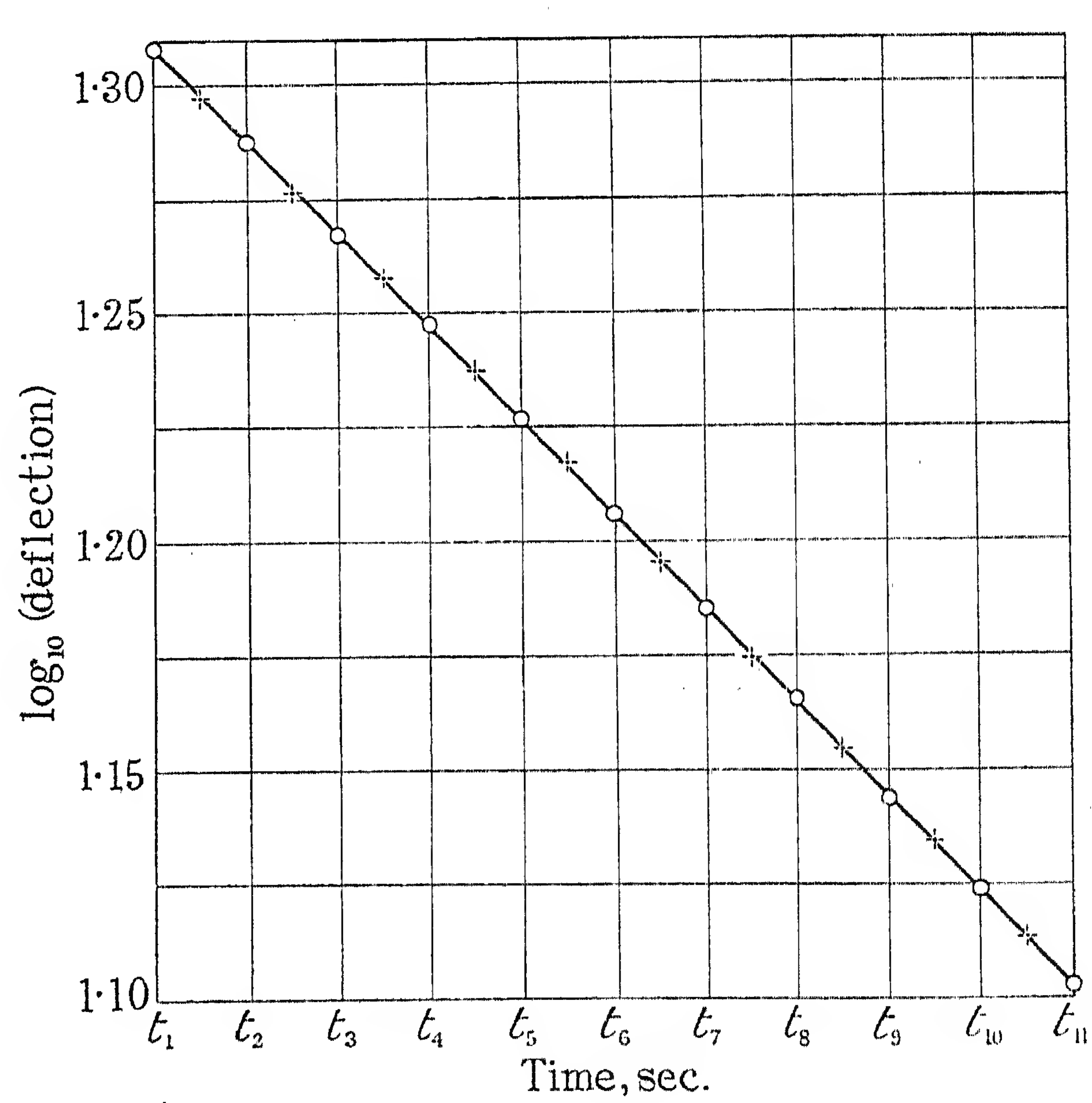


FIG. 15.

○ Deflections to left.
+ Deflections to right.
$t_{11} - t_1 = 208 \cdot 7$ sec.

A galvanometer mirror was fixed on a slight flat filed on the spindle, and, with the aid of a galvanometer lamp and scale, oscillations of the suspended system could be conveniently observed. The suspended rotor was caused to oscillate, and, with the meter cover in

position, successive deflections to left and right were read. The observed deflections were corrected to correspond to circular-scale readings, and the logarithms of the corrected deflections are plotted against time in Fig. 15. Using the information provided by this curve, it is found that the damping factor has no appreciable effect on the periodic time.

(i) *Moment of inertia of rotor.*—A light annular lamina of aluminium was prepared having a calculated moment of inertia ($\mathbf{I}_1$) of $65 \cdot 18$ g-cm². The periodic time (T_A) for the rotor alone was $20 \cdot 87$ sec., and that for the rotor and ring together (T_B) was $23 \cdot 47$ sec. For the moment of inertia of the rotor we have

$$\mathbf{I} = \mathbf{I}_1 T_A^2/(T_B^2 - T_A^2)$$
$$= 246 \cdot 2 \text{ g-cm}^2.$$

current within the range of our low-load tests, we are justified in writing $\mathbf{T}_3 = 0$.

For the low-load torque tests, the zero reading of the torsion pointer, as well as that of the rotor, was first noted. A 200-volt 50-cycle supply was then connected to the voltage circuit of the meter. The rotor was deflected owing to the driving torque of the shunt flux, and it was turned back to its zero position by turning the torsion pointer. The angle through which the torsion pointer was turned, viz. $15 \cdot 7°$, thus gave a measure of the shunt torque for the zero position of the rotor. This torque was subtracted from the load torques in each subsequent measurement. The voltage was maintained at 200 volts, and different values of current were passed through the meter series circuit. The watts were measured by the standard wattmeter,

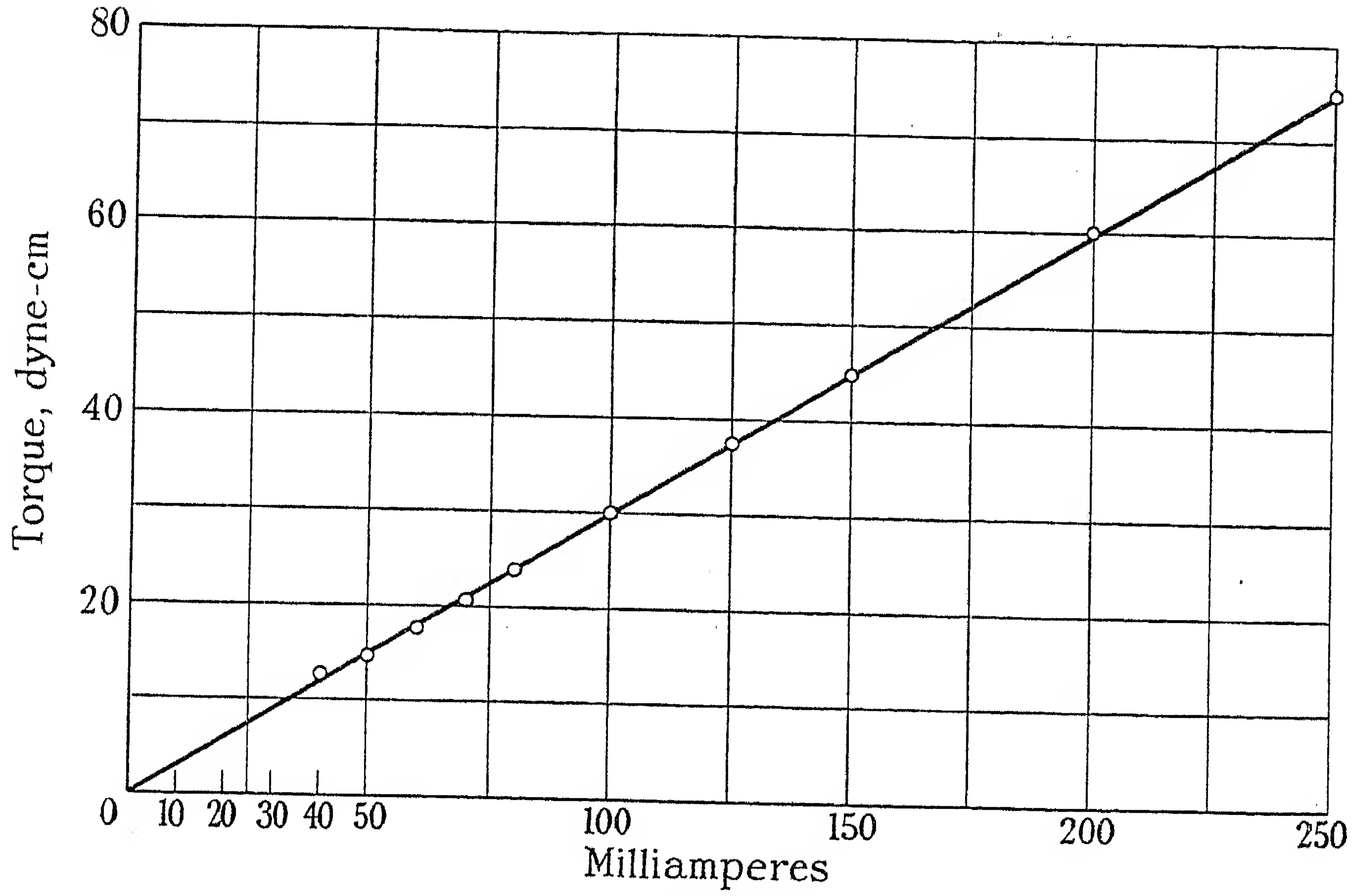


FIG. 16.

(ii) *Torsion constant of suspension.*—From

$$T_A = 2\pi\sqrt{(\mathbf{I}/a_2)},$$

where a_2 is the torsion constant, we get

$$a_2 = 4\pi^2 \times 246 \cdot 2/(20 \cdot 87)^2$$
$$= 22 \cdot 3 \text{ dyne-cm per radian deflection.}$$

(*e*) *Measurement of Main Driving Torque on Low Loads.*

The meter was retained in the same condition as that in which it was used for the tests described in the previous section. It was connected in circuit with the standard wattmeter, and the torques for very low loads were measured by means of the torsion control.

This arrangement of the apparatus provided an excellent opportunity of detecting the presence of an independent torque due to the series flux alone ($\mathbf{T}_3$). The voltage circuit of the meter was left disconnected, and a current of 1 ampere was passed through the current coil, but there was no resulting deflection of the rotor. As 1 ampere is greater than the maximum value of the

and the torsion angle for each value of power was read after the rotor had been turned back to its zero position by means of the torsion control.

The curve in Fig. 16 shows the relation between load current and torque. The mean value derived from the graph is $74 \cdot 8/250 = 0 \cdot 299$ dyne-cm per milliampere, and therefore K_1 = torque per ampere = 299 dyne-cm. This compares very favourably with the value 298 dyne-cm which was obtained by the suspended-weight method in Section 4(*c*), and the close agreement serves to confirm the experimentally determined value of the torsion constant. The torque per ampere at 200 volts will be taken as 299 dyne-cm.

(*f*) *Measurement of Torque Due to Shunt Flux.*

Having determined the torsion constant of the suspension, we can proceed directly to make tests which will establish a relationship between the torque $\mathbf{T}_2$ and the angular deflection θ for all positions of the rotor throughout one complete revolution. As has already been explained, the rotor disc is marked in degrees and the

deflection is read with reference to a specially fitted fixed pointer.

A description of one measurement will explain the procedure. The voltage circuit of the meter was opened and the torsion head was turned until the rotor reading was 90°, the simultaneous reading indicated by the torsion pointer being 340·2°. A voltage of 200 volts at 50 cycles per sec. was applied to the voltage circuit, when the rotor was deflected in an anticlockwise direction. By a clockwise rotation of the torsion head the rotor reading of 90° was restored, when the torsion pointer indicated 355·5°. The torsion angle was therefore 355·5° − 340·2° = 15·3°. Hence when the 90° mark on the rotor disc was under the fixed pointer, the shunt flux exerted a positive torque on the rotor equal to $15 \cdot 3 \times 22 \cdot 3\pi/180 = 5 \cdot 95$ dyne-cm, the torsion constant of the suspension being $22 \cdot 3\pi/180$ dyne-cm per degree deflection. Similar measurements were made for all positions of the rotor, and the results were used to construct the curve shown in Fig. 17.

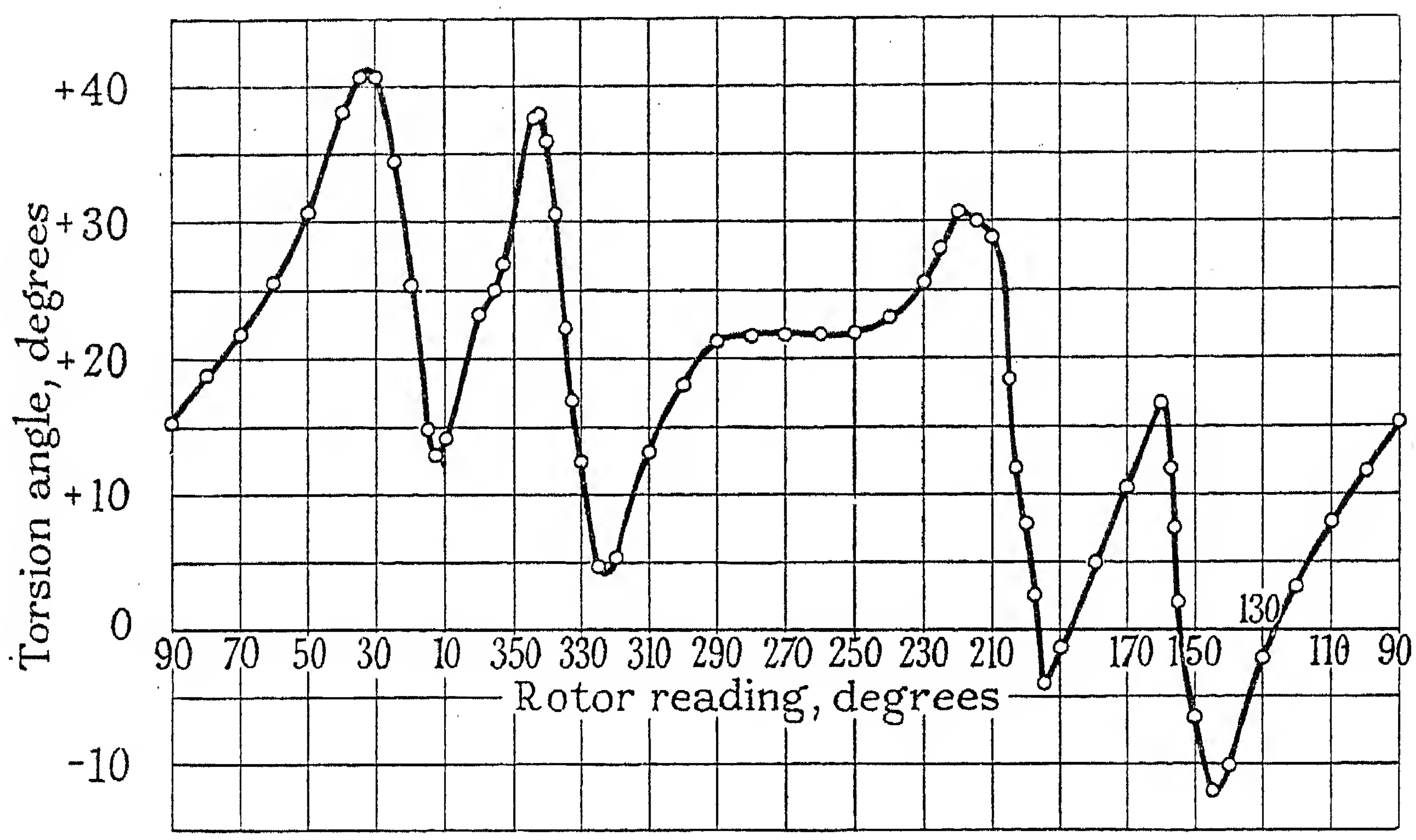


FIG. 17.—Driving torque due to shunt flux; at 200 volts, 50 cycles per sec.

A plan view of the rotor and shunt stator poles is shown in Fig. 17A, where the two anti-creep holes (1) and (2) are seen to be on the diameter joining the 0° and 180° marks on the disc. By examining this diagram in conjunction with the curve of Fig. 17 the shunt-torque variations can be related to the various positions of the anti-creep holes with reference to the shunt stator poles. Obviously the torque fluctuations corresponding to rotor readings from 90° to 270° are due chiefly to the effects of hole (1), whilst the torque variations for the rotor readings 270° to 90° are caused chiefly by hole (2). As we should expect, the two halves of the torque curve bear some resemblance in shape but are not identical. This, however, is hardly unexpected, for a perfect repetition of the torque curve per half-revolution would necessitate, at least, a homogeneous disc of uniform resistance, and holes of exactly equal size situated at equal distances on the same diameter on either side of the disc centre. Where two or more holes or slots are used to prevent creeping on shunt, it is a matter of common experience to find that one hole in particular is usually more effective than the others in preventing rotation.

The irregular outline of the curve of Fig. 17 suggests as a first consideration that $f(\theta)$ might be most suitably represented by a Fourier series, but if $f(\theta)$ is replaced in equation (1) by a Fourier series a solution is by no means obvious. We may, however, derive some assistance from the application of harmonic analysis if we can accomplish first some modification in the differential equation itself. Let it be assumed tentatively that the

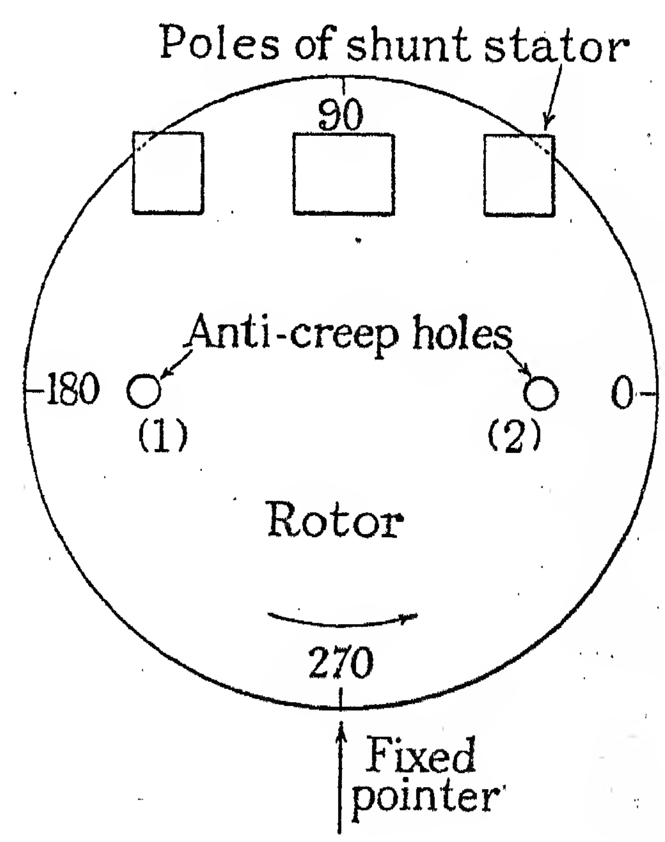


FIG. 17A.

term $Id^2\theta/dt^2$ is negligibly small compared with $Nd\theta/dt$. Then equation (1) can be transformed to

$$Nd\theta/dt = U - F + f(\theta) \quad . \quad . \quad . \quad (2)$$
$$= S + f(\theta)$$

where $S = U - F$.

Then

$$\frac{d\theta}{dt} = \frac{S}{N} + \frac{1}{N}f(\theta)$$

or

$$\frac{dt}{d\theta} = \frac{1}{X_1 + X_2 f(\theta)} \quad . \quad . \quad . \quad . \quad (3)$$

where $X_1 = S/N$ and $X_2 = 1/N$.

Suppose we take a given ordinate of the curve in Fig. 17 and multiply it by X_2, to the product add X_1, and find the reciprocal of the result. If a sufficient number of values thus obtained are plotted against corresponding values of θ we shall have constructed a curve representing a different function of θ, which may be called $f_1(\theta)$.

Thus

$$dt/d\theta = f_1(\theta)$$

For any given load a curve $f_1(\theta)$ may be drawn, and if from the mean curve as drawn and was focused more on the disposition of the experimentally determined points. It was soon realized that the points plotted from experiment could be quite accurately joined by a number of straight lines, as shown in Fig. 18. Each line ascribes to $f(\theta)$ a form $a + b\theta$ within appropriate limits. Taking any given straight line and inserting the corresponding expression $a + b\theta$ in the differential equation, we can completely represent the motion of the rotor by

$$\mathrm{I}d^2\theta/dt^2 + Nd\theta/dt = U - F + a + b\theta \quad . \quad (5)$$

that is,

$$\frac{d^2\theta}{dt^2} + \frac{N}{\mathrm{I}}\frac{d\theta}{dt} - \frac{b}{\mathrm{I}}\theta = \frac{U - F + a}{\mathrm{I}}$$

or

$$d^2\theta/dt^2 + pd\theta/dt + q\theta = r \quad . \quad . \quad . \quad (6)$$

where $p = N/\mathrm{I}$, $q = -b/\mathrm{I}$, and $r = (U - F + a)/\mathrm{I}$

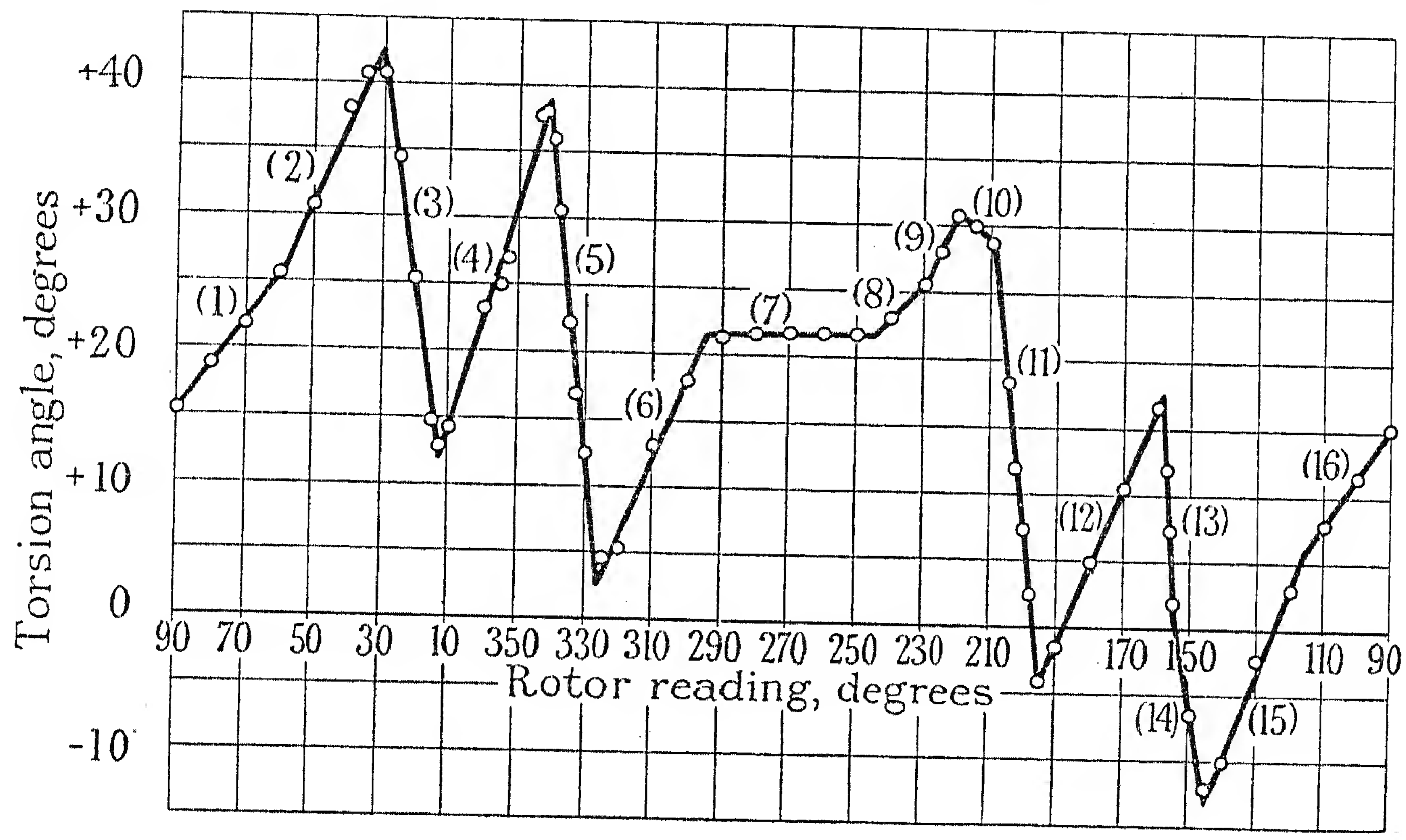


FIG. 18.—Driving torque due to shunt flux: alternative graph. Values obtained at 200 volts, 50 cycles per sec.

the curve is analysed so as to yield a Fourier series we obtain an equation such as

$$dt/d\theta = a_0 + a_1 \sin(\theta + \phi_1) + a_2 \sin(2\theta + \phi_2) + \dots$$

$$\therefore t = \int_0^{2\pi} [a_0 + a_1 \sin(\theta + \phi_1) + a_2 \sin(2\theta + \phi_2) + \dots] d\theta$$

$$= 2\pi a_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

giving the time for one complete revolution of the rotor for a given load.

This treatment leads to a very simple expression for the time, and calls for the determination of the constant term only in the harmonic analysis; but unfortunately it rests on the assumption that the angular acceleration of the rotor is negligibly small, and an experimental demonstration of this is not easy to achieve. In view of this, the course described in Section (5) was followed.

(5) Interpretation of $f(\theta)$, and Solution of the Differential Equation of Motion.

As the expediency of applying harmonic analysis to the curve in Fig. 17 was doubtful, attention was diverted

The solution of equation (6) is

$$\theta = e^{-\frac{1}{2}pt}(Ae^{at} + Be^{-at}) + r/q \quad . \quad . \quad . \quad (7)$$

or, alternatively,

$$\theta = e^{-\frac{1}{2}pt}(C\cosh at + D\sinh at) + r/q \quad . \quad (8)$$

where A, B, C, and D are arbitrary constants, and $a = \sqrt{(\frac{1}{4}p^2 - q)}$. The ultimate choice of the equation to be used will obviously be governed by the suitability of the mathematical tables available.

The graph in Fig. 18 consists of 16 straight lines; if we include in our equation the values for a and b given by each straight line in turn we shall obtain 16 values of time, which, when added together, will give the time for one revolution of the rotor for a given constant load. Hence the beginning of each straight line can be regarded as corresponding to zero value of both time and angle of rotation.

Substituting in equation (7) the condition that when $t = 0$, $\theta = 0$, we get

$$A + B = -r/q \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Modifying equation (7), we have

$$\theta = Ae^{(a-\frac{1}{2}p)t} + Be^{-(a+\frac{1}{2}p)t} + r/q \quad . \quad . \quad (10)$$

Therefore

$$d\theta/dt=\omega=A(a-\tfrac{1}{2}p)e^{(a-\frac{1}{2}p)t}-B(a+\tfrac{1}{2}p)e^{-(a+\frac{1}{2}p)t} \quad . \quad (11)$$

When $t = 0$ we may write $\omega = \omega_0$, and hence from (11) we have

$$A - B = \frac{\omega_0}{a} - \frac{rp}{2qa} \quad . \quad . \quad . \quad . \quad (12)$$

where ω_0 is the velocity at the beginning of the particular straight line under consideration.

From (9) and (12) we obtain

$$A = \tfrac{1}{2}\left(\frac{\omega_0}{a} - \frac{rp}{2qa} - \frac{r}{q}\right)$$

$$B = \tfrac{1}{2}\left(\frac{rp}{2qa} - \frac{r}{q} - \frac{\omega_0}{a}\right)$$

The numerical values of the constants for each of the sixteen straight lines which compose the graph in Fig. 18 are given in the Table.

TABLE.

Line	Limits of θ	Torque $(=a+b\theta)$
	radians	dyne-cm
1	0 to $0\cdot52 = \theta_1$	$5\cdot95 + 7\cdot58\theta$
2	0 to $0\cdot50 = \theta_2$	$9\cdot92 + 13\cdot07\theta$
3	0 to $0\cdot32 = \theta_3$	$16\cdot53 - 36\cdot64\theta$
4	0 to $0\cdot54 = \theta_4$	$4\cdot71 + 19\cdot35\theta$
5	0 to $0\cdot27 = \theta_5$	$15\cdot17 - 52\cdot37\theta$
6	0 to $0\cdot54 = \theta_6$	$1\cdot01 + 13\cdot81\theta$
7	0 to $0\cdot87 = \theta_7$	$8\cdot48$
8	0 to $0\cdot26 = \theta_8$	$8\cdot48 + 5\cdot5\theta$
9	0 to $0\cdot17 = \theta_9$	$9\cdot92 + 11\cdot37\theta$
10	0 to $0\cdot17 = \theta_{10}$	$11\cdot91 - 2\cdot68\theta$
11	0 to $0\cdot26 = \theta_{11}$	$11\cdot44 - 49\cdot65\theta$
12	0 to $0\cdot64 = \theta_{12}$	$-1\cdot56 + 13\cdot14\theta$
13	0 to $0\cdot05 = \theta_{13}$	$6\cdot92 - 117\cdot4\theta$
14	0 to $0\cdot15 = \theta_{14}$	$0\cdot78 - 38\cdot16\theta$
15	0 to $0\cdot54 = \theta_{15}$	$-5\cdot21 + 14\cdot1\theta$
16	0 to $0\cdot43 = \theta_{16}$	$2\cdot41 + 8\cdot12\theta$

For line No. 7 the torque is constant, and as a result equation (6) does not represent the motion during this interval, because the term $b\theta$ is zero. Putting $q\theta = 0$ in equation (6), we have

$$d^2\theta/dt^2 + pd\theta/dt - r = 0 \quad . \quad . \quad . \quad (13)$$

If when $t = 0$ the velocity $\omega = \omega_0$, then the solution of equation (13) is

$$\omega = \frac{r}{p} + \left(\omega_0 - \frac{r}{p}\right)e^{-pt} \quad . \quad . \quad . \quad . \quad (14)$$

If the torques are suddenly applied at time $t = 0$ and the rotor is initially at rest, then

$$\omega = (r/p)(1 - e^{-pt}) \quad . \quad . \quad . \quad . \quad (15)$$

The order of magnitude of p is $5\cdot8$, so that the exponential term is negligible compared with unity after a very short interval of time. The minimum time for the interval represented by line No. 7 is about 4 sec., and hence, from (14),

$$\omega = \frac{r}{p} + \left(\omega_0 - \frac{r}{p}\right)e^{-23}$$

Since $\omega_0 - (r/p)$ is at most of the order of $0\cdot0002$, the velocity at the end of line No. 7 is accurately given by $\omega = r/p$.

The velocity at the end of line No. 7 is obviously the velocity at the beginning of line No. 8, and so ω_0 is known for this line. Fortunately, it was discovered

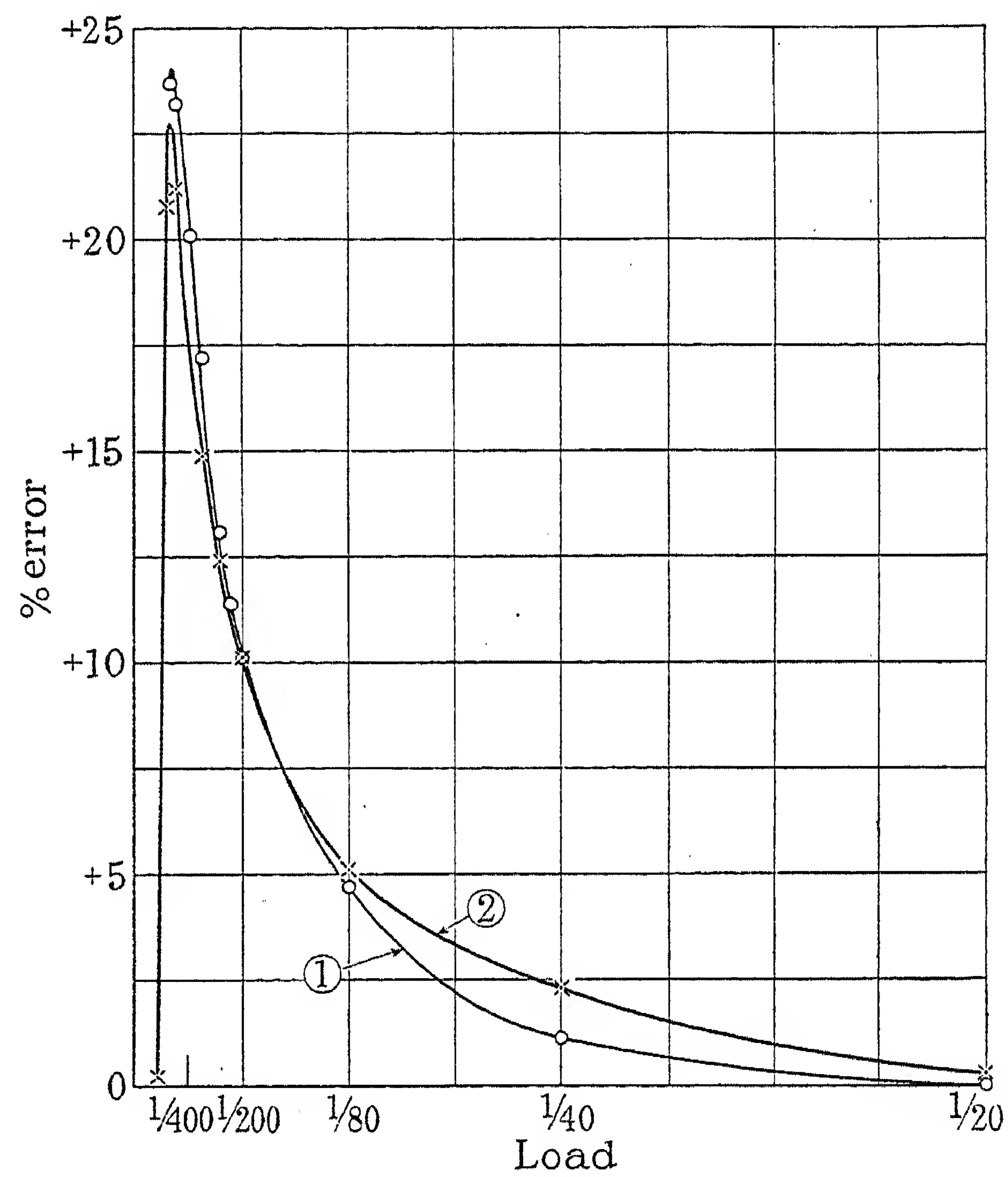


FIG. 19.—Low-load curves. At full load and unity power factor, voltage = 200 V, current = 20 A, frequency = 50 cycles per sec.

○ By actual test (curve 1).
× By calculation (curve 2).

that the term $Be^{-(a+\frac{1}{2}p)t}$ in equation (10) was in every case so small that it could be neglected. Hence the time for rotation through the angle appropriate to line No. 8 was found by substituting the theoretically determined value of ω_0 in the equation

$$\theta = Ae^{(a-\frac{1}{2}p)t} + r/q \quad . \quad . \quad . \quad . \quad (16)$$

The velocity at the end of line No. 8 was also found from equation (16), thus providing the value of ω_0 at the beginning of line No. 9. This method was followed for each of the sixteen lines in Fig. 18, giving the total time for a rotation of 2π radians for a given load.

It was thus possible to determine the theoretical time for one or more complete revolutions of the rotor for any chosen value of load, and in this way the theoretical percentage errors were calculated for various selected

loads. The theoretical curve derived from the calculations is drawn in Fig. 19, and for comparison the curve which was obtained by actual test on the meter is drawn in the same figure.

(6) Conclusion.

In producing a theoretical load curve, and presenting it as in Fig. 19 along with the experimental load curve, we have accomplished the main purpose of the investigation. It is true that the curves do not coincide throughout, but the degree of agreement is sufficient to confirm our confidence in the mathematical treatment of the problem; and in view of the number of constants depending on individual measurement it is gratifying to find ultimately such small differences between the experimental and theoretical results.

There is, however, one serious difference between the curves. By experiment it was found that rotation ceased at about 1/500 of full load, but the theoretical curve indicates that rotation should continue until the load has been reduced to about 1/800 of full-load value. It will be realized that the friction of the registering mechanism may vary slightly, and it is quite conceivable that any small deformity in a tooth engaging with the worm on the rotor spindle might have a pronounced influence when the driving torque is so low as that which corresponds to 1/500 load. This is the probable explanation of the cause of the marked difference between the curves below 1/500 load, but it will be noticed that the discrepancy is confined to a very restricted load range, and can hardly be regarded as detracting from the validity of the theory.

From a cursory examination during the calculations, the acceleration appears to be always small compared with the velocity, and this revives some confidence in the method of harmonic analysis which was abandoned. This is referred to merely as a possible alternative which might prove less tedious than the method used.

Reverting to the adopted method of treating the problem, it will be recalled that the most comprehensive statement of the conditions is represented by equation (5), which is here reproduced for convenience of reference.

$$\mathbf{I}d^2\theta/dt^2 + Nd\theta/dt = U - F + a + b\theta$$

The term $b\theta$ is due to the anti-creep device, and if this effect were absent we should have

$$\mathbf{I}d^2\theta/dt^2 + Nd\theta/dt = U - F + a \quad . \quad . \; (17)$$

Now a may be regarded as a constant torque due to the shunt flux in the absence of the anti-creep device, and this is really the torque which is provided for the purpose of compensating for friction. If we write $a = F$, then equation (17) becomes

$$\mathbf{I}d^2\theta/dt^2 + Nd\theta/dt = U \quad . \quad . \quad . \quad . \; (18)$$

Ignoring transient effects at switching on, we obtain as the solution of (18), $\omega = U/N$, giving the speed of rotation as directly proportional to the load torque U. Under such conditions the meter would register without error for all loads within the given range.

If the term a exceeds F in magnitude, then we get a constant speed on any given load, but the meter registers with a positive error. Finally, when the term $b\theta$ is introduced in addition to a, we have a varying torque which is mainly positive and greater than F throughout a complete revolution of the rotor. As a consequence, the registration error on any given load within the low-load range is positive. Moreover, the lower the value of the load torque, the greater is the comparative magnitude of the varying torque $a + b\theta$, and so the lower the load on which the meter is tested the larger is the positive error disclosed. This applies until the load has been decreased almost down to the value at which rotation ceases.

It will be seen in Fig. 18 that during certain intervals of rotation the torque $a + b\theta$ is negative. When U is approaching its lowest effective value the total driving torque during such intervals may just exceed the friction torque, and the excessive time of rotation for these intervals will reduce the magnitude of the positive errors. This is seen to occur in both the curves in Fig. 19.

It is clear, therefore, that the anti-creep device is responsible for the positive errors on very low loads, such as those chosen.

As explained in the Introduction, the primary motive for this research was a desire to discover the cause of the intolerable low-load errors, and so prepare the way for a possible extension of the useful working range of the meter. Whether practical difficulties are likely to prohibit the realization of this aspiration is a matter which is not included within the scope of this inquiry, but it is clear that careful consideration should be given to the design of the anti-creep device, in an endeavour to reduce the wide and unnecessary fluctuations in the magnitude of the independent shunt torque.

It is readily admitted that such an improvement as that suggested is conceived without any regard whatever for the influence of changing friction due to the registering mechanism and worn pivots and bearings, but these are separate problems, the solution of which would be of no avail for the purpose proposed unless a satisfactory design of anti-creep device were first accomplished.

The author wishes to record his indebtedness to Prof. W. Cramp, D.Sc., Member, who provided—in the Department of Electrical Engineering, Birmingham University—all the facilities for carrying out the work described in this paper.

[The discussion on this paper will be found on page 368.]

DISCUSSION BEFORE THE METER AND INSTRUMENT SECTION, 1ST FEBRUARY, 1935.

Mr. E. W. Hill: I find myself in complete disagreement with the conclusion to which the author commits himself in Section (2), namely, that the large positive errors at low load are attributable to the anti-creep holes in the disc. In the study of nature there is possible a particular kind of fallacy, old as the human race, which apparently never loses its plausibility. It is the erroneous assumption that if in some "event" there are two phenomena A and B occurring in association, then there must necessarily be a causal relation between them. Section (2) deals with a meter which happened to be in a particular state of low-load calibration (unaltered throughout the tests) such that large positive errors at very low loads (phenomenon A) were present at the same time as anti-creep holes in the disc (phenomenon B). The simultaneous existence of A and B is, as far as I can see after very careful examination, the sole basis put forward in the paper for the contention that B causes A.

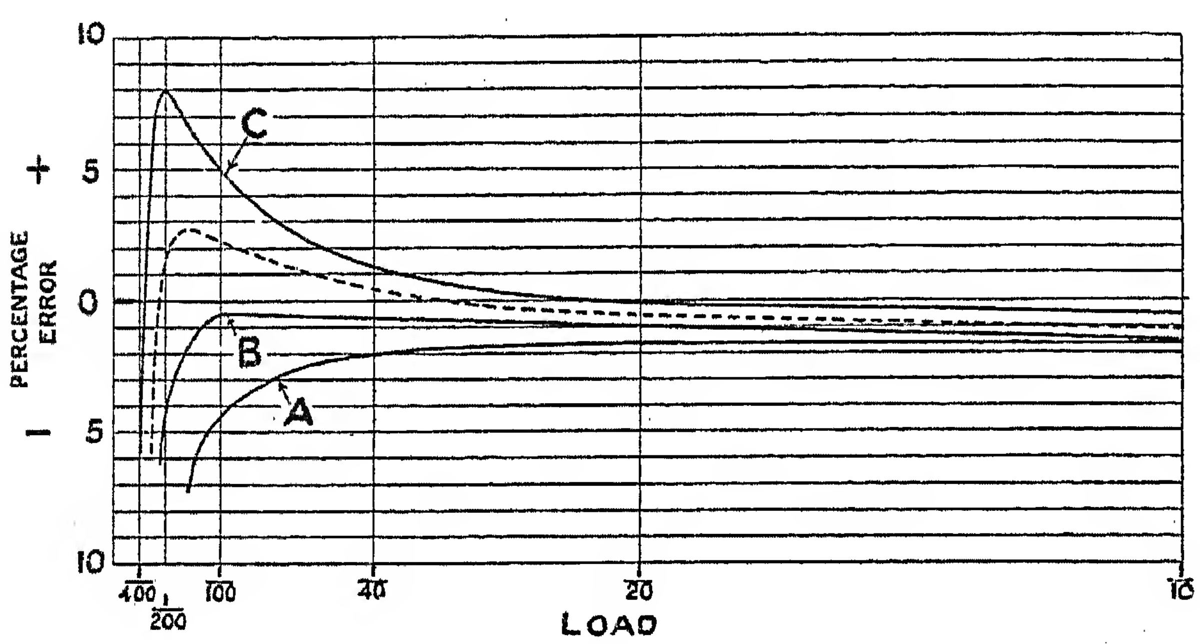


FIG. A.—Curves obtained for disc with anti-creep holes.

But if simultaneity of existence is to be the criterion, it would be just as logical (and still consistent with anything that appears in the paper) to argue that the anti-creep holes are caused by the large positive errors. I think the author should at least have informed himself whether B is *invariably* accompanied by A, and whether the elimination of B would cause the disappearance of A. I find in the paper no vestige of any such discriminatory tests, which of course are really indispensable for the complete establishment of the author's proposition.

I do find, however, that the author supplements what in my view is a false assumption, with a remarkable *non sequitur*.

The alleged false assumption is this: that meters necessarily have the characteristic curves drawn in Fig. 5, where a steep rise is shown from about 1/80th load downwards. This assumption is actually made; otherwise the author's proposition would be emptied of any significant content, because immediately it is admitted that a meter with anti-creep holes can have a curve that does not rise steeply as shown, then it is evident that anti-creep holes are not likely to be the cause of the rise when it does occur.

The question whether this rise is an inescapable feature can be comparatively easily settled by a few tests, which would also settle the other contested points. For the latter, one important experiment is that where a meter is tested down to very low loads, first with a disc without holes, and then with the same disc with anti-creep holes drilled in it. The setting of the low-load adjustment, and all the other conditions, should be kept the same for this pair of tests.

Tests of this sort were included in some I have recently made on a perfectly normal sample of a modern meter. The results I obtained are shown in Figs. A and B, for discs with and without anti-creep holes respectively. In each diagram each curve shows the meter errors for a different setting of the low-load adjustment. The family of curves thus obtained shows the effect, well known, of the use of the low-load adjustment on the character of the extremity of the error curve. From the results shown in these figures it is quite obvious that: (1) The curve of the kind of meter I tested need not necessarily have a sharply rising characteristic (this matter anyhow depends on the setting of the low-load adjustment). (2) By appropriate setting of the low-load adjustment quite small errors at very low loads are compatible with the presence of anti-creep holes. It must be noted too that the curves marked "A" in the two figures were both the result of the same setting of the low-load adjustment; a slightly "faster" setting was responsible for the two B curves. From this we can venture to lay down a reasonable principle, namely, that for any setting of the low-load adjustment the disc with the holes will at very low loads give results relatively *less* fast than the disc without holes. This, of course, is in direct conflict with the author's main proposition "that the anti-creep device is responsible for the positive errors on very low loads." For this would apparently mean that a meter by reason of the anti-creep holes is made fast at very low loads when it otherwise would not be.

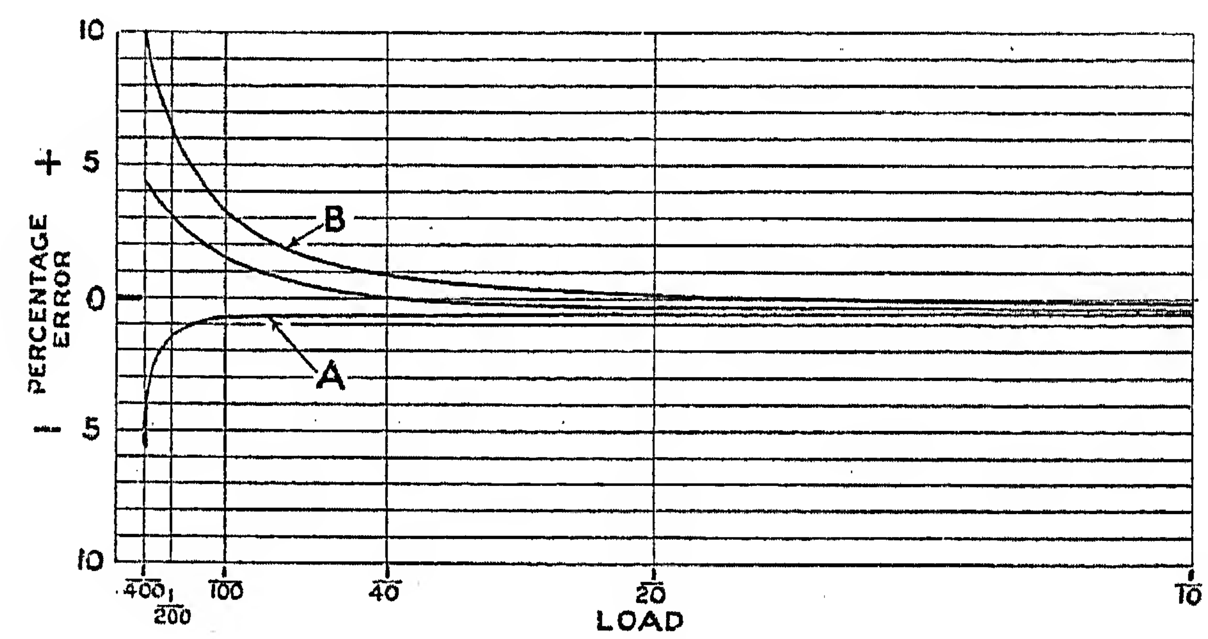


FIG. B.—Curves obtained for disc without anti-creep holes.

In face of the experimental evidence I have submitted I find it very difficult to accept the author's proposition as a valid generalization. Further, I do not see how it derives any support from theoretical considerations. All the experimental facts are susceptible of a very straightforward interpretation. Of the observed facts put forward in the paper, the most significant set is that represented by Fig. 17. It is important for my argument, however, to emphasize that this curve in Fig. 17 is true only for one particular low-load adjustment

setting, namely for that which happened to exist when the meter was undergoing its tests. For other settings the curve will in general be expected to retain (for that particular meter, of course) its same characteristic zigzag shape, but will assume different horizontal positions on the graph in relation to the zero line, the positions being the higher the "faster" the setting. For each different position the average height of the curve above the zero line will be the sole determining factor for the main characteristics of the extremity of the error curve; it will determine whether the curve will rise steeply or not. The anti-creep device plays a well-defined but rather subordinate part of a different kind, in that it influences the stage at which the somewhat sudden drop occurs in the curve near zero load.

It is quite easy to picture the complete set of possible conditions. A sufficiently "fast" setting of the low-load adjustment will locate the torque curve in Fig. 17 wholly above the zero line (signifying that the anti-creep devices are then ineffective), and the extremity of the error curve will rise indefinitely. Less "fast" settings will make the torque curve assume lower positions, so that, as in Fig. 17, parts of the zigzag will fall below the zero line, meaning that the anti-creep devices are holding the disc at these points. Then so long as the average height of the torque curve represents a shunt torque in excess of the frictional retarding torque the extremity of the error curve will rise; it will, however, droop when the average shunt torque is less than the friction torque. In either case there will be a sharp drop in the error curve at some stage of the load when the anti-creep devices begin to arrest the disc's rotation.

When there are no anti-creep devices the shunt torque curve will theoretically be a horizontal straight line, obviously occupying a position on the graph depending on the setting of the low-load adjustment. The extremity of the error curve will correspondingly rise or droop according to whether the shunt torque is in excess of, or less than, the frictional retarding torques.

One may now inquire as to the validity of the apparently rigorous mathematical demonstration the author provides for his proposition. I suggest that this is not a demonstration of a general truth at all; it is indeed merely a verification of a particular instance which gives no warrant for the complete induction (to use the philosophers' term) the author has attempted to build on it. It is really important to appreciate what the author's mathematical analysis actually does. It is based largely upon the torque curve in Fig. 17, and we must recognize that this curve and the "actual test" error curve (1) in Fig. 19 are only two different aspects, experimentally determined, of but one picture, namely the low-load calibration state of the meter as the author tested it. The state of the meter is exhibited in Fig. 17 by one method of examination, and in Fig. 19 by another. The author has chosen to attempt to derive the second way of depicting the facts from the first by a process of mathematical analysis, and to compare the result so obtained with that obtained by experiment. To do this he has dissected the experimentally ascertained curve in Fig. 17, and collected together the other experimentally determined physical constants of the meter; and then he has re-integrated these constituents to find that he has arrived at a theoretical error curve (in Fig. 19) agreeing with the experimental one. In the Summary the author says that this demonstrates his proposition. I think it does not. In my view he has merely supplemented the experimental picture by a parallel mathematical picture, which, being only a kind of paraphrase, adds no demonstrative value to the experimental picture. Suppose there had been complete disaccord between the calculated curve and the test curve in Fig. 19. Would he then have said that consequently the anti-creep holes were *not* the cause of the large positive low-load errors? Such a result would, however, merely have demonstrated that some muddle had occurred in the mathematics, or that some error had arisen in evaluating the meter's physical constants. In the absence of any such mishaps the two curves ought in any case to agree, quite independently of any assumptions as to the anti-creep holes making the meter fast, slow, or anything else, these assumptions being in my view not only unnecessary but irrelevant in analyses of the kind we are considering. The truth of this can be shown by ignoring or deliberately denying any assumption as to the effect of the anti-creep holes while making an analysis in another way. Let us assume for this purpose that the shunt torque is quite uniform during one revolution of the disc, and maintains the average value represented by the curve in Fig. 17 which can be taken as approximately 15 torsion degrees, equivalent to 5·85 dyne-cm. Let us also use the author's own values for the other constants in his equation (1), which I will restate (for steady conditions, and for this particular meter only) as:—

$$\text{Disc speed} = (K_1 I - F + 5\cdot 84)/N$$

From Section 4(*a*) we find that $F = 2\cdot 1$ dyne-cm, and from Section 4(*c*) we find that $K_1 = 298$ dyne-cm per ampere.

Thus:—

Disc speed at full load (20 amperes)	$= (5\,960 + 3\cdot 74)/N$
Disc speed at 1/40 load	$= (149 + 3\cdot 74)/N$
Disc speed at 1/80 load	$= (74\cdot 5 + 3\cdot 74)/N$
Disc speed at 1/100 load	$= (59\cdot 6 + 3\cdot 74)/N$
Disc speed at 1/200 load	$= (29\cdot 8 + 3\cdot 74)/N$
Disc speed at 1/400 load	$= (14\cdot 9 + 3\cdot 74)/N$

The departures from strict proportionality between the disc speed and the load, can be seen to be successively —from 1/40 load downwards—2·5, 5·0, 6·3, 12·5, and 25·1 per cent. These represent the meter's percentage errors, assuming that it was correct at full load. Referring to Fig. 19 again, an error curve drawn from these figures would agree with the test curve as closely as the author's calculated curve does, so far as the large positive errors are concerned. One would therefore be entitled to say, borrowing some of the author's words, that "there is sufficiently close agreement between the curves to leave little or no doubt as to the validity of the mathematical treatment, and this in turn shows clearly that the anti-creep device" has nothing whatever to do with the large positive errors on very low loads, since any anti-creep device hypothesis was expressly excluded. Here I must leave the author to choose which horn of the dilemma he prefers. If my mode of reasoning is invalid his, being the same, is invalid too; but if it is

valid it produces a valid conclusion which completely contradicts his own.

I feel it necessary to point out that the modern induction-type meter has an overall range of accuracy much exceeding that of all other comparable instruments, and, anti-creep devices or no anti-creep devices, can be adjusted as I have shown to integrate within very close tolerances down to 1/100th or less of its nominal full load. From the point of view of its possible effect on the outside world, therefore, the use of the term "intolerable low-load errors" seems to me to be unfortunate; all the more so as a meter, if its size is only reasonably proportioned to the loads it has to measure, will register so small a fraction of its aggregate registrations at loads less than 1/40th that errors at lower loads than these are almost completely submerged.

Mr. A. H. Gray: Mr. Hill has already drawn attention to the phrase (page 367) "the cause of the intolerable low-load errors"; I suggest that the phrase should have read "the cause of the errors at the intolerable low loads." I do not agree that an electricity meter is an inaccurate mechanism, and in this connection I would refer the author to his own curve (Fig. 4), wherein he shows that the meter is capable of a high degree of accuracy, from the low loads met with in practice up to 3 times full load. Even at the extremely low loads used in his experiments one is bound to admit that the performance is highly creditable, seeing that the driving torque for meters running at 1/200th full load is of the order of 19·5 dyne-cm.

The author states that the shunt compensation is 5·9 dyne-cm, whilst the frictional torque is 2·1 dyne-cm. Does he consider these to be representative of an average modern meter? I consider that these errors are greater than one generally expects; this may be due to over-compensation.

On page 356 the author mentions that the compensating devices used to control the overload characteristics vary with the power factor of the load. Can he quote any specific figures with regard to these? I believe that this variation is very small, particularly when the common form of magnetic shunt fitted to the series magnet is used.

On the same page he mentions that low-speed meters provide good overload characteristics; this is perfectly true, but they are also rather prone to erratic behaviour at extremely low loads, since the momentum of the rotor system is less.

In Fig. 17 the author plots the shunt torque against the position of the anti-creep holes, and points out that the two holes do not produce symmetrical effects: might not this discrepancy be due to mechanical unbalance? This unbalance will produce a decrement curve very similar to those reproduced in the paper, and I should like to know what precautions the author has taken to prevent such errors arising.

Whilst it is agreed that anti-creep devices produce variation in the torque, I am afraid that meter users will be very loath to abolish them, since not only will the calibration of a meter become more difficult but, what is perhaps even more important, the removal of these devices might lead to controversy between the supply engineer and his consumer.

Finally, I would suggest that there are other fields of exploration than those dealt with in the paper which might produce more practical solutions of the problem of low-load errors. To quote only one example, the low-load characteristics of a meter are dependent to some extent on the series-magnet circuit; a study of this might provide a practical solution to the problem.

Prof. W. Cramp: As a piece of careful analysis in which delicate tests have had to be invented, delicate apparatus constructed, and delicate measurements made, the author's work will stand critical examination. It is an investigation that will not have to be done again, and one cannot say anything higher than that. There are no less than 59 points upon the curve reproduced in Fig. 17, and great labour and patience were involved in obtaining each one of those points. Moreover, they are situated exactly at those places where one wants to know what the function is doing, i.e. wherever there is a slight bend in the curve. To anyone who is doing experimental work, Fig. 17 will stand as a model of how the work ought to be done in order to carry conviction.

The anti-creep devices consist of little holes through the meter disc or little slots in its edge, and no one seems to know precisely where they should be placed or of what size they should be. It is clearly necessary to have these devices, but it is also desirable that we should understand what they do and how to design them. The paper ought to be followed up by work showing a proper basis for the calculation of these details.

What happens if a meter at low load has, owing to anti-creep devices, a negative torque? If the disc comes to rest at a place where the torque has a negative value, then, when power is slowly put on to it again, presumably the negative torque will still exist. Will the meter for a fraction of a revolution run the wrong way round? In other words, ought negative torque to be avoided? Mr. Hill has suggested that the proper way to improve the low-load curve is to bring the zigzag curve lower down. Does the author agree that, under those conditions, there will be a tendency at certain positions for the discs to run backwards?

Turning to the question of the pole-face shape, there is in this country one who knows a great deal about this problem, in Dr. D. K. Morris. From his analysis of what takes place in the eddy-current brake it is possible to predict to some extent those changes to which the author refers. The meter manufacturers might make use of a paper,* written for an entirely different purpose, in order to solve the problem raised by the present author.

With reference to Mr. Hill's criticisms, I should like to say one word about the general meaning of a paper of this kind. The author sets out to find an equation to fit the behaviour of a difficult piece of apparatus. He finds that equation, and he shows what each term means. He does not by so doing say that every meter made on this principle will have the same constants, but he does show the meter maker what term he must alter to obtain a desired change in the characteristic curve. This is the object which the author had in view, and this, I think, he has very successfully achieved.

In view of the magnitude of the errors which the author is attempting to correct, it is interesting to com-

* *Journal I.E.E.*, 1905, vol. 35, p. 445.

pare the present-day accuracy of electricity meters with corresponding figures for gas meters. The author's typical curve for 1934 (Fig. 4) shows a maximum error of about $\frac{1}{4}$ per cent from 1/10th to 3 times full load. The error which he regards as "intolerable" is 1 per cent at 1/20th full load. The statutory error allowed in gas meters is 5 per cent at full load, i.e. 2 per cent fast in favour of the supply authority and 3 per cent slow in favour of the consumer. I recently found that my gas meter was "certified as correct" although the official test showed that, actually, it was reading 1·63 per cent fast.

Mr. O. Howarth: While Prof. Cramp was talking it occurred to me that there is a rule which determines the size of the holes or slots in the meter disc, namely the greater the friction in the working parts of the meter the larger the holes.

Figs. 17 and 18 would have conveyed more information if, instead of being plotted against torsion angle, they had been plotted against the torque expressed, say, as a percentage of normal torque on the meter. The curves would then have given some idea of the variation in percentage error for different positions of the disc during rotation. One point about these curves is that the average height above the zero line determines the shape of the curve below 1/20th load and also the starting current. It is absolutely essential that there should be a position of negative torque. While the errors may or may not be intolerable, it would certainly be intolerable to have meters which were in the habit of creeping. It would seem that if this curve could be flattened to some extent, and the highest and lowest points brought nearer together, better results could be obtained at lower loads. There would possibly be a lower starting current, and it would certainly necessitate less friction.

We have made use of an ordinary induction meter as a device for timing switches and relays by dividing the disc into 100 divisions and arranging that the load is on during the period we want to time, namely from the inception of current to the opening of the switch. We have found, by comparing it with the cycle counter, that the accuracy of the device is quite good within the limits of accuracy of such measurements.

I hope the work which has been done by the author will be extended. The errors of meters of low power factor are quite important nowadays in view of the popularity of all-mains wireless sets. One would like to know whether the results would be the same at low power factor as with unity power factor. From the information given in the paper about the effect of the series flux one would expect that the power factor would not influence the shape of the error curve between 1/20th and 1/400th load, provided the quadrature adjustment did not introduce any error. It would be interesting to have Fig. 17 plotted for various voltages, both for normal voltage and for values differing from it by ± 10 per cent.

Mr. J. W. Carter: With reference to the author's conclusion that the anti-creep device is responsible for the positive errors shown in the case investigated, it is interesting to note that the complete meter curves given in the paper show a change in the direction of slope at about 1/8th load. The tendency, immediately before this point is reached, is for the curve to droop, and the question appears to be whether the anti-creep device has on the average a retarding effect which contributes to the drooping characteristic, and thus makes it necessary to have a strong shunt torque to give accuracy at 1/40th load and entailing large positive errors lower down. I have had practical experience with meters in which the anti-creep device consists of four equally spaced holes in the disc, and one of the features which distinguished these meters was an inclination to develop a marked creep after they had been in service for a period. I thought at the time that this was due to an alteration in the electromagnetic system, but it now appears that the meters may have been suffering from too much anti-creep device, and were illustrating the fact that the disc holes may be responsible for positive errors extending even to infinity. It would be interesting to have the author's opinion on the effect of the disposition of the holes in reference to the brake magnet, and also the effect of the variation in disc speed on stroboscopic testing.

Mr. G. F. Shotter: Whilst congratulating the author on his experimental and mathematical investigation, I should like to criticize some of his statements.

On page 356 he says that "in general, compensation devices to obtain better overload characteristics are not desirable" and also that "one of their defects is that the correcting action is dependent upon the current." The latter statement is perfectly true and in a badly designed meter considerable difference can exist between, say, 150 per cent load at unity power factor, and 300 per cent load at 0·5 power factor, i.e. with the same watt loading. On the other hand, the statement that they are undesirable may leave a false impression as to what can be done with such compensation. As an illustration, a meter which I tested showed an actual difference at the above load condition of 1·2 per cent, the calculated figure being 1·5 per cent. The meter in question was accurate to within 1 per cent up to 300 per cent load, at both unity and 0·5 power factor.

With reference to the curves on page 357, I am of the opinion that it is necessary to get a different sense of proportion with regard to meters A and B. We have from time to time done a considerable amount of work on the low-load characteristics of meters, and have never found any difficulty (where time is no object) in obtaining fairly flat curves down to the lowest loads taken by the author. To illustrate this point, Fig. C gives the characteristics of a modern meter down to 1/400 load. Curve (1) is with the instrument set 0·3 per cent fast at 1/20 load, which brings the meter within $\pm$ 1 per cent from 1/400 to 1/20 load. Curve (2) gives the characteristics of the meter set 1 per cent fast at 1/20 load, showing that a difference of 0·7 per cent + at 1/20 load makes the meter approximately 14 per cent fast at 1/400 load.

The second point I wish to illustrate is shown by curves (1*a*) and (2*a*) in Fig. C, where the effect of adding a friction torque of 3 dyne-cm to each meter is shown. To emphasize our sense of proportion, this is equivalent to an error of just over 1 per cent at 1/20 load on a meter having a full load torque of 5 gramme-cm. This value of 3 dyne-cm is obviously not an abnormal increase in friction in bearings and clockwork over a period of 7 years in service. To illustrate further the

effect of a small increase in friction I have corrected in Fig. D curve A in the author's Fig. 5 by the addition of 3 dyne-cm.

The above illustrations will, I think, serve to show what I believe to be the correct sense of proportion required in the author's study of the low-load properties of a meter.

With regard to the author's statement on page 367 that "the anti-creep device is responsible for the positive errors on very low loads," I think that he is not justified in making this a general statement. The resultant torque produced by the anti-creep device taken over one complete revolution is zero, and consequently it cannot fundamentally be responsible for the low-load errors. To emphasize this point let us imagine an anti-creep device purely frictional in operation and which is released at the smallest possible load or, say, below 1/200 load, which is the B.S.S. starting-load. This would be equally as effective and would be the ideal anti-creep device for a meter, as it would entail setting the friction compensating torque to a minimum. From this it will be seen that the cause of any large positive errors at very low loads is due to excessive use of the low-load friction-compensating torque, and is in no way due to the anti-creep device. The necessity for the use of an anti-creep device is due to (1) variable friction, (2) the difference between starting and running friction, and (3) varying voltage, frequency, etc. As these three points are beyond the control of the meter manufacturer, it is essential to provide some device to prevent the meter from running at no load, and if this device is badly designed the errors at very low loads can become extremely high, due to its masking the use of large friction compensation. For example, we can adjust a meter to 2 per cent slow at full load and $2\frac{1}{2}$ per cent fast at 1/20 load, and have the anti-creep device designed to allow the meter to start at 1/200 load. This arrangement will give the maximum errors at very low loads and yet the meter will be within the B.S.S. accuracy. It will be seen from this that the fundamental curve of a meter might have some effect on the ultimate design of the anti-creep device.

Mr. Albert Page: What is an eddy current? The whole performance of the meter depends on the answer to this question, and yet we do not know what it is. An eddy current cannot be a flow of electricity in the ordinary sense, otherwise the whole charge causing the currents would immediately be dissipated throughou the disc, which, according to Fig. 17, evidently does not occur. The phenomenon has certain characteristics akin to electricity, but it has several decided distinctions. To my method of reasoning there is no such thing as eddy currents, nor local electricity generating them. They consist of a palpitation of the constituent of the material in which they are formed, causing a dissipation of energy through a rapid change of energy levels of the atoms. This subject is one we must understand thoroughly before we can proceed further.

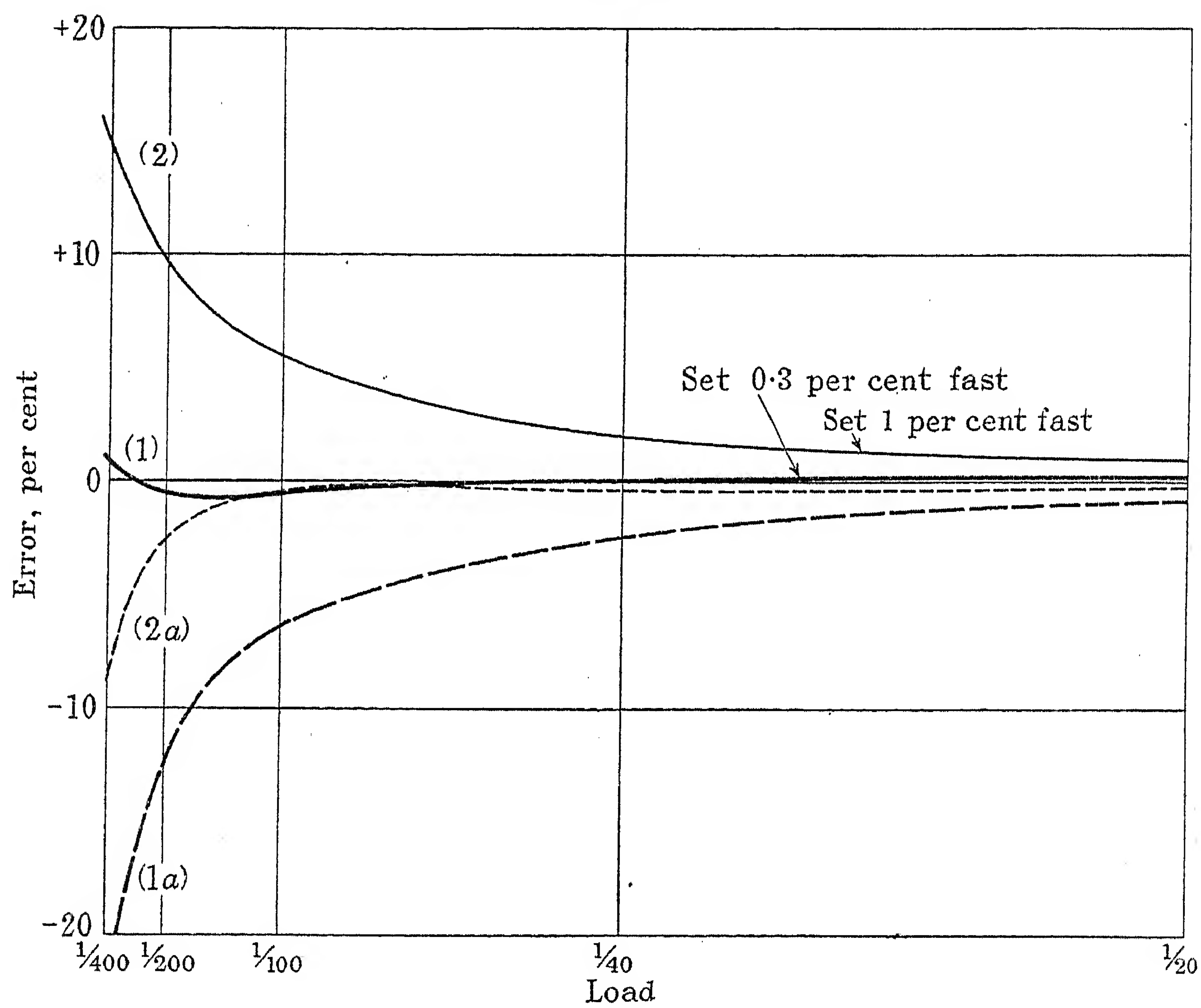


Fig. C.

The author in his analysis has made several assumptions, conducted experiments on the basis of these assumptions, and, as one would expect if his analysis were correct, has proved that as the anti-creep device is the only variable driving force independent of the load, it is the cause of the errors on light loads. I think this conclusion could have been equally well

arrived at by common-sense reasoning, without any experimental analysis. Equally well one could evolve the design of a meter, based on the author's conclusions, which would not have the inherent fault he describes. It would be a meter so arranged that the shunt flux would drive it at a constant speed, at no load, and a deduction would require to be made from the registrations directly proportional to the interval between the meter readings. I do not put forward this suggestion seriously, however, as I know it would not be accepted by the industry.

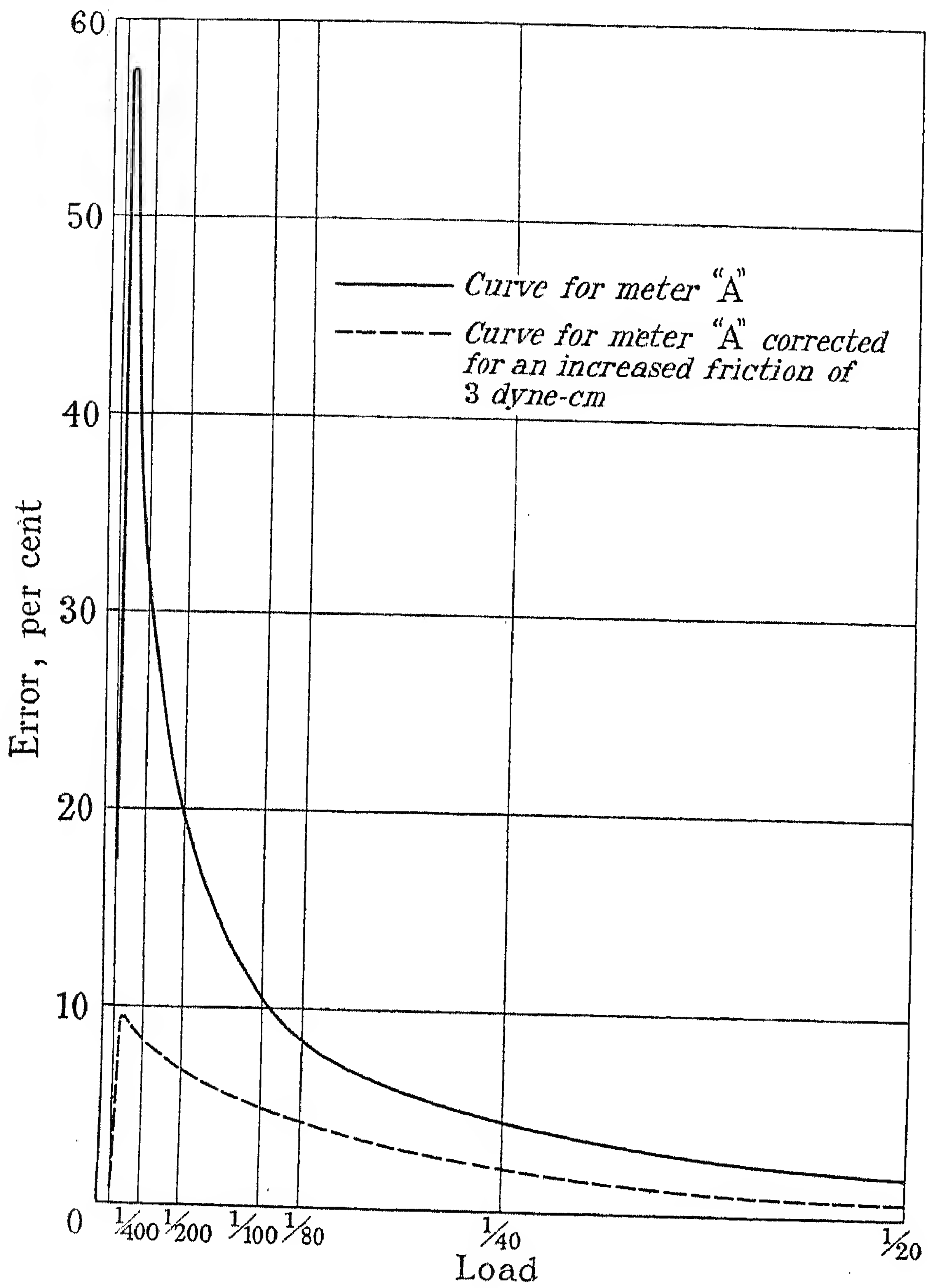


FIG. D.

Load	1/20	1/40	1/100	1/200	1/400
Meter "A"	2·0+	4·6+	10·8+	19·0+	32·2+
Meter "A" + 3 dyne-cm friction	0·8+	2·2+	4·8+	7·0+	8·2+
Difference	1·2−	2·4−	6·0−	12·0−	24·0−

When analysing any problem it is desirable, now and again, to review the position generally in order to steer a well-balanced course. How easy it is to overlook the object in view is illustrated in Figs. 10, 11, 13, and 16. These are all exactly the same, and lead to the same results. Thus Fig. 10 gives $6 \cdot 14 \times (60/40) \times (1/6 \cdot 28) = 1\,438$ dyne-cm per radian. Fig. 11 gives the same result (in this figure I recommend that relative amperes should be placed along the axis of abscissæ). Fig. 13 gives the following: 1 amp. = 2 r.p.m. = $0 \cdot 209$ radian per sec. Therefore the torque is equal to $3 \cdot 04 \times 981 \times (1/0 \cdot 209) = 1\,438$ dyne-cm per radian. Fig. 16 gives $(74 \cdot 8/250) \times (1\,000/1) \times (60/2) \times (1/6 \cdot 28) = 1\,438$ dyne-cm per radian. All the diagrams referred to have the same fault, namely, that they cover a range outside that of the present investigation. The useful part of the curve, namely loads below 1/20th full load, is represented by less than $\frac{1}{8}$-in. square. In passing, I would mention that Fig. 12 could be used to advantage for measuring the shunt- and series-flux torque. Further, the retardation test is not quite as accurate as it is represented on page 359, at the foot of col. 1. The degree markings are of practically no use except in the final half-revolution, as will be seen from Fig. 7, which covers 45°. The time-intervals soon become so short that the author, in constructing Fig. 8, has deemed it advisable to plot points only per revolution.

I now come to the important part of my remarks. In commencing the investigations the author makes certain assumptions: firstly, a constant voltage for the driving torque which corresponds to the conditions of the test; and secondly, that the shunt torque varies with the angle θ. It is equally true, however, that if the shunt torque varies with the angle θ on account of the holes in the disc, so also must the main driving torque, the series driving torque, and the retarding torques of the permanent magnet, shunt flux, and series flux respectively. The whole paper is devoted to the analysis of one curve, namely Fig. 17, whereas for each heading there should be a family of curves relating to each of the test loads. Apart from this point the mathematical exercises given in the paper are correct, and also their solution. The elaboration of the arithmetical part is, however, enormously laborious. As the differences met with in evaluating equation (16) are extremely small, it is necessary to employ 7-figure logarithms. Since the experimental data are not accurate to this degree, the value of the final result is questionable. Further, as the calculations for each line of Fig. 18 take approximately 1 hour and 16 such calculations are necessary to secure one point on Fig. 19, I am most sceptical about the location of the two left-hand crosses in the latter figure. It is also indeed fortunate, for the author, that line (7) of Fig. 18 was horizontal, to form a starting-point in the calculations. Indeed this is the only part of the curve which is not as one would expect, and if correct it shows the presence of some foreign substance in the disc causing a local disturbance at precisely the point in the curve where it would prove of greatest advantage.

Mr. A. Felton (*communicated*): I find myself at variance with the author as regards both his premises and his conclusions.

The shape of a meter curve above $\frac{1}{4}$ full load is inherent in the design, and to this extent the curve of Fig. 4 may be accepted as typical of most modern induction meters. The shape of that portion of the curve below $\frac{1}{4}$ load can be varied, however, in any given meter by manipulation of the low-load adjustment. It is the general practice of manufacturers to adjust meters to run as fast as possible (within the limits of B.S.S. No. 37) at 1/20th load: this is done, no doubt, in order to allow a large factor of safety on starting current and to allow for the anticipated increase of friction as the bearings

become worn. The consequence is that, at very light loads, the positive errors, expressed on a percentage basis, become quite large. If in a particular case these errors are regarded as "intolerable" they can be reduced or even reversed in sign by means of the low-load adjustment which is provided for the purpose.

In order to verify this point, experiments were carried out on a 20-ampere meter of a good modern type chosen at random. The curves of Fig. E were obtained. Curve A represents the light-load characteristic of the meter as adjusted by the manufacturer. The characteristic was modified to curve B by one turn of the low-load adjustment in the positive direction, and to curve C by two turns in the opposite direction. In each condition the meter started and continued to run at a load less than 1/300th full load.

It is clear, therefore, in this case, that the cause of the large positive errors of curve B and of the negative errors of curve C is the setting of the low-load adjustment.

There is fair evidence for believing that friction in a

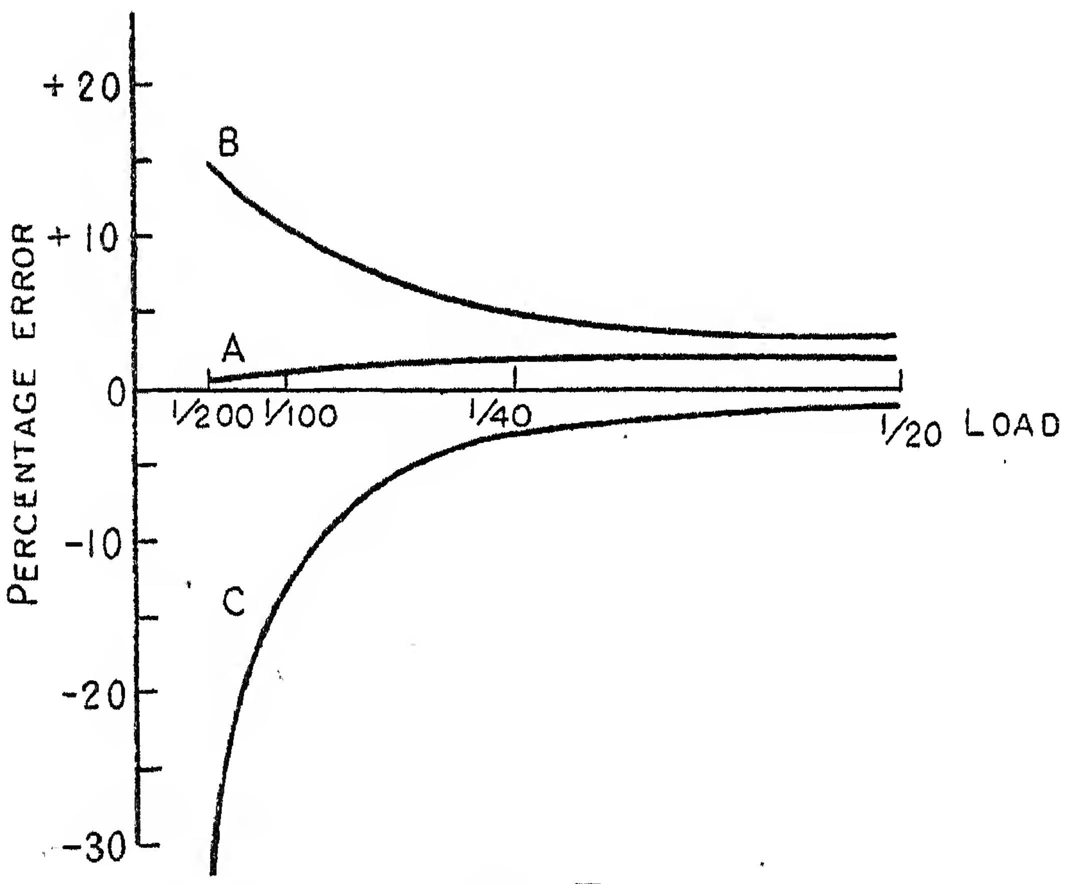


Fig. E.

meter is independent of speed. In order to compensate for this effect, therefore, it is necessary to provide a constant forward driving torque, and this is accomplished in modern meters by a movable vane or other device on the shunt element. If the "shunt torque" is made greater than the frictional torque the meter will run forward at zero load, and in order to prevent this the anti-creep device is provided. At any load great enough to overcome the anti-creep device, however, there will be an effectively constant forward torque added to the series torque, and the meter will exhibit a positive error. There is nothing remarkable, therefore, about the character of the low-load curve in Fig. 5: it is roughly the rectangular hyperbola to be expected from the addition of a constant component to a component proportional to the load.

The curve of Fig. 17 is interesting as showing the variation of shunt torque with the position of the rotor. The shape is broadly what might have been expected: over one complete revolution of the disc, the mean torque is positive, but the presence of negative peaks prevents rotation below a certain minimum load. The value of the experiment would have been very much enhanced at small expense of time and trouble by repetition tests: (a) at other settings of the low-load adjustment, and (b) using a disc without anti-creep holes.

In the latter case it is to be expected that the curve would be wholly above the zero line, and slightly undulating on account of accidental non-homogeneity of the disc. In the former case either of two effects is possible: the scale of the curve might alter (with perhaps minor modifications of shape) or the curve might remain the same but with a zero shift depending on the setting of the adjustment. There is some practical evidence for believing that the latter effect actually occurs.

Assuming this to be so, the general behaviour of the meter may be predicted without mathematical analysis.

If the zero of Fig. 17 be shifted upwards by an amount equal to the friction, the resulting curve about the new zero represents the variation of net torque during a revolution due to the combined effects of friction and shunt torques. Assuming the acceleration of the disc to be negligible, if the positive area between the curve and the new zero line is greater than the negative area, the meter will run fast at any load greater than the starting load, which is represented by the depth of the lowest negative peak below zero. In order to make the meter register correctly, it is necessary to shift the zero line up until the positive and negative areas become equal. By hypothesis, which is borne out by experience, this can be done by manipulation of the low-load adjustment. The upward shift of zero results in a deepening of the lowest negative peak, and in consequence the starting load becomes greater, and adjustments in practice must be made with due regard for this fact.

The author has, by means of his mathematical analysis, arrived at a very similar conclusion, which is represented by his equation:

$$\mathbf{I}d^2\theta/dt^2 + Nd\theta/dt = U - F + a + b\theta$$

It is to his deductions from this point onwards that I take strong exception. In order to make the meter register correctly the author suggests that $b\theta$ should be made zero and that a should be made equal to F. This solution has been obvious from the first introduction of low-load compensation, but it is impracticable on account of the impossibility of obtaining exact equality of friction and shunt-running torques. The anti-creep device, represented by the term $b\theta$, was then purposely introduced.

The practical solution is to make the mean value of $a + b\theta$ over one revolution of the disc equal to F. The means of effecting either solution is manipulation of the low-load adjustment.

There is not the slightest evidence for the statement that the anti-creep device is responsible for the positive errors on very low loads. The cause is the shunt torque, which has been adjusted to too high a value.

Mr. R. S. J. Spilsbury (*communicated*): I should like to refer to a paper* published a few years ago which gives a detailed theoretical and experimental study of the induction meter, and includes the characteristics of twelve meter types. The curves are not given for loads

* S. Jimbo: "The Characteristics of the Induction Watt-hour Meter," Researches of the Electrotechnical Laboratory, Tokyo, No. 320, 1931. (In English.)

below 1/20th of rated load: the fairly large errors at this load are, however, correctly given by calculations which do not take account of any effect of the anti-creep device.

With regard to the main conclusion of the present paper, namely that positive errors at low loads are due to the anti-creep device, it seems clear that so long as the disc speed remains substantially constant during a revolution the local fluctuations of torque due to the anti-creep device can have no effect on the time of rotation, and the effective value of the torque shown by the curve of Fig. 17 can be estimated by averaging the positive and negative areas: the value of this average is, of course, determined by the setting of the low-load compensator, and any positive errors at low loads will be due to an incorrect setting. If, however, the disc speed varies significantly during a revolution, the effect, as the author mentions, will be that the time of travel for the regions of low torque will be excessively great, and will more than compensate for the high speed during intervals of high torque: hence the meter speed will be lower than the value calculated from the average torque of Fig. 17, and the errors due to the anti-creep device itself will be negative.

In conclusion, I think the author is to be congratulated on a combination of great care in his experimental work with great good fortune in his selection of a meter. When it is considered that each of the errors of Fig. 19 represents a torque of the order of 1/2 000th of the full-load value, it is evident that a very small variation of friction could have completely destroyed the correspondence between experiment and calculation.

Dr. T. Havekin (*in reply*): Mr. Hill's elegant exposition of the "fallacy of antiquity" merely serves to show that he has an aptitude for describing, rather than for discerning, the fallacy. That my conclusion is based solely on the simultaneous existence of A and B is Mr. Hill's assumption, and when he shows, by the logical development of his theme, that the positive errors cause the holes in the disc, he does at least exhibit an unwavering confidence in his own assumption.

Mr. Hill is apparently too firmly convinced of the soundness of his case to be satisfied with a mere denial of the truth of my conclusion, for he hastens to provide what is described as experimental evidence. A meter is submitted to two tests. For the first test a plain disc is fitted, and for the second test the same disc is used after anti-creep holes have been drilled in it. It is insisted that the setting of the low-load adjustment, and all the other conditions, should be kept the same for both tests. The words "all the other conditions" suggest the meticulous methods of the scientific investigator, but the test results presented in Figs. A and B fail to support this impression. I regret that I cannot regard the test as an experimental demonstration of Mr. Hill's claims. What it does clearly show is a resignation to the tyranny of the anti-creep device, and a failure to comply with a condition which must be imposed. Of all essential conditions associated with low-load operation, a satisfactory value of starting load is the *sine qua non*. The calibration indicated by curve A, Fig. A, shows very little promise of complying even with B.S.I. recommendations, and it must be agreed that all good meters start on a load which is more than likely to be of the order of 1/400 of full load.

We shall probably see the matter in better perspective if we suppose that I am a prospective purchaser. As such, I would approve curve A, Fig. B, except that there is no anti-creep device. With the latter included, I cannot accept curve A, Fig. A, because the error on 1/10 load is now $-1\cdot 5$ per cent, and the starting load is quite unsatisfactory. I have noted that curve A, Fig. B, shows an error of $-0\cdot 5$ per cent on 1/10 load, and starting load 1/400 of full load, and I am not unreasonable when I insist that these two points, at least, shall be the same after the anti-creep device has been provided. It will therefore be essential to advance the low-load adjustment, and the errors in general between 1/10 and starting load will then be found to be more positive than they were without the anti-creep device. Mr. Hill may complain that this is the result of advancing the low-load adjustment; but he must remember that the influence of the anti-creep device left him with no alternative.

There is rather a querulous note behind Mr. Hill's references to my having drawn conclusions from tests on one meter, in one particular state of low-load adjustment. If I had tested dozens of meters I am confident that it would have made no difference to the conclusions

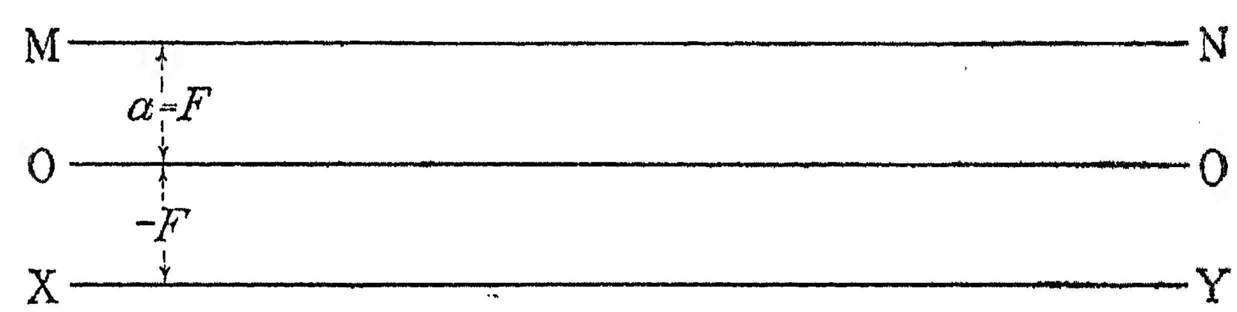


FIG. F.

laid down in the paper, but in order to bestow that generalized form which I had confidently left to the imagination I will present the matter in this way:

(1) All induction meters are subject to bearing and counter friction.

(2) The anti-creep device always imparts an irregular form to the independent shunt torque as the rotor turns.

(3) All induction meters must begin to register on a load not exceeding 1/200 of full load.

Let the line OO in Fig. F represent the line of zero torque, and let the line XY indicate the friction torque $-F$ during one complete revolution. In the absence of any anti-creep device the low-load adjustment can be set so that a presumably steady independent shunt torque a, indicated by line MN, can be made equal in magnitude but opposite in sign to F. Under these conditions the meter will begin and will continue to register accurately on the slightest load. Just before rotation starts, the friction is rather greater in value than it is after rotation has been established, and in any case it is rather too much to expect that a can be adjusted to be exactly equal to F. It is more than likely that a will be slightly greater than F, so that after the load has been removed rotation continues. Rotation on no load cannot be permitted, so that we are compelled to rely on some form of anti-creep device as a remedy. The supposed effect of the anti-creep device is shown in Fig. G, where it will be seen that the torque variation

is leniently assumed to alternate, during a limited range of rotation, between the values $\pm F$ for each complete revolution of the rotor. The starting load will be determined by the torque conditions at the point Z, and will obviously have a value corresponding to a torque slightly in excess of F. If the meter is to start and continue to register on the application of the slightest load as it did without the anti-creep device,

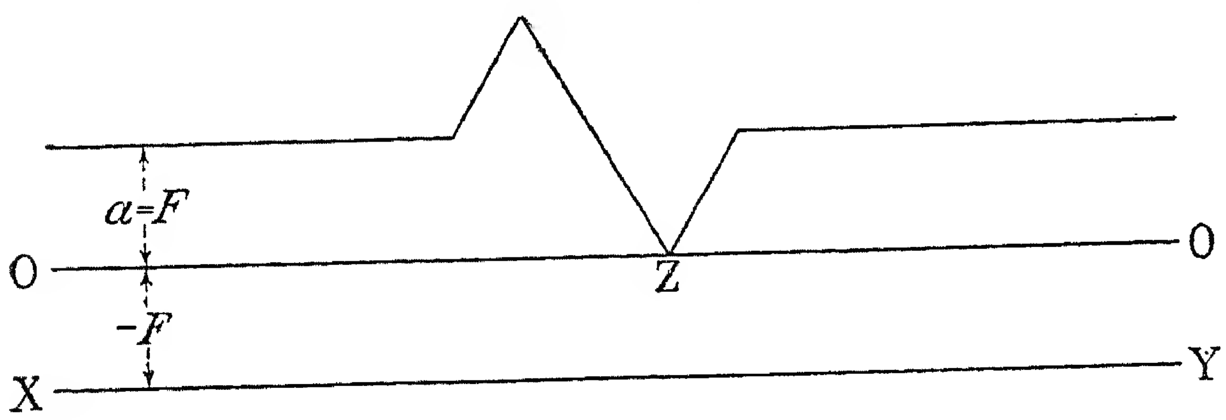


FIG. G.

then the low-load adjustment must be advanced to fulfil the conditions shown in Fig. H. This adjustment, however, results in a shunt torque whose mean value a is equal to $2F$, from which it follows that the low-load errors are now greater and more positive than they were before the advent of the anti-creep device. This conclusion is unaltered no matter what value of starting load is decided upon, provided it is the same both with and without the anti-creep device.

Mr. Hill describes the mathematical demonstration as " merely a verification of a particular instance." Though the phrase appears to have slipped in as a perfunctory comment, it is, nevertheless, an admission that he accepts equation (5) as the true equation of motion for the particular meter tested. This is the truly significant point in his criticism, for the remainder is simply a denial that the analysis furnishes any information about the effects of the anti-creep device on the low-load curve. As he agrees that the mathematical

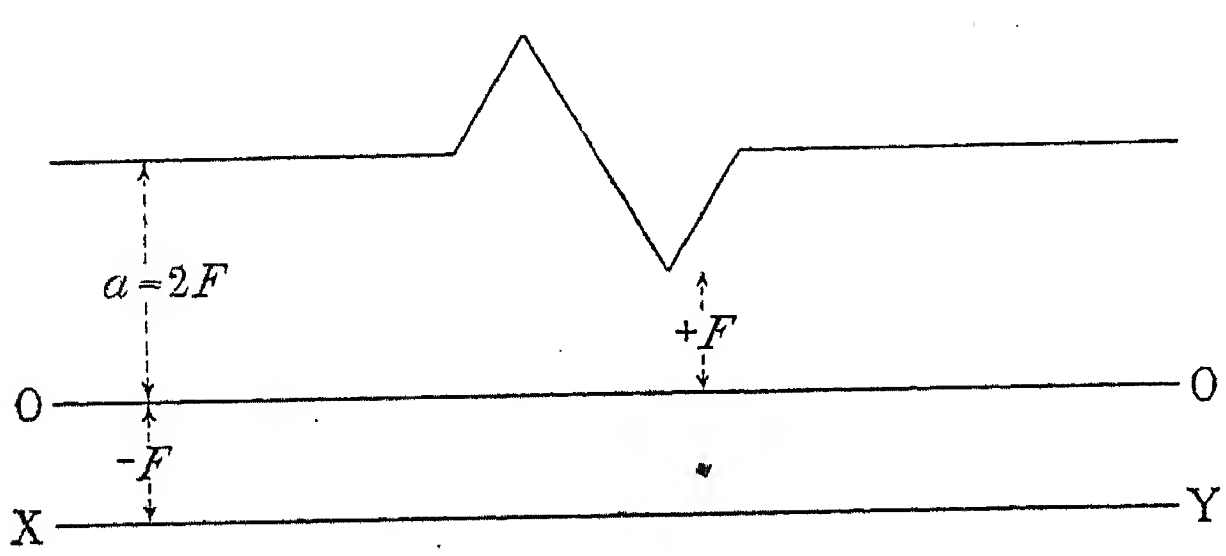


FIG. H.

treatment verifies this particular case, it follows that he accepts the expression $a + b\theta$ as an accurate representation, over a given range, of the independent shunt torque. Moreover, he must admit that when a is greater than F the error on a given load will be positive. What he denies is that $b\theta$, due to the anti-creep device, has any influence in determining the magnitude of a. The starting load in our special case is of the order of 1/500 of full load, and this could only be achieved by increasing the value of a, in order to reduce the torque corresponding to the maximum negative peak, in the shunt-torque curve, due to $b\theta$.

While this reasoning is concerned with a particular case, it provides all that is necessary as a basis for general conclusions. If Mr. Hill has a meter which registers satisfactorily in all respects on very low loads in spite of the presence of the anti-creep device, he may now realize that such a meter is a tribute to the designer, and not a contradiction of my thesis.

I would assure Mr. Gray that I regard the modern induction meter as a highly accurate instrument over its stated range of operation. After the successful overload improvement that we have noticed during recent years, however, it is quite reasonable to examine low-load conditions. After all, the meter must start on 1/200 load, and yet it is not expected to register accurately until the load has been increased to 1/20 of full load.

With reference to the friction torque, I should think that 2·0 dyne-cm is quite a good figure. While I can give no definite figure for the average shunt compensation torque, there is abundant evidence in the steeply-rising curve below 1/40 load that some of the most modern meters have considerable disparity between the average value of the shunt torque and the friction torque.

I am afraid I can give no figure for the variation with power factor of the overload compensating effects. The compensation depends in general on the value of the main current, and must therefore vary with power factor. I believe, however, that the overload improvement is due largely to lower speed, and so the demands on compensating devices are now lighter than they were formerly.

The suspended rotor turned truly about the one axis of rotation during the tests for Fig. 17. If there had been any lateral movement during the test it would have been easily detected by observing the side movement of the pivot over the slightly lowered bearing.

I do not recommend the abolition of the anti-creep device.

I am indebted to Prof. Cramp for his appreciative remarks. Perhaps the most satisfactory way to answer the question concerning negative torque will be to refer to the meter dealt with in the paper. Suppose that when the load is removed the disc is in the position corresponding to the negative peak at the junction of lines 14 and 15, in Fig. 18. At this point the negative torque is greater in value than the friction torque, and, as a consequence, reverse rotation is imparted to the disc. As the disc turns, however, the shunt torque becomes less negative, and so the disc comes to rest when the shunt and friction torques are equal in value. If power is now applied, a positive torque tends to cause positive rotation, but before continuous positive rotation is established the positive torque must be at least equal to the friction torque plus the shunt torque at the maximum negative peak. Thus the starting load of the meter is determined; and if this is to remain at 1/500 load, as found by test, the low-load curve cannot be improved by lowering the zigzag curve but only by reducing the wide differences between the positive and negative peaks in Fig. 18.

In reply to Mr. Howarth, I have not carried out any tests on very low loads at low power factor, but I see no reason to expect that the results of such tests would be different from those for unity power factor. I commend

Mr. Howarth's contribution to those who disagree with my conclusion.

It is rather curious that Mr. Carter should refer to the change of slope at or about $\frac{1}{8}$ load, because it was a desire to eliminate this low-load droop that first led me to suppose, some years ago, that a thorough investigation of conditions on very low loads might suggest a solution. I think it is now safe to say that the anti-creep device is in no way responsible for the low-load droop. In some meters this droop is influenced considerably by the shape of the core of the series element, and also by the disposition of the winding on the core. This alone suggests that the operating conditions on $\frac{1}{8}$ load are likely to resemble the conditions on normal, rather than on very low, loads.

With reference to the meters which have been found to creep on shunt after being in service for a time, I am afraid I cannot suggest other than obvious possible causes. Given proper initial calibration I know of no reason to suspect that the anti-creep device alone could be the cause of this condition. If Mr. Carter will give me particulars of the structural details of the meters I shall be pleased to discuss the matter with him.

I have no experimental results which would show completely how the brake torque is influenced by the location of the anti-creep holes in relation to the permanent magnet. Without experimental evidence, I feel that an expression of opinion on this matter would be of little value.

Considering the meter dealt with in the paper, the magnitude of the speed variation is perhaps sufficiently clearly indicated by the following figures, which are based on calculation.

Load	Appropriate max. and min. speed during one revolution
1/20	Mean speed $\pm$ 3 per cent
1/40	Mean speed $\pm$ 7 per cent
1/80	Mean speed $\pm$ 14 per cent
1/200	Mean speed $\pm$ 30 per cent

The mean speed is based on the calculated time for one revolution on the particular load concerned. It is evident that the speed variation on half or full load will be quite imperceptible, and stroboscopic testing on such loads will be in no way jeopardized by the presence of the anti-creep device.

Mr. Page assures us that the whole performance of the meter depends on the answer to the question, What is an eddy current? I would suggest that the answer to this question would have no bearing on the present investigation, because we are concerned with well-known effects rather than with the essence of what we should still have to refer to as an eddy current.

Dealing with Mr. Page's contention that my experimental analysis is unnecessary, it is true that the low-load errors are attributed to the anti-creep device, but not necessarily because this is the cause of the only variable torque independent of the load; for one can easily visualize a meter which is subject to no low-load errors over the greater part of the low-load range, in spite of the existence of an independent variable torque due to the anti-creep device.

However appropriate may be the reproof suggested by Mr. Page in referring to the desirability of an occasional review during the analysis of a problem, he is most unfortunate in his selection of the examples on which he bases this admonition. From Fig. 10 is derived the constant N. No constant or figure of any description is determined from Fig. 11. The whole purpose of this test is to show that the braking effect of the series flux is negligible on very low loads. The constant K_1 is found either from Fig. 13 or from Fig. 16, but by entirely different methods of test, and so provides a check on the value of the torsion constant found by the experiment referred to in Fig. 15. In addition, Fig. 16 shows a straight-line relationship between driving torque and very low values of current, a condition which I considered could hardly be assumed without experiment. Moreover, the current values in Fig. 16 are all within the low-load test range.

The only torque which varied with the angle θ was the independent shunt driving torque. The curve in Fig. 16, for example, was repeated for different positions of the rotor. The braking effect of the permanent-magnet and shunt flux is not influenced perceptibly by the anti-creep holes, as was proved by adding a counter weight and timing the oscillations of the rotor, suitably suspended, for different mean positions.

I agree with Mr. Page that the calculations are laborious, but I do not for one moment recommend this as a test-room method of calibration.

In reply to Mr. Felton, the shape of curve A, Fig. C, is probably as near an approach to the ideal as one could desire, but from my answers to previous speakers it will be clear that this curve really indicates a meter in which the positive and negative peaks in the shunt-torque curve have been brought nearer together, as recommended in my Conclusion. I do not suggest as a practical solution that we should dispense with the anti-creep device. The term $b\theta$ was made equal to zero merely in the process of examining the general equation of motion. Furthermore, I cannot agree that the complete solution is to make the mean value of $a + b\theta$ over one revolution of the disc equal to F. This is certainly an essential condition, but it is not sufficient; because a satisfactory value of starting load demands that some limit be imposed on the magnitude of the maximum negative value of $a + b\theta$.

Dealing with Mr. Spilsbury's contribution, it will be seen on referring to Fig. 19 that the calculated error on 1/20 load is very small, indicating that the influence of the anti-creep device at this point is not at all pronounced. The relative importance of certain terms obviously depends on the magnitude of the load, and from this we could rather loosely say that the operating conditions on normal loads are different from those on very low loads. The point of transition may be in the vicinity of 1/20 or 1/10 load.

At first sight, it does seem right to say that the low-load positive errors are due to a fast setting of the low-load compensator, but if a large negative peak in the shunt-torque curve renders this fast setting in-

evitable, in order to adjust for a given value of starting load, then I think it is correct to say that the positive errors are due to the anti-creep device. I appreciate Mr. Spilsbury's concluding remarks.

Replying to Mr. Shotter, my reference to overload compensation is made in the introductory part of the paper, and it should really be considered in its relation to the review of overload improvement. In such a résumé we start with the consideration of the meter as it existed before overload accuracy was called for; a full-load speed of at least 40 r.p.m. and a registration error on double load of the order of $-3\cdot0$ to $-5\cdot0$ per cent. As the demand for overload improvement became more insistent various types of compensation were tried and used, but it was soon realized that the desired improvement could not be satisfactorily achieved, entirely, or even chiefly, by compensating devices. A lower full-load speed was found to provide a much more reliable method of correction, and there is on all sides at the present time ample evidence of this. I am inclined to regard Mr. Shotter's test results as an indication of an appropriate example. I do not know, of course, what type of meter he tested, but I would guess that its full-load speed is much less than 40 r.p.m.; and, since the accuracy up to 300 per cent load is within $1\cdot0$ per cent on both unity and $0\cdot5$ power factor, it is obvious that the compensating device is relied upon to provide only a small correcting action.

While all this may be accepted as showing the true place of compensation in the history of overload improvement, it may be insisted that the desirability of compensation should be judged in relation to its use in the modern meter. Even so, considering the common form of magnetic shunt fitted to the series element, the fixing calls for special care; for a small movement, in some cases even of the order of $0\cdot0005$ in., causes considerable change in the compensating effects. This seems to me to make exacting demands on the manufacturer, who, after all, is dealing with a mass-production article which is expected to be available at a low purchasing price. Again, any change, after calibration, in the magnetic properties of the compensating bridge might have very pronounced effects on the meter curve, and I am not sure that we can always ignore the possibility of such changes occurring during the life of the meter. On the whole, I still consider compensating devices to be undesirable.

The curves in Fig. D are rather similar in principle to those in Fig. C, and in this connection I would refer Mr. Shotter to my reply to Mr. Felton. I agree that at present it is advisable when calibrating to make provision for a subsequent increase in friction, but I purposely excluded all consideration of increasing friction because this is quite a separate problem from the one under discussion.

In answer to Mr. Shotter's objection that the anti-creep device cannot be responsible for the low-load errors, I would direct his attention especially to Figs. F, G, and H, which give a clear indication how an anti-creep device, which produces excessive fluctuations in the shunt-torque curve, will of necessity cause low-load positive errors.

SMALL SELF-STARTING SYNCHRONOUS TIME MOTORS.

By W. Holmes, Member, and E. Grundy, B.Sc.Tech., Associate Member.

(*Paper first received 20th November*, 1934, *and in final form 3rd June*, 1935; *read before the* Meter and Instrument Section 1*st March*, 1935.)

Summary.

The paper deals with small self-starting synchronous motors such as are employed in time switches, meters, recording instruments, and other timing devices.

In order to keep the paper within reasonable bounds it was felt generally advisable to confine the notes to self-starting motors, since they are of more general application. The small hand-starting motors as used in domestic clocks, and any device which requires external starting means, are excluded.

The paper gives a résumé of some designs available, and includes general test-results obtained.

It then continues with detailed data on the hysteresis type of motor recently investigated by the authors, and endeavours from the results obtained to substantiate Steinmetz's theory of this type of motor expounded in 1900. Various new materials, such as cobalt steel, aluminium-nickel (Al-Ni) steel, and magnetic silver alloys, are considered.

The hysteresis coefficients for cobalt steel and aluminium-nickel steel were determined experimentally and are given.

The nine designs described are chosen to give typical examples of the designs available, and in the short description of each some of the difficulties of design and manufacture are explained.

The question of running-in torque, which is a very important factor to be considered, has been thoroughly investigated.

The use of permanent magnets as synchronous-motor rotors has been studied from theoretical and practical viewpoints, and the conclusions are given.

The requirements of the motor bearings are stated and typical examples are shown.

The test methods used in the laboratory for measuring the various torques are explained.

The test figures and curves given are those obtained by the authors from the particular models available in the laboratory. They may not, however, represent the maximum figures obtainable, as it is quite conceivable that variable results may be obtained due to inconsistencies in manufacture.

In conclusion, brief notes will be found of some specialized applications of these small motors.

Table of Contents.

Table of Contents—*continued.*

(1) Introduction.

With the advent of the national grid system in this country, which eventually will give time-controlled frequency throughout the whole area, the synchronous motor has found ready acceptance as a means whereby accurate timekeeping may be obtained.

Self-starting synchronous motors are now very important parts in most meters and instruments requiring a time element.

Time-controlled frequency is supplied voluntarily by the various supply authorities, and the close relation it bears with respect to standard time is really remarkable.

Fig. 1B is a typical frequency record of a large supply undertaking connected to the grid in this country, and Fig. 1C indicates the frequency error in seconds from standard time during one day of the same supply.

Continuity of supply has been observed independently in Cheshire, Lancashire, and Yorkshire, at private residences having synchronous clocks of the hand-starting type. At the first, over an observed period of 18 months, one interruption took place three months from the commencement of the observation; its duration was a few seconds. An observation over a similar period at the second showed one interruption of a few hours' duration, occurring 11 months from the commencement. Lastly, in the third case, over a period of three years, three interruptions have taken place at approximately yearly intervals.

(2) Classes of Motors.

Before actually describing in detail any known types, it may be advantageous to outline the classes in which they may be allocated according to their manner of

operation. We may therefore divide them into three fundamental or primary classes according to the manner in which the synchronous tendencies are obtained:—

A. A rotor system of permanent-magnet material

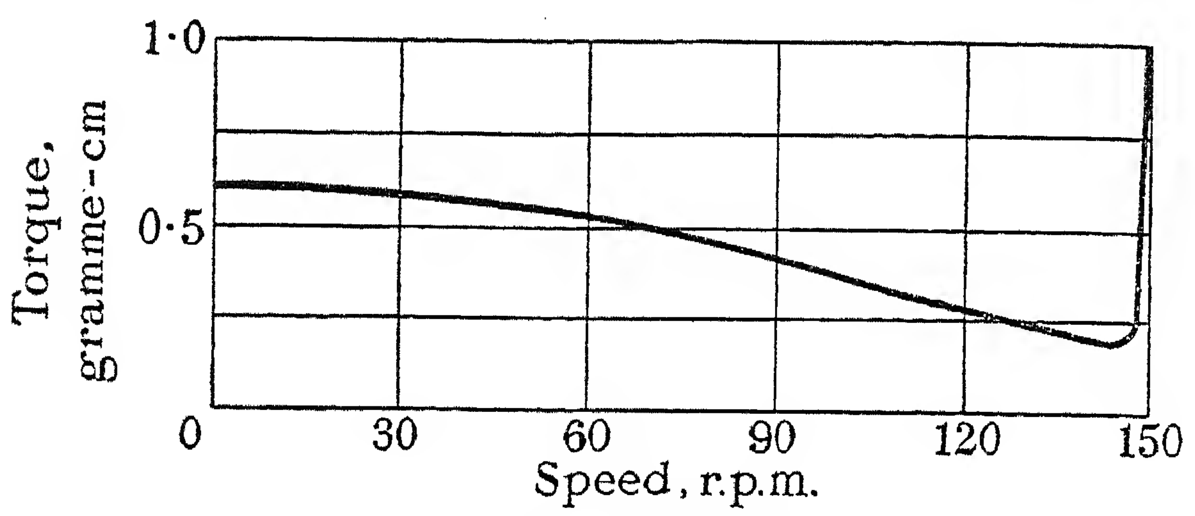


FIG. 1A.—150 r.p.m., 230 volts, 50 cycles.

magnetized to a high degree of intensity by external means before being applied to the motor.

B. A rotor system of permanent-magnet material which, however, derives its degree of magnetization from the effect produced by the stator system.

C. A rotor system of soft iron or magnetic material.

Again, each of these may be subdivided into two groups:—

(1) The rotor element acting as, and comprising, both starting and synchronous components with only one rotor solidly coupled to its shaft.

(2) The rotor element actually comprising two rotors—one for starting, the other for synchronizing effect—either spring- or friction-coupled, and generally operating in separate fields.

(3) REQUIREMENTS.

The following are the main requirements of the perfect synchronous motor:—

(i) The starting torque should allow of a generous safety factor for all stated applications.

(ii) The running-in torque should not be less than the starting torque.

(iii) The synchronous torque should be high to allow for any variable increased friction or load after the motor has been running for some time.

(iv) Overspeeding or underspeeding should not be possible.

(v) The watt loss and resultant heating should be low.

(vi) The speed of the rotor should not be too low or too high.

(vii) The direction of rotation should be predetermined by design.

(viii) It should attain synchronous speed in the minimum space of time.

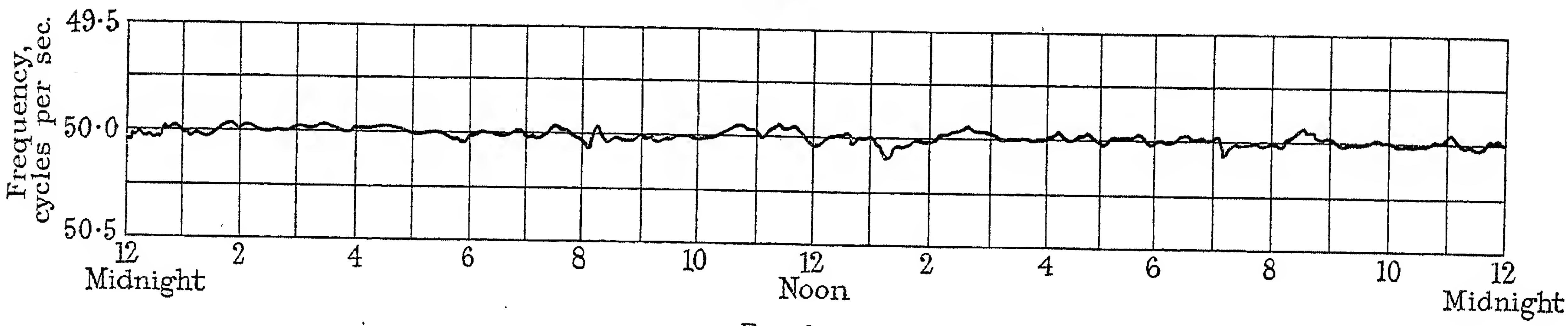


FIG. 1B.

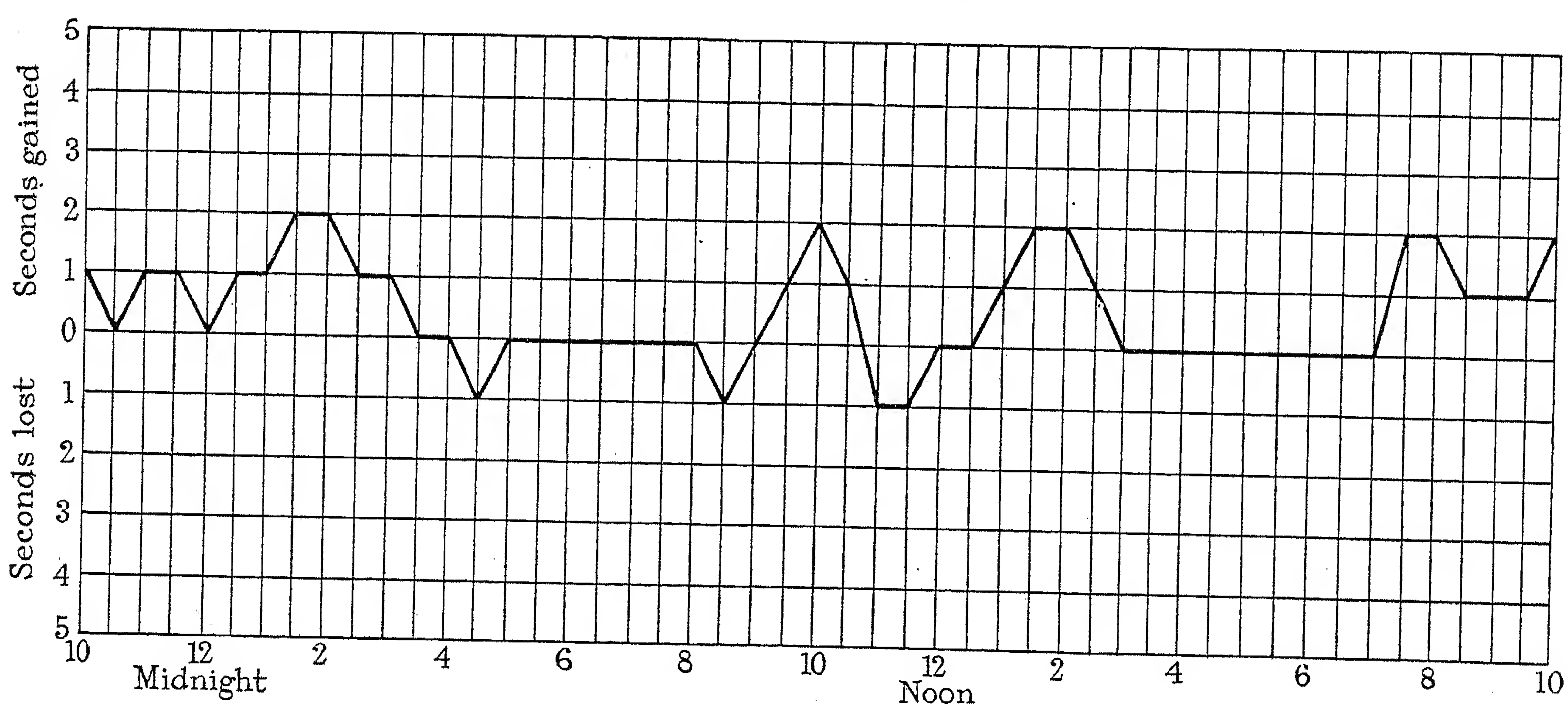


FIG. 1C.

(ix) The rotor should be of low mass.

(x) The bearing design should be such as to allow of continuous running for many years without attention, cleaning, or oiling.

(xi) The design should be compact and overall dimensions not too large.

(xii) Its construction should be simple, and its manufacturing costs low.

(4) DEFINITION OF TORQUES.

The magnitude of the following factors is very important:—

(1) *Starting Torque.*—This is the maximum torque the motor can give out at its rotor shaft, available to move or try to move any apparatus connected to it from rest.

(2) *Synchronous Torque.*—The torque necessary to cause the rotor to pull out of step when it is running at synchronous speed.

(3) *Running-in Torque.*—This is the minimum torque of the motor which is available when the rotor is revolving below synchronous speed. In the case of a motor whose rotor element comprises two rotors, or one rotor with the two effects, one to provide the starting and the other the synchronous effect, the running-in torque is the torque of the starting element at any speed below synchronous speed minus any back force or braking torque produced by the synchronous element.

If $\mathbf{T}_1$ = torque of starting element at any speed (or generally just below synchronous speed),

$\mathbf{T}_2$ = back torque or brake torque of synchronous element,

then running-in torque $= \mathbf{T}_1 - \mathbf{T}_2$.

The Practical Considerations of the Various Torques.

Starting Torque.—Later in the text it will be seen that the measurement taken is that of the maximum available starting torque which can be given out at the rotor shaft after overcoming the rotor bearing friction. It is therefore important that this torque should be reasonably high in order to overcome any starting friction or load of the apparatus to which it may be connected.

Synchronous Torque.—As stated in the definition, this is the maximum torque available at synchronous speed for external use after allowance has been made for the running friction torque in the rotor bearings. It is incorrect to work out the power of a self-starting synchronous time motor by taking the synchronous torque as a base; there are many synchronous motors with very large synchronous torques which have barely sufficient power to run up to synchronous speed. The authors would like to draw particular attention to this point, as there is a great tendency to compare self-starting synchronous time motors by judging the torques at synchronous speed.

Running-in Torque.—This means the torque available, after allowing for friction in the rotor bearings, to run the motor into synchronous speed. In many cases it is generally the torque just before the synchronous torque (Fig. 1A); in other motors it is the torque just after the starting torque (Fig. 9C). In this latter case it may be taken as being equal to the starting torque. Running-in torque is the maximum torque at the lowest part of the speed/torque curve, and is the torque between starting and synchronous torque which is available to move the mechanism to which the motor is coupled. Consequently it represents the strength of the weakest link in the chain, and it is, therefore, the torque on which all calculations for power and efficiency should be based.

Hence,

Maximum safe load torque = running-in torque (g-cm)

Maximum safe work done in g-cm per minute

$$= \text{running-in torque (g-cm)} \times 2\pi \times \text{r.p.m.}$$

Maximum safe b.h.p.

$$= \frac{\text{running-in torque (g-cm)} \times 2\pi \times \text{r.p.m.}}{453 \cdot 6 \times 2 \cdot 54 \times 12 \times 33\,000}$$

where r.p.m. is equal to the synchronous speed.

This figure is not to be confused with the maximum b.h.p. corresponding to synchronous torque, and it is not the b.h.p. taken at the lowest part of the speed/torque curve.

(5) Description and Test Results of Some of the Available Types.

For the purpose of the following description, the various motors, typical examples of their class, will be enumerated as shown in Table 1.

Table 1.

Type I.	An early design of Class A1
Type II.	An impulse starter, Class A1
Type III.	An impulse starter, Class A1
Type IV.	An induction starter, Class A2
Type V.	Very early high-speed hysteresis, Class B1
Type VI.	Induction starter, high-speed, Class C1
Type VII.	Induction starter, low-speed, Class C1
Type VIII.	An induction starter, Class C2
Type IX.	New low-speed hysteresis, Class B1

A. *Type I.*

This is an example of an early attempt to manufacture a low-speed self-starting synchronous motor.

It consists of a copper disc located in the field of a bipolar electromagnet having shaded poles. The rotor has round its extreme edge a number of permanent-magnet steel pins to act as synchronizing poles, and defined poles are formulated on the stator system to give a pulling-in effect when the induction disc has run up to synchronous speed. In another form a stamped, thin, spider-shaped magnet is used instead of the pins on the rotor.

The difficulties in this motor were in the proportions of the stator synchronous poles, for if they were well-defined locking took place at starting due to any rotor pins which might be located in the vicinity of the stator synchronous poles. To reduce this effect it became necessary to soften or blunt the effect of the stator poles, but this had the disadvantage that the synchronous torque available at synchronism was insufficient to pull the rotor into step, with the result that the induction element took it beyond synchronous speed. In an endeavour to overcome the locking action and yet retain the well-defined synchronous poles, a small permanent magnet was fixed to the stator and adapted to co-operate with one of the rotor pins when the rotor was at rest, and hold it away from the influence of the stator synchronous poles. The stator poles were so shaped that the eddy currents produced in the disc at synchronous speed actually kept the magnetic pins correctly magnetized.

This design of motor ran very successfully for some time, but it was found in practice that although it would start up from rest at any time with the supply switched on at any part of the wave it was liable to lock should

the supply be switched on at a particular part of the wave whilst the disc was coming to rest following the supply having previously been interrupted. For instance, one motor would stop if the current were switched on again 12·3 seconds after it had been switched off. It was considered that this was a technical fault and the design was abandoned. The example is given here in order to point out some of the possible pitfalls in design.

It is from the results obtained on this motor that the authors recommend the aforementioned definition of running-in torque.

A speed/torque curve of this motor is given in Fig. 1A.

B. *Type II.*

This motor is known as the impulse starter type, and depends for its action on the following considerations.

Although the rotor is perfectly balanced in its mass the magnetic poles are arranged, by being slightly distorted, to be out of balance magnetically as soon as the alternating field is applied. These conditions are obtained by constructing the rotor with the pitch of the poles unequal and out of symmetry with the poles on the stator. Referring to Fig. 2A, it will be seen that the stator A is so arranged that, starting at a particular point and travelling round the circumference, the polarity of the poles at any instant will be N, S, N, S, etc., and the pitch of these poles is uniform throughout.

in step with the frequency, and thereafter will run synchronously.

The starting torque of this motor will be seen to be rather a variable quantity. The test results given in Table 2 show the figures obtained, and these may be

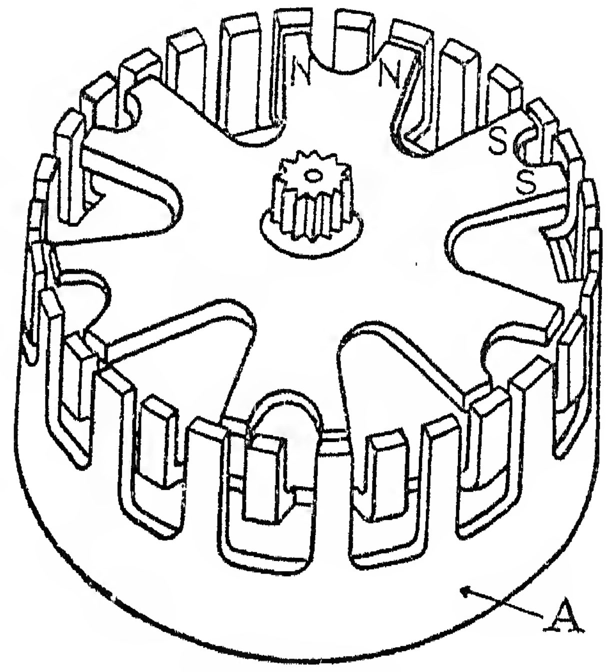


Fig. 2A.

taken as the minimum value for the particular watts input to which the reading corresponds. Owing to the quality of the permanent-magnet material used and the very high degree of magnetization attained, the synchronous torque of the motor is very high, and with $1\frac{1}{2}$ watts input the synchronous torque of 19 gramme-cm is easily obtained. By closing the air-gap the torque may be increased to double this figure.

Table 2.

Results on Type II Motor at 200 *r.p.m.*, 230 *volts*, 50 *cycles.*

Volts	Watts	I^2R	Starting torque	Synchronous torque	Running-in torque	Work done	B.H.P. × 10^5	Efficiency
			g-cm	g-cm	g-cm	g-cm/min.		per cent
180	0·8	0·26	1·4	15·0	1·4	1 758	0·385	0·360
210	1·15	0·39	1·95	17·7	1·95	2 450	0·537	0·350
230	1·5	0·48	2·3	19·0	2·3	2 890	0·633	0·315
250	1·85	0·56	2·5	20·6	2·5	3 140	0·688	0·277
280	2·45	0·85	2·65	22·8	2·65	3 328	0·729	0·222
*230	1·5	0·48	2·0–19·0	19·0	19·0	23 800	5·22	2·60

* These are the test figures obtained with the motor embodying the points mentioned in Section (5) B (i) (page 383).

The rotor is made of a high-percentage cobalt steel, and its formation is a 6-legged spider which is permanently magnetized so that the poles have polarity as indicated; in the interests of good magnet design certain poles on the rotor are omitted, but the pitch of the theoretical poles is equal, or almost equal, to the pitch of the stator poles. The pitch between certain rotor poles is made intentionally slightly less or greater than the corresponding pitch of the stator poles, and, when the alternating current is applied, this will cause magnetic unbalance as previously stated. The actual amount of magnetic out-of-balance is predetermined by design, and it will be appreciated that the effect of the stator poles becoming intermittently polarized N and S will cause the rotor to move one way or the other, or to make a slight vibration. The magnitude of this movement or vibration will increase after a brief interval of time from the rest position until, when it becomes greater than the pole-pitch between stator poles, the rotor will run

It will be seen that the direction of rotation of this motor cannot be predetermined by design, for at the instant that it reaches synchronous speed it may be rotating clockwise or anti-clockwise, and it becomes necessary to provide mechanical means of preventing rotation in the wrong direction.

This is achieved by pivoting a spring-controlled pawl on the plate of the motor, the pawl acting on a cam mounted on the underside of the rotor. The cam is so arranged with its drop that, if the rotor should rotate in the wrong direction, the projection will come up against the pawl and the rotor will rebound and thereafter rotate correctly. The controlling spring on the pawl is very weak and only applies a very small friction load on the rotor.

In addition to the general tests on the motor a series of tests was taken of starting and synchronous torques measured against varying frequencies, and it was seen that as regards starting the motor operates only between

certain limits, generally ± 10 per cent change of frequency, although its running characteristics remain substantially unaltered; it becomes necessary to have a different design of rotor to suit the different frequencies.

A point worthy of note in the design of this motor is that it can be made to hunt for about 1 sec. at the instant of switching on, and as there is generally backlash in all gearing this feature is a very useful one as it gives a running-in torque, against load, of nearly half synchronous torque.

Experiments have been made to prove this fact; a standard motor, of starting torque 2·3 gramme-cm and synchronous torque 19 gramme-cm, was geared by means of spur wheels with backlash or shake of about 0·005 in., corresponding to an angular movement of the rotor of 4 degrees. The second motion shaft was then loaded up to a value representing a torque of 9 gramme-cm on the rotor, and it was found that the rotor started against this load with the stated backlash. It may thus be claimed that the running-in torque of this motor when acting with backlash would be 9 gramme-cm. However,

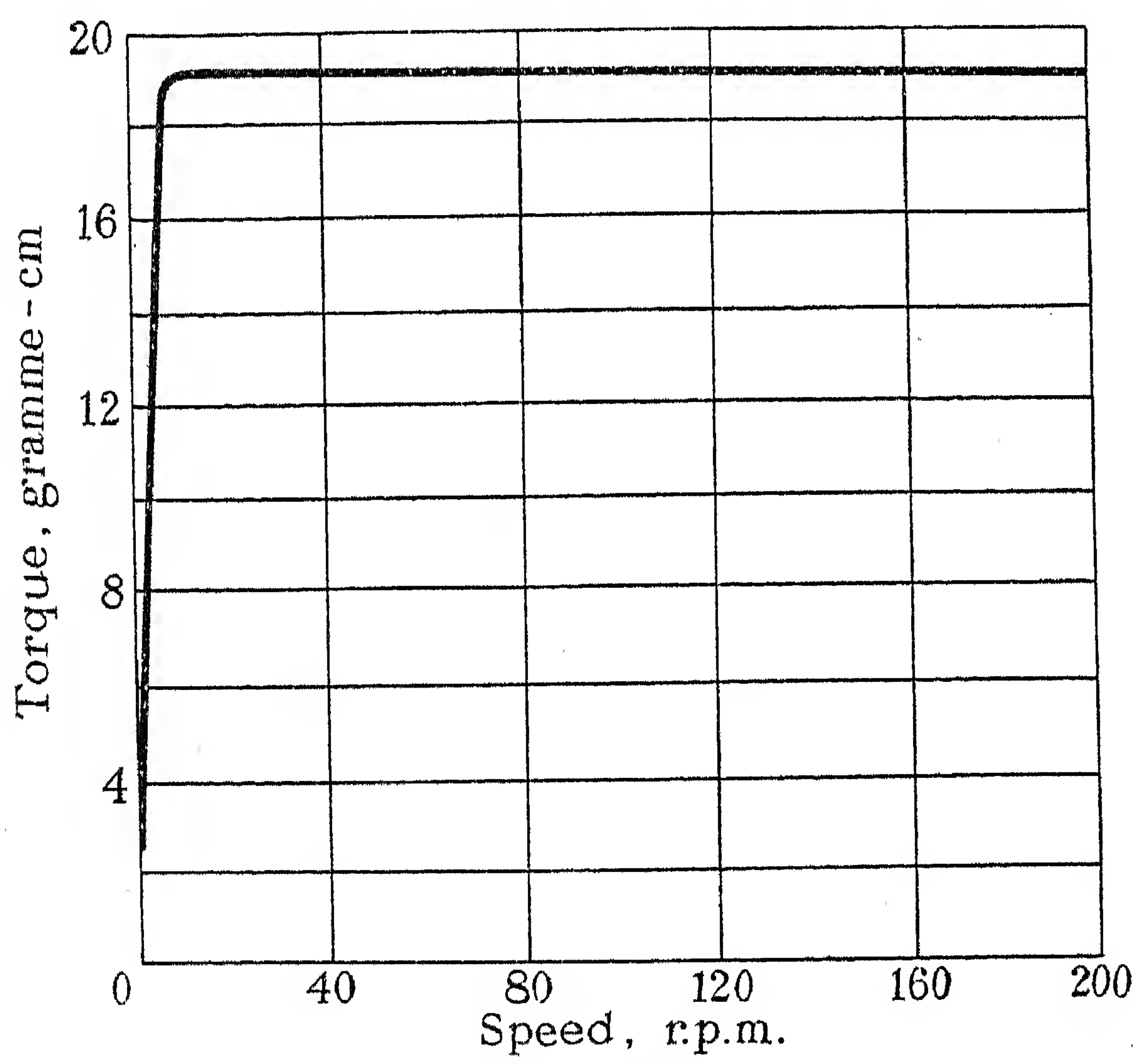


FIG. 2B.—200 r.p.m., 230 volts, 50 cycles.

the value given in the Table is 2·3 gramme-cm, representing the measured torque on the rotor spindle without backlash.

The test results and characteristics of this motor are shown in Table 2 and Fig. 2B.

(i) *An Arrangement to Increase Running-in Torque of Type II.*

The running-in torque on the Type II motor may be still further increased by using a spring-mounted driving pinion. It is usual in motors of this type to fix the pinion firmly on the rotor spindle, but in this modification the pinion is mounted loosely on the shaft and connected thereto through a spiral spring. When the motor starts up, the rotor and its spindle can rotate without rotating the pinion and the connected load, until it has wound up the spring several degrees. Thus, when the spindle has rotated through an angle of 12 degrees, which is equal to one pole-pitch of the stator, it will attain its full synchronous torque of 19 gramme-cm. The meaning of this from a practical point of view is that the motor when so connected has a starting torque of 19 gramme-cm, a running-in torque of 19 gramme-cm, and a synchronous torque of 19 gramme-cm. The only disadvantage of the arrangement is the added friction of the loose pinion on the rotor shaft; this may interfere with the available starting torque on the rotor if the former is not accurately mounted. Tests have been conducted on this design of motor, and whilst the starting torque is 2·3 gramme-cm the increased friction due to the loosely mounted pinion on the rotor is less than 0·3 gramme-cm, leaving more

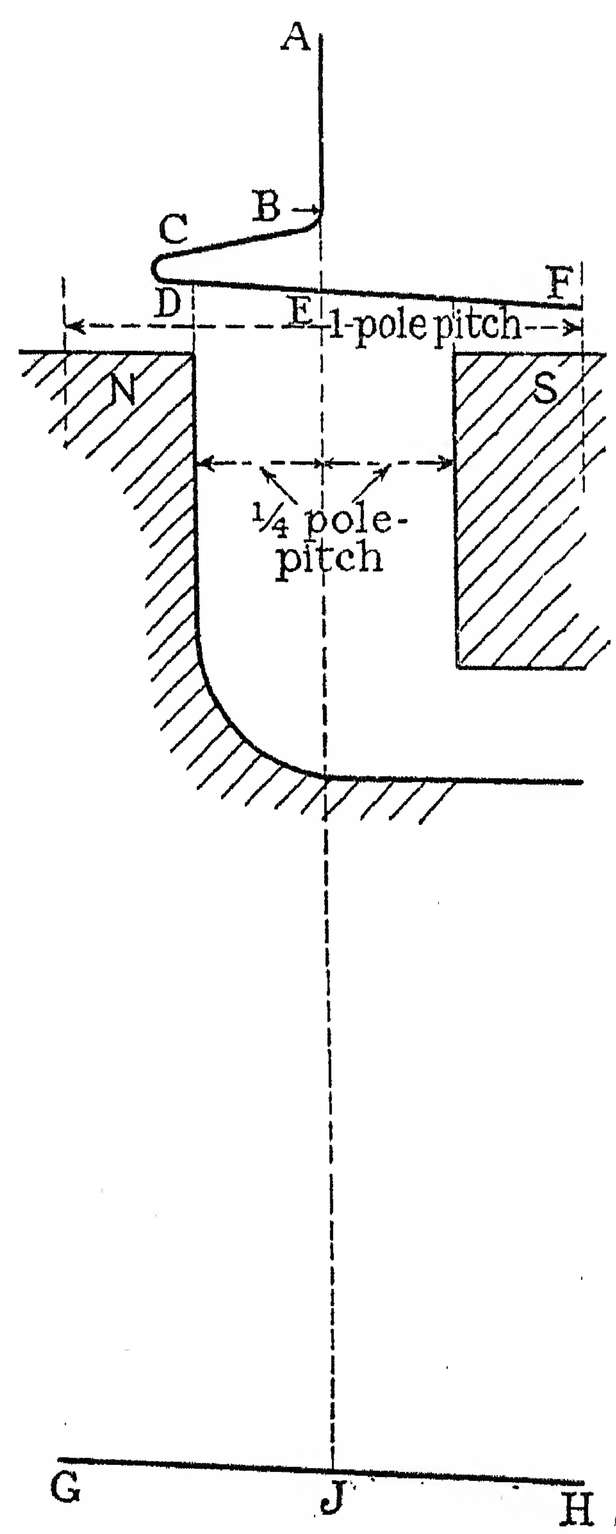


FIG. 2C.

than 2·0 gramme-cm for further variation in the friction of the combination. It is important to note that, owing to the spring connection, the starting friction and starting load of the external mechanism is overcome by the motor, which, at the particular instant, can develop a synchronous torque of 19 gramme-cm.

(ii) *Measuring Starting Characteristics by High-Speed Camera.*

During the design and development of this motor particular tests were conducted to observe exactly how the rotor behaved at the precise instant of starting.

First, a high-speed cinematograph camera was utilized, the number of pictures which could be taken per second being approximately 100. Consequently, with a 50-cycle supply applied to the motor, allowing the film to reach a steady speed before supplying the motor, a record of the motor was obtained which showed the position of the rotor with respect to a fixed point

for every maximum value of the flux, that is, every 1/100th sec. It was, however, decided to find out what was occurring in the brief interval between successive pictures, and it became necessary to employ more complete apparatus.

To this end, the outer casing and part of a Duddell oscillograph were introduced, equipped with the moving film attachment, by means of which it was possible to arrive at a continuous record of the movement of the rotor. Accordingly, a minute mirror was fitted on the rotor spindle and a beam of light was directed on the centre of it, the whole being so arranged with respect to the film, and in combination with lenses, that a pencil of light was deflected to the film, generally as shown in Fig. 2G. To obtain a measure of the movement of the rotor, the film was set in motion by means of a special time-lag switch about 1/50th sec. before the alternating current was applied to the motor, and, as the rotor moved to right and to left before commencing to rotate, the path of the beam was traced on the film. By accurate measurement the angular movement of the rotor was obtained, and by calculation from the speed of the film the time taken was also derived. These records are shown in Figs. 2C to 2F, marked with these corresponding values.

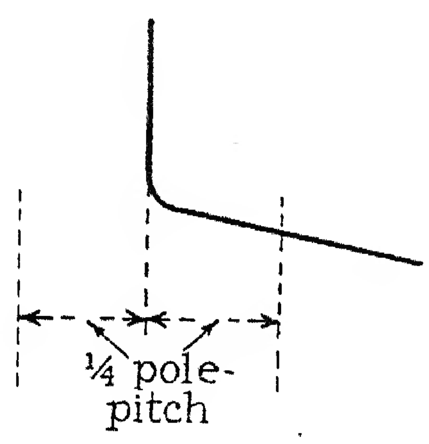


FIG. 2D.

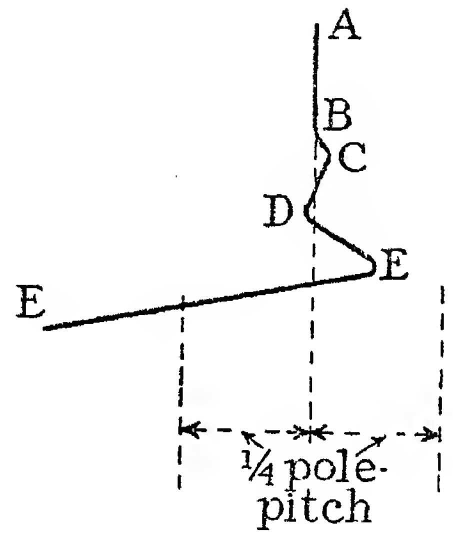


FIG. 2E

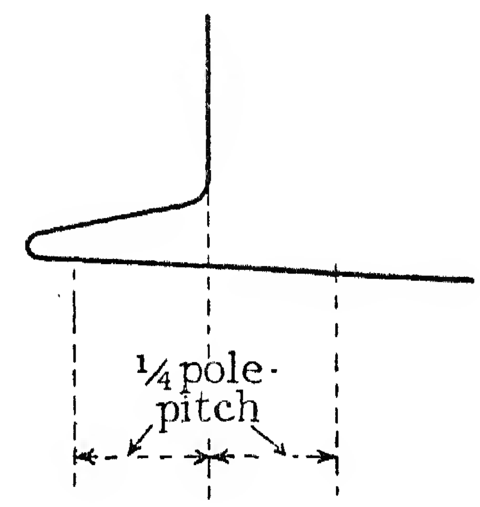


FIG. 2F.

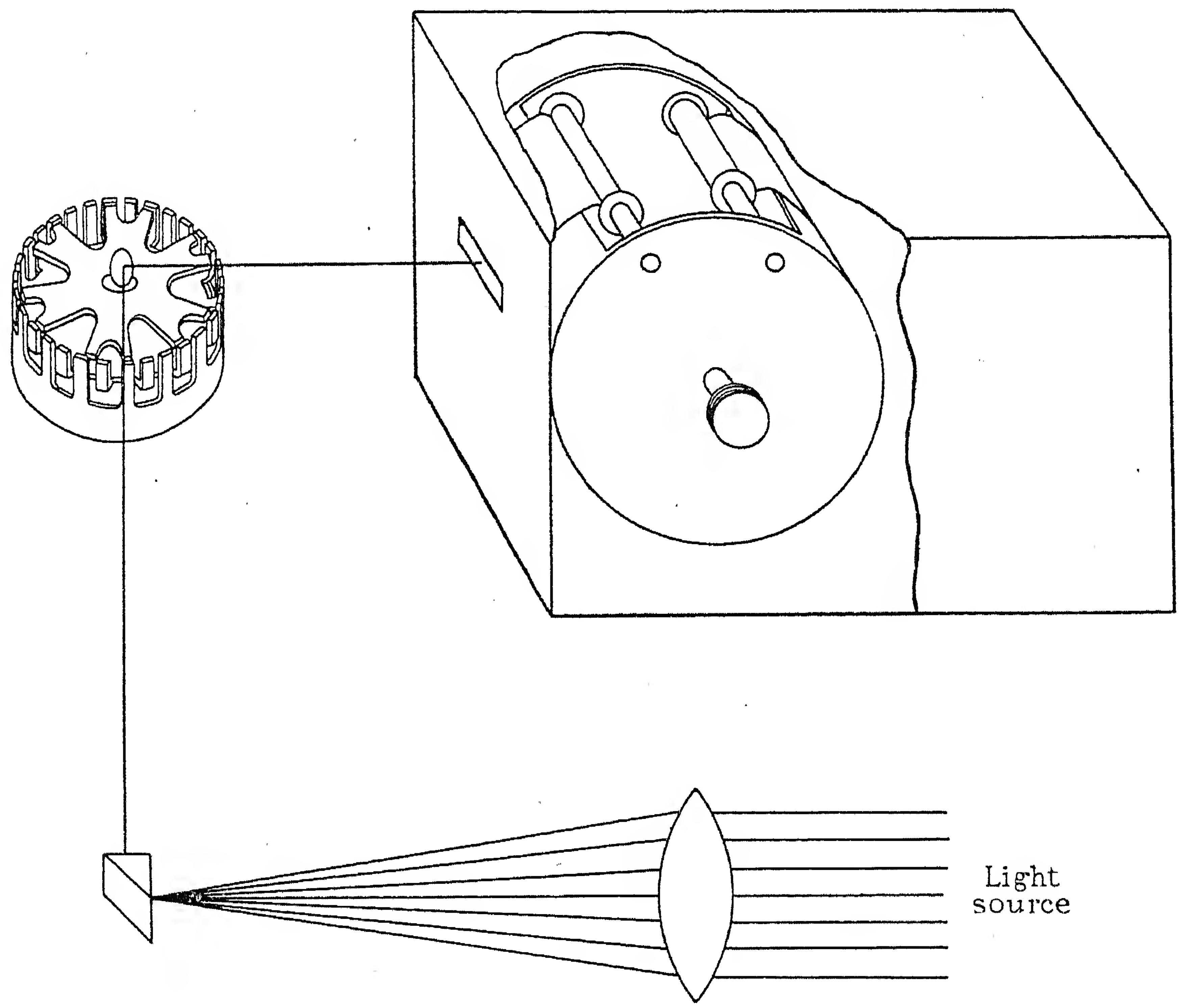


FIG. 2G.

Referring to Fig. 2C, the film was set in motion at the point A, and at B the supply to the motor was applied. Since the rotor moved to the left, the mirror traced a line from B to C; it returned to D, for clearly the rotor had reversed its direction, and at F went outside the limit of the film. It is seen to return at G, leaving again at H.

It is interesting to observe the relation between the distances BE and EJ. Since there are 30 poles on this

particular motor, causing the rotor to rotate at 200 r.p.m. when on a 50-cycle supply, it will take 15 cycles for the rotor to perform one revolution. The distance BE, since it represents a move to the left and then one to the right, is equivalent to 1 cycle, since each half-cycle of the wave attracts the rotor in turn.

As the beam of light leaves the film at F and returns again at G, the rotor having performed one revolution at synchronous speed, the distance EJ must be equivalent to 15 cycles. Therefore, as BE represents 1 cycle and EJ 15 cycles, the ratio BE/EJ expressed as a ratio of number of cycles is 1/15, and if the rotor in moving from B to E via C and D is in step with the frequency, then the ratio BE/EJ expressed as a ratio of length measured on the film will equal 1/15. This is actually found to be so, thereby verifying the statement, made earlier, that the rotor does vibrate in step with the frequency before commencing to rotate at synchronous speed.

The angular movement of the rotor before commencing to rotate synchronously may be seen on reference to the stator pole-pitch which is indicated.

Figs. 2D, 2E, and 2F, show other film records of the same motor, where, before commencing to rotate uniformly at F, it vibrates a number of times depending upon the position of the rotor at start and at what part of the curve the current is switched on. In Fig. 2D the rotor attained synchronous speed immediately on switching on, whilst in Fig. 2E it vibrated a few times before settling down to synchronous running. In Fig. 2F the rotor moved in one direction, then reversed, and afterwards dropped into step.

The importance of the information given by these films will be appreciated from a study of the theory of the operation of this type of motor, for the mass of the rotor plays an important part. Secondly, the proportion of rotor pole-face width to stator pole-face width influences the starting properties to a considerable degree. The very thorough testing and measurement described here resulted in the most favourable proportions being obtained.

C. *Type III.*

The motor to be described operates in a similar manner to Type II, but the method of obtaining this result is applied in a different way. The stator is in theory the same, for referring to Fig. 3A the stator poles will be seen to be arranged in circular form, and going round the circle the polarity of the pole projections will be N, S, N, S, at any instant: thus the rotor is located inside the coil surrounding which are the stator iron and poles.

Fundamentally, the rotor shown at A in Fig. 3A is of similar design to Type II except that the pitch of its

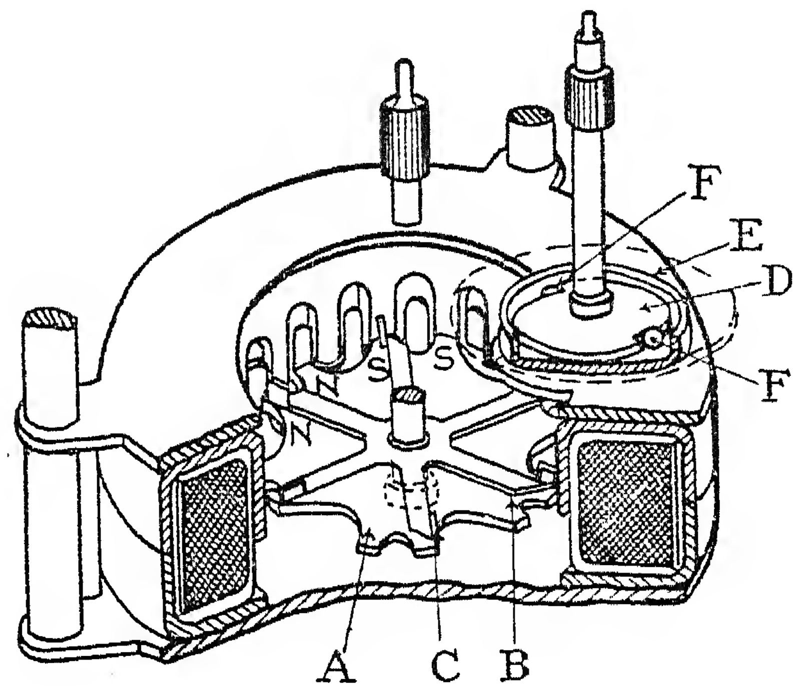


FIG. 3A.

poles is uniform, and also that it carries a spider B of soft iron, the projections C of which are so placed that they partially overlap the one permanent-magnet pole and partially overlap the space between that pole and the

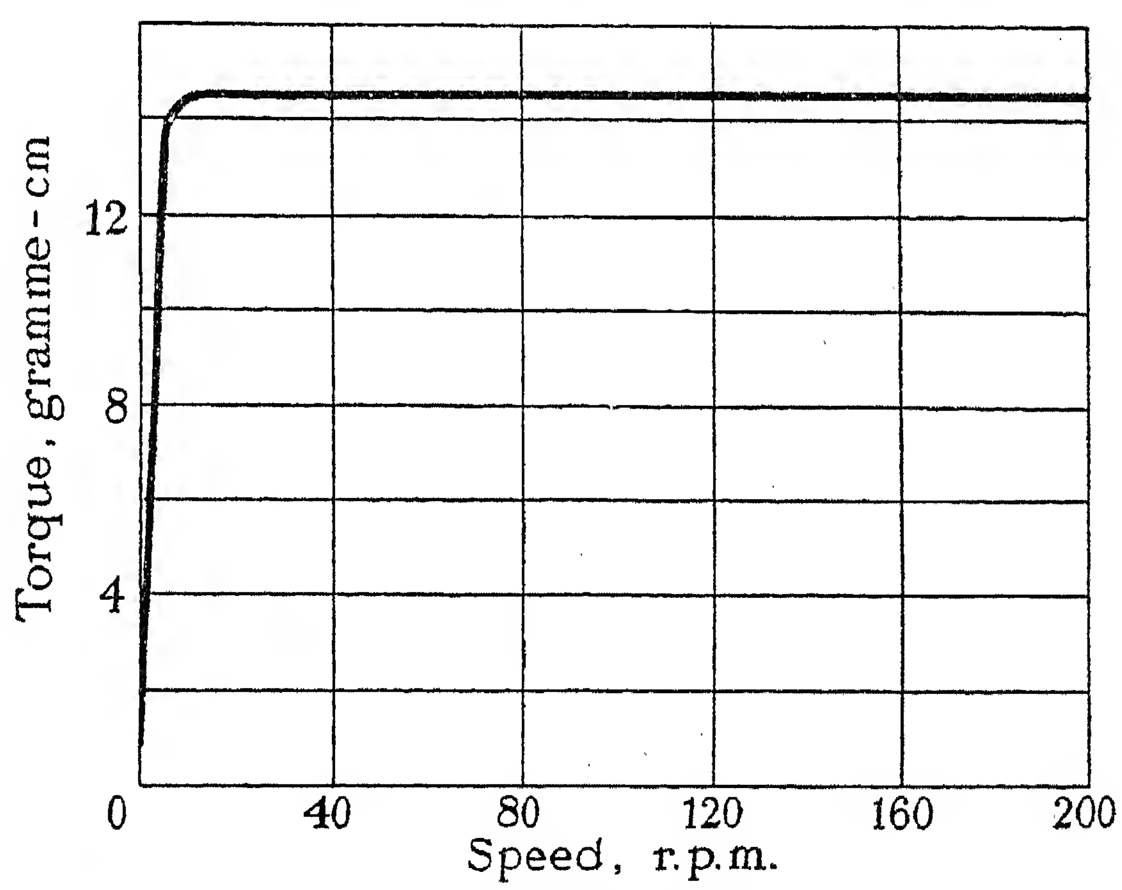


FIG. 3B.—200 r.p.m., 230 volts, 50 cycles.

next pole of the rotor; the spider has 6 legs, equally spaced 60° apart. When the rotor is at rest and the stator is unenergized, owing to magnetic induction the permanent-magnet poles will naturally take up positions directly opposite to the stator poles. Immediately,

TABLE 3.

Results on Type III Motor at 200 r.p.m., 230 volts, 50 cycles.

Volts	Watts	I^2R	Starting torque	Synchronous torque	Running-in torque	Work done	B.H.P. × 10^5	Efficiency
			g-cm	g-cm	g-cm	g-cm/min.		per cent
180	0·95	0·85	1·0	11·6	1·0	1 256	0·275	0·216
210	1·45	1·1	1·3	13·2	1·3	1 630	0·358	0·184
230	1·75	1·31	1·6	14·4	1·6	2 010	0·440	0·187
250	2·2	1·54	1·7	15·5	1·7	2 135	0·468	0·159
280	3·1	2·03	1·9	16·8	1·9	2 386	0·523	0·126

however, the alternating current is applied, the stator poles are polarized, and for an instant, or actually for the period of less than one half-cycle, the soft-iron spider legs will, due to the higher reluctance of the permanent magnet, tend to take up a position directly opposite to the stator poles, and consequently during this period of time the permanent-magnet poles approach midway between two stator poles of opposite polarity. Here, then, at this instant, the rotor is in a position of magnetic out-of-balance, and as the stator poles become intermittently polarized N and S they will cause the rotor to move one way or the other or to make a slight vibration; if, and when, the vibration or movement becomes greater than the pole-pitch the rotor will jump into step and thereafter run synchronously.

The fact that this motor operates in a similar way to Type II gives rise to the situation that it is liable to commence rotation in either direction, and a method of preventing incorrect rotation is provided in this construction on the second motion shaft. This consists of a double snail D (Fig. 3A) inside a cylindrical recess E; between the snail and the peripheral wall of the recess are two balls F. If the rotor starts in the correct direction it is free to rotate. If, however, it starts in the incorrect direction it will be checked, but, as there will be a rebound, a tendency to rotate in the correct direction will be produced.

The test figures of this motor are shown in Table 3, and the speed/torque curve in Fig. 3B.

D. *Type IV.*

This synchronous induction motor has an unusual stator construction, particularly as applied to the disposition of the lagged and unlagged poles producing the rotating field.

Referring to Figs. 4A and 4B, the stator consists of a number of iron pieces, A, B, C, and D, fastened to one end of the core, and E and F to the other end. After being bent over, the extremities are curved and trued

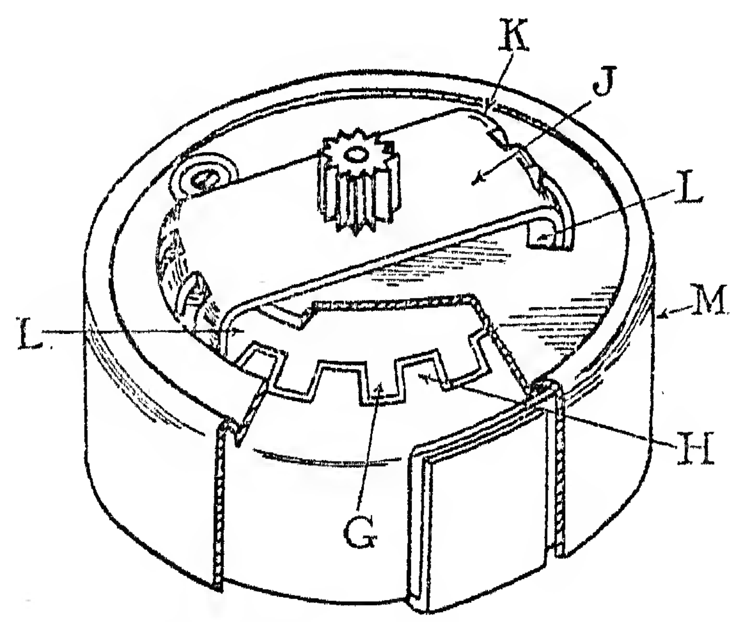


FIG. 4A.

up so that they lie in circular formation. Considering the arrangement, without reference to lagging for the moment, the fact that A, B, C, and D are secured to the same end of the core will mean that at any instant they will have the same polarity, and E and F, being at the opposite end, will have at the same instant opposite polarity to A, B, C, and D. The poles A and B have copper shading which causes the flux in them to lag behind that in poles C and D; the circumferential width of poles E and F is made twice as great as each of A, B, C, and D, and no shading is embodied therein due to structural design.

However, this gives the stator system for the induction-drive effect, and in order to obtain the synchronous control the path of the poles E and F is interrupted by a small air-gap of the shape shown in Fig. 4A. This has the effect of producing a number of

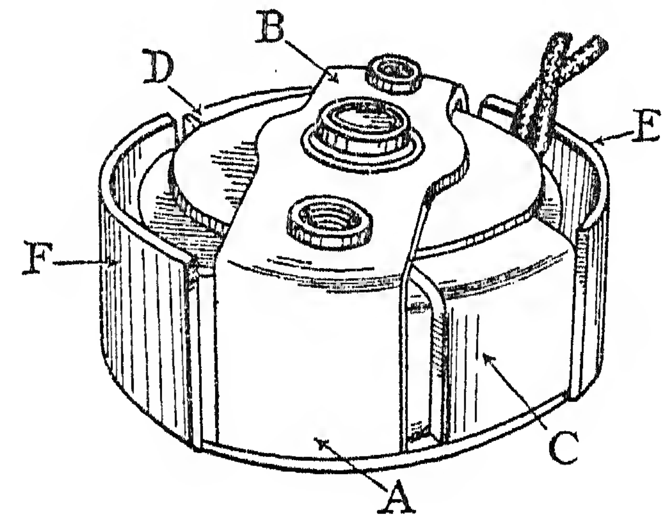


FIG. 4B.

poles G and H in circular formation; these, going round the circle, have alternate polarity at any instant. The induction-drive element consists of an aluminium or copper bell rotor M having the outer cylinder of iron or steel to form the return circuit for the main flux. Mounted loosely on the bell is a small permanent-magnet synchronous rotor J with driving pinion K;

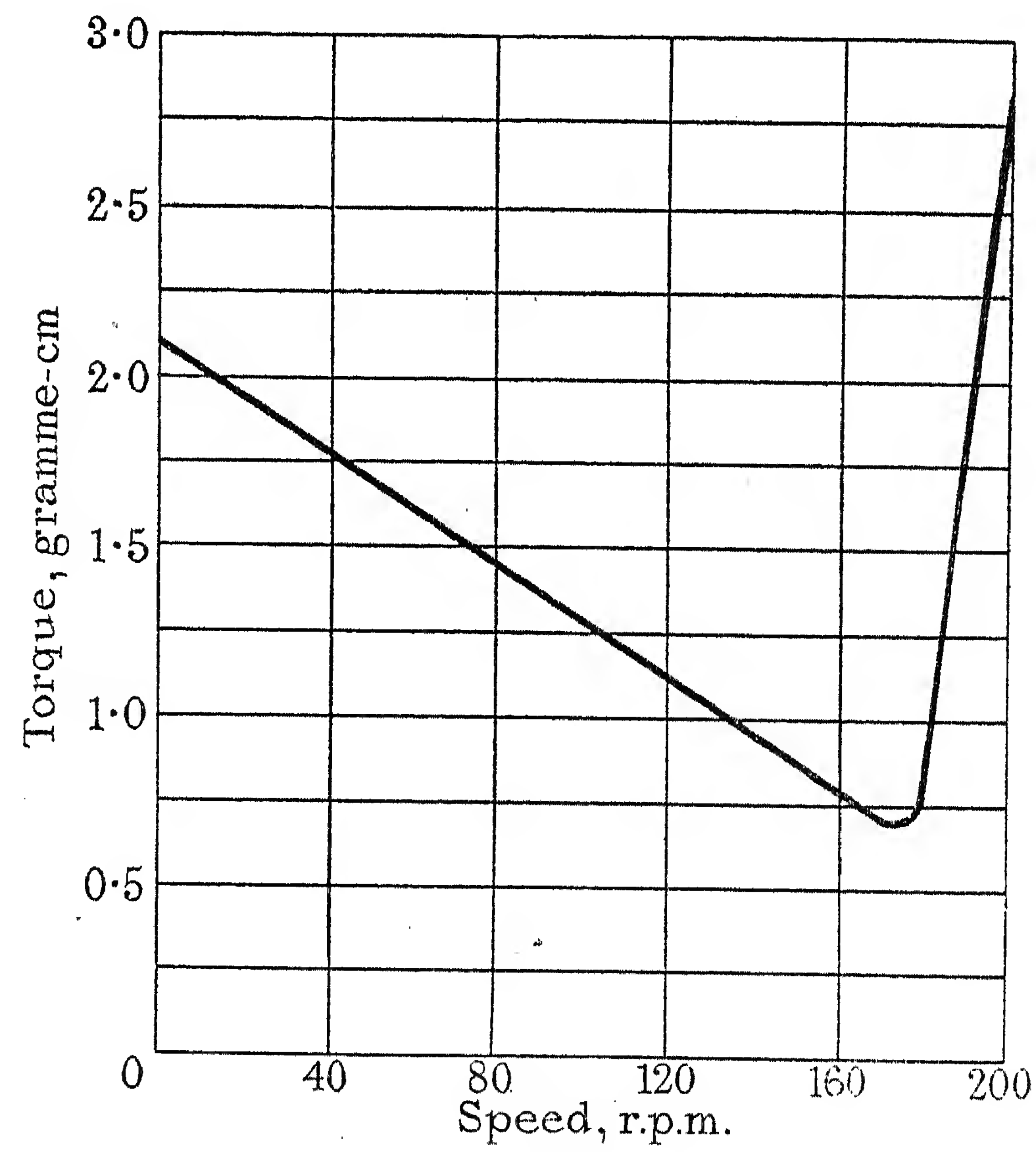


FIG. 4C.—200 r.p.m., 230 volts, 50 cycles.

the polar projections of the permanent magnet J extend through arcuate slots L provided at diametrically opposite sides of the flange of the bell, thereby enabling the magnet poles to rotate under the influence of the leakage flux produced from the poles at the interruption of the main magnetic circuit.

The extremities of the permanent magnet are slotted, forming a number of teeth, the pitch of which corresponds to the pitch of the teeth formed by the shape of the interruption to the main magnetic circuit. Relative

TABLE 4.

Results on Type IV Motor at 200 r.p.m., 230 volts, 50 cycles.

Volts	Watts	I^2R	Starting torque	Synchronous torque	Running-in torque	Work done	B.H.P. × 10^5	Efficiency
			g-cm	g-cm	g-cm	g-cm/min.		per cent
180	1·1	0·65	1·4	2·1	0·5	628	0·137	0·093
210	1·9	0·97	1·87	2·5	0·65	816	0·178	0·070
230	2·3	1·18	2·12	2·8	0·75	942	0·206	0·066
250	2·8	1·38	2·35	3·15	0·85	1 068	0·234	0·063
280	3·6	1·77	2·75	3·5	0·98	1 232	0·270	0·057

circumferential motion is allowed between the permanent magnet and the slots in the induction element to facilitate the falling into synchronism of the former with the alternating flux, and also permits it to continue running at synchronous speed against the action of torque changes.

When a supply is given to the motor the rotating field acts on the induction element, causing rotation of the latter, and exerting sufficient torque to overcome the stationary locking tendency of the synchronous rotor. The theoretical synchronous speed of the induction element is considerably greater than synchronous speed of the permanent-magnet rotor, from which there will be a sufficiency of induction torque to take the system as a whole up to actual synchronous speed, when the synchronous element will take control. The synchronous torque exceeds the induction torque at this speed and thereby prevents the motor from overspeeding.

Fig. 4c gives the torque/speed curve of this motor and shows the value of running-in torque and synchronous torque for its rated voltage and frequency: general test figures are given in Table 4.

E. *Type V.*

This motor, generally known as the Warren motor from its American inventor, is well known, but for comparison purposes it may be useful to give a short description here. Its speed is 3 000 r.p.m. when connected to a 50-cycle supply, and it operates owing to magnetic hysteresis effect in the rotor. A 2-pole stator system is employed, having half of each of the poles shaded with copper rings to give a rotating field effect, and in this field runs the rotor, which consists of a combination of hardened steel discs. When the alternating current is applied, at the first half-cycle induced magnetism is set up in the rotor, and around the periphery magnetic poles are formed; in this instance one N and one S pole are induced and, owing to the hysteresis of the steel, they will not die away immediately the stator flux diminishes. Accordingly, the rotor poles will tend to follow the rotating field in the stator, and had the rotor no mass the motor would run in step with the frequency from the very instant of switching on; instead, however, a fraction of a second will be taken in attaining speed, but

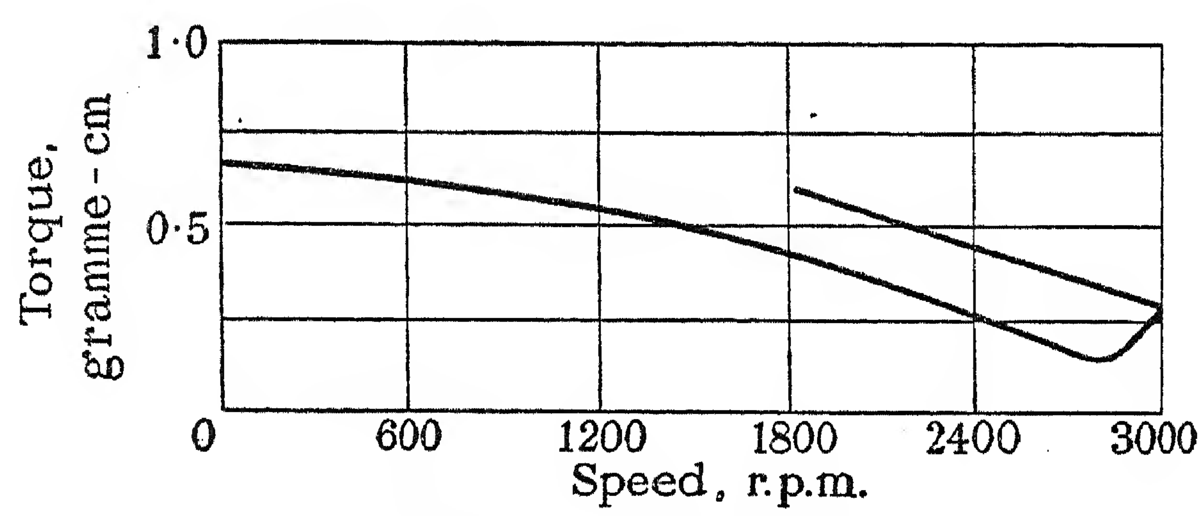


FIG. 5.—3 000 r.p.m., 230 volts, 50 cycles.

once having reached that value it will continue thus with quite an appreciable running torque.

It will be seen from the test figures given in Table 5 that this motor develops quite an appreciable b.h.p. due, in the main, to its high speed.

Referring to Fig. 5, which is the speed/torque curve of the motor, the loop on the upper part of the curve gives the fall in speed as a gradually increasing torque greater than synchronous torque is applied.

The increased torque above synchronous torque causing the motor to run at 1 875 r.p.m. is apparently the maximum torque of the motor at any speed, since any torque in excess of this value brings the rotor to rest.

TABLE 5.

Results on Type V Motor at 3 000 r.p.m., 230 volts, 50 cycles.

Volts	Watts	I^2R	Starting torque	Synchronous torque	Running-in torque	Work done	B.H.P. × 10^5	Efficiency
			g-cm	g-cm	g-cm	g-cm/min.		per cent
180	1·8	1·3	0·37	0·16	0·06	1 130	0·248	0·103
210	2·6	1·77	0·54	0·23	0·09	1 695	0·372	0·106
230	3·35	2·1	0·72	0·29	0·13	2 450	0·537	0·119
250	3·95	2·5	0·85	0·39	0·16	3 020	0·665	0·126
280	5·0	3·26	1·0	0·69	0·19	3 580	0·785	0·117

The description and test results refer to the normal pattern of this motor; there are two other types available, a larger one having more power for use in heavy work, and one in which the stator is so arranged that reverse direction of rotation can be obtained at will.

F. *Type VI.*

The motor in this case operates on the synchronous induction principle, i.e. it is run up to speed by an induction element, and when synchronous speed is attained a synchronous rotor takes control. The rotor as a whole is a single unit and comprises a squirrel-cage induction motor where the rotor bars give rise to the induction torque, and the iron between slots forms synchronous poles. It is known that the induction motor has zero torque at synchronous speed and that the pure synchronous element when composed of ordinary commercial iron has for practical purposes zero starting torque; consequently the induction drive is used to take the rotor as a whole up to synchronous speed.

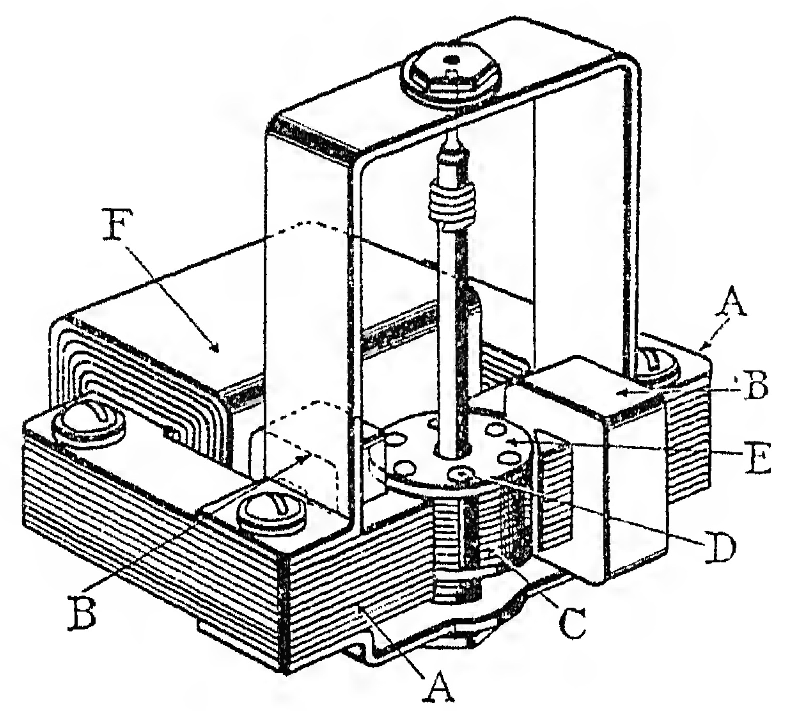


FIG. 6A.

Looking at Fig. 6A, the field system consists of the usual bipolar arrangement, and one half of each of the poles AA is shaded with copper rings BB in the usual manner. Free to rotate between these poles is the rotor C with copper bars D, and the iron between the bars forms the poles E; F is the coil. The fact of the induction system being bipolar, coupled with the fact that, owing to there being 6 copper bars, there will be 6 rotor iron teeth, the theoretical synchronous speed of the induction system will be 3 times greater than synchronous speed of the synchronous element. This is so devised in order that the induction torque available at synchronous speed of the motor as a whole shall be sufficient to take it up to synchronous speed. It is, however, necessary that the induction torque at synchronous speed should be less than the synchronous element torque, otherwise the motor will overrun to a speed above synchronous. From this angle, the shape, width, and length, of the synchronous poles had to be designed to give the correct value of synchronous torque, and yet these proportions required careful consideration from the point of view of starting, otherwise the synchronous characteristics would tend to lock the rotor at standstill and give the motor low effective starting torque or prevent it from starting at all.

Fig. 6B gives the speed/torque figures for this motor, from which the value of running-in torque can be seen in addition to the actual torque when running at synchronous speed, and a study of this curve will clearly show the factors to be taken into consideration in the design of a motor of this type, i.e. the running-in torque must have sufficient latitude to take the rotor up to synchronous speed to combat extra friction which may occur, and yet it must not be so great as to cause the rotor to run through synchronous speed.

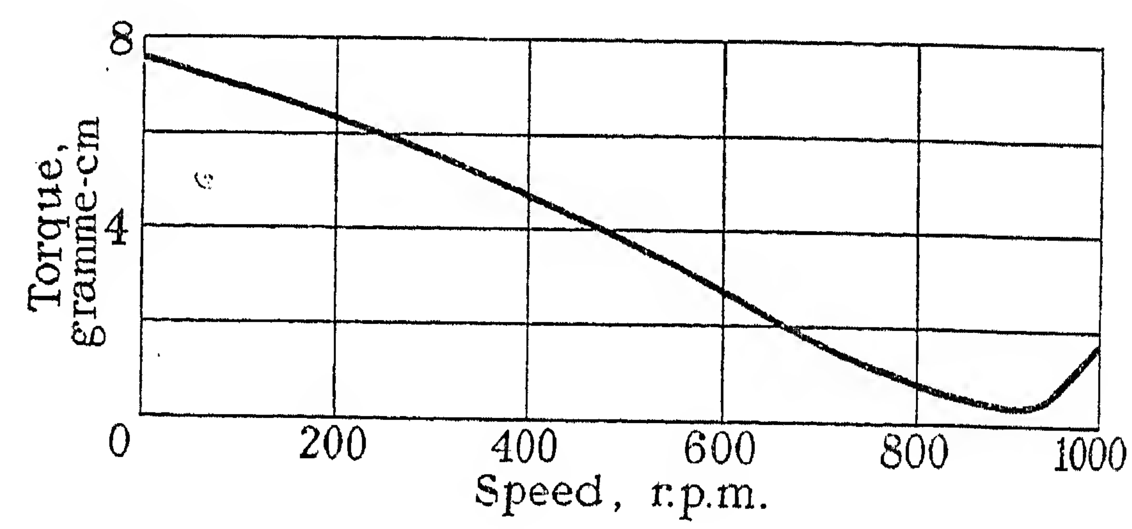


FIG. 6B.—1 000 r.p.m., 230 volts, 50 cycles.

This motor, running at a speed of 1 000 r.p.m., has given good results in many applications. It is intended to run with the rotor spindle vertical, and under working conditions the rotor is floating; owing to variations in voltage and fluctuations in the load the rotor varies up and down, resulting in the oil being distributed over the bearing surfaces.

The general test results are shown in Table 6.

G. *Type VII.*

This motor operates in a similar manner to Type VI for it has a squirrel-cage rotor for the induction element with iron poles formed around to give it synchronous tendencies, and runs at 200 r.p.m.

The stator consists of a cylindrical centre core, and

TABLE 6.

Results on Type VI Motor at 1 000 *r.p.m.*, 230 *volts*, 50 *cycles.*

Volts	Watts	I^2R	Starting torque	Synchronous torque	Running-in torque	Work done	B.H.P. × 10^5	Efficiency
			g-cm	g-cm	g-cm	g-cm/min.		per cent
180	1·55	0·64	4·5	1·2	0·15	943	0·207	0·0995
210	2·2	0·86	6·8	1·6	0·2	1 256	0·276	0·0935
230	2·65	1·06	7·7	1·8	0·25	1 570	0·342	0·0965
250	3·15	1·25	8·4	2·2	0·45	2 828	0·620	0·146
280	3·95	1·6	9·5	3·2	0·98	6 160	1·35	0·255

iron pieces are secured at each end and turned up to form poles in circular disposition. Referring to Fig. 7A, the pole A divided into two halves B and C, together with D and E and F and G, are all taken from the bottom of the core, and the remaining poles H, J, K, L, M, and N, are taken from the top end. Consequently, omitting shading for the moment, the poles B, C, D, E, F, and G, will all be of the same polarity and of opposite polarity to H, J, K, L, M, and N, all of which have the same polarity. The shading rings O are made in stampings, one stamping shading all the poles required, and the quantity to be used is governed by the amount

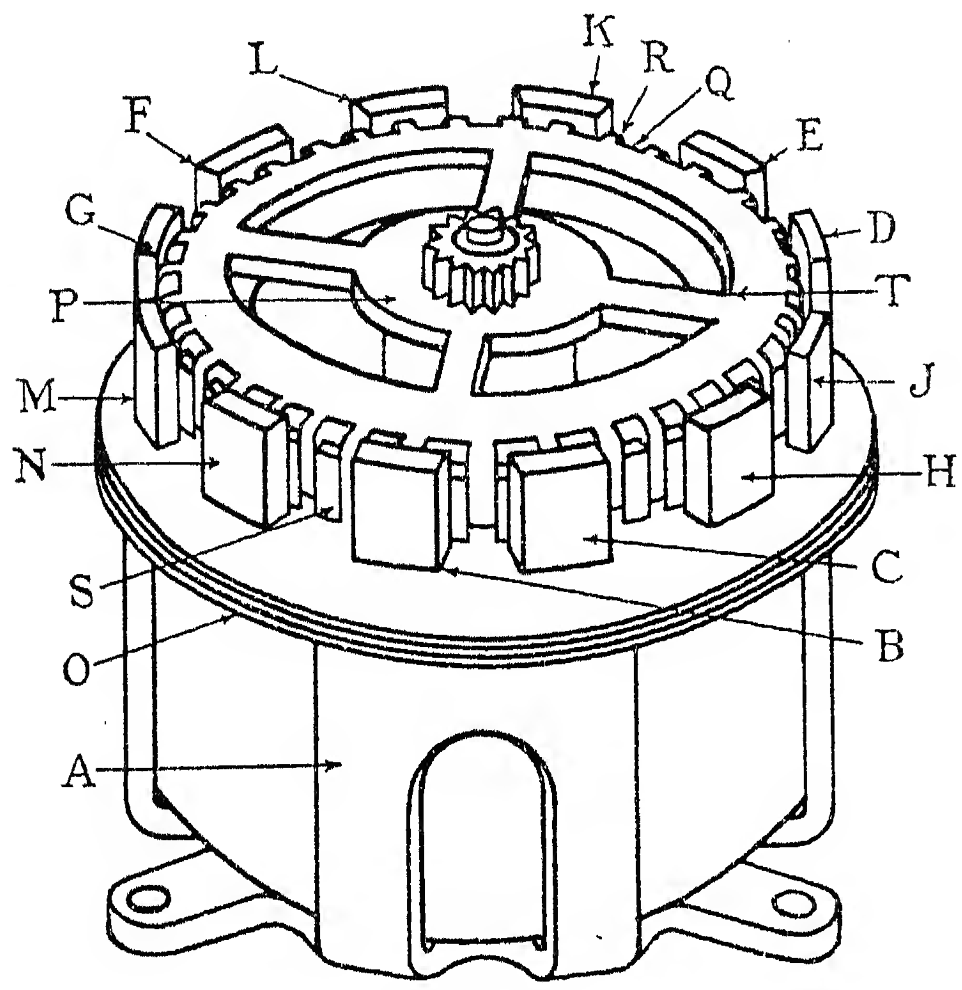


Fig. 7A.

of lagged flux required. In the actual motor three rings each about 1/32 in. thick, and one about 6 mils thick, are employed, the latter constituting an adjustment and so arranged that it may be used to add to or subtract from the result achieved by the three main rings. Expanding this, the motor will be assembled with its three main shading rings, but with the thin one omitted, and the position of the thin ring will depend on whether the lagged flux is in excess or in a minority.

The rotor P is made from stampings having a phonic wheel form, i.e. it has a large number of polar projections R with slots Q between. In the latter, copper pieces S are inserted which are short-circuited by means of copper discs T, one at each end. The whole is then soldered solid, after which the rotor is turned true and balanced.

The number of poles on the rotor is 36, and the synchronous speed of the motor will therefore be 1/6th the theoretical synchronous speed of the induction element.

With this motor the running-in torque will, on reference to Fig. 7B, be seen to be very small. It follows that any excessive friction in the bearings would prevent the motor from attaining synchronous speed; were it too high, however, the motor would run through synchronous

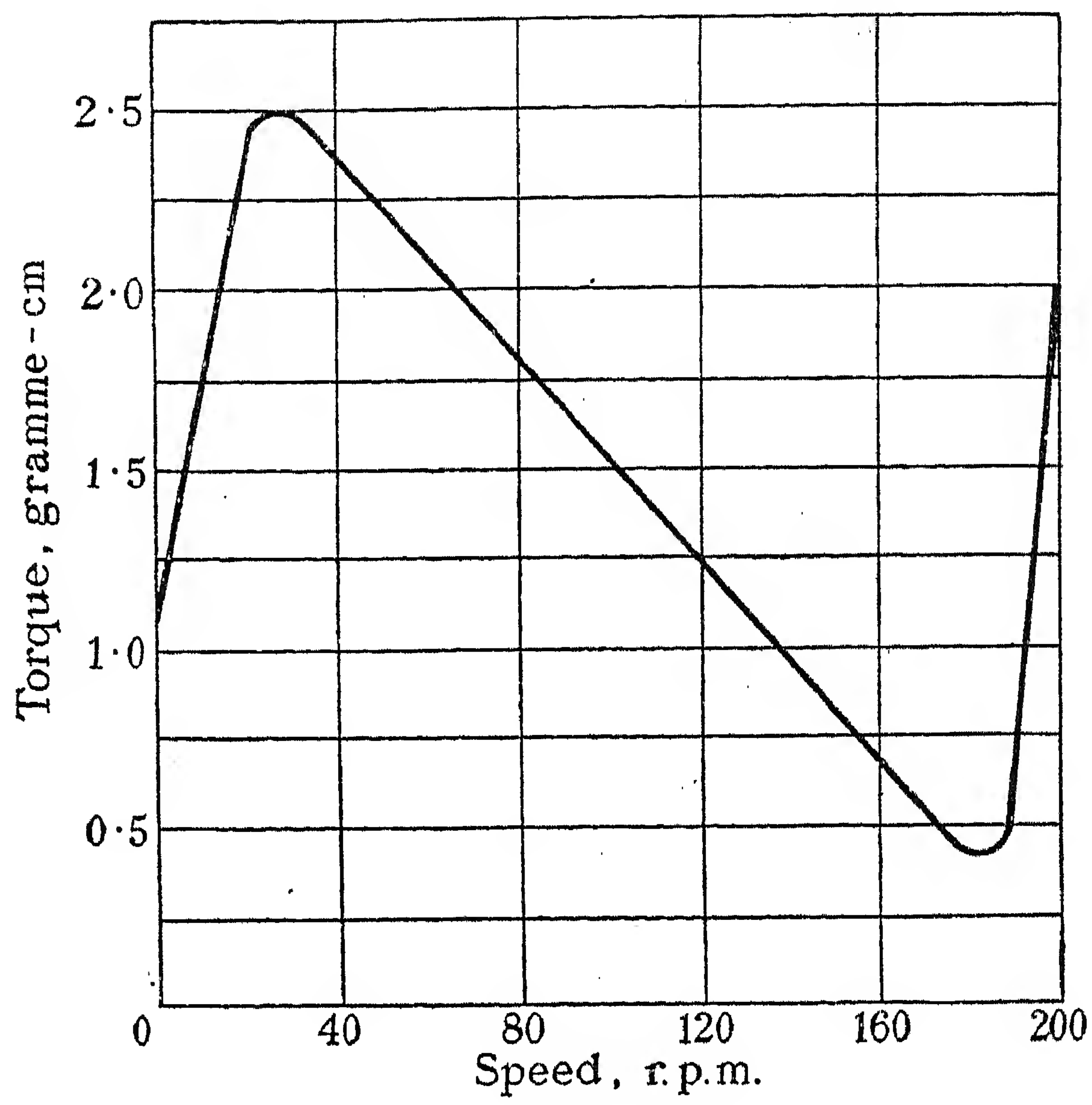


Fig. 7B.—200 r.p.m., 110 volts, 60 cycles.

speed and settle down to induction running at a speed greater than synchronous. Consequently the position of the final adjusting ring cannot be determined until the magnitude of the induction torque at synchronous speed has been verified, and the fact that the induction and synchronous elements cannot be separated even for an instant as in Type IV makes adjustment more critical.

The results are given in Table 7; the starting torque in all the results actually varies with the position of rest of the rotor, and the results given are when it is in a position of minimum torque. This fact gives rise to the unusual form of the speed/torque curve, for although it will be seen that the starting torque is 1 gramme-cm due to the locking action of the iron pieces when the rotor is at rest, immediately this is overcome the torque

Table 7.

Results on Type VII Motor at 200 *r.p.m.*, 110 *volts*, 60 *cycles.*

Volts	Watts	I^2R	Starting torque	Synchronous torque	Running-in torque	Work done	B.H.P. × 10^5	Efficiency
			g-cm	g-cm	g-cm	g-cm/min.		per cent
90	1·5	0·67	0·65	1·25	0·1	125	0·0275	0·014
100	1·9	0·84	0·9	1·6	0·25	314	0·0687	0·027
110	2·4	1·04	1·0	2·0	0·35	440	0·0963	0·030
120	2·9	1·25	1·1	2·4	0·45	565	0·1237	0·032
130	3·5	1·49	1·25	2·75	0·6	753	0·1650	0·035

rises to 2·5 gramme-cm and then follows the usual form of curve.

H. *Type VIII.*

The synchronous induction motor under this heading has two rotors, one synchronous, the other induction, which are coupled together through the medium of a spring. The fields for the two rotors are obtained from the same energizing coil, but the rotors operate under distinct and separate poles as in the case of

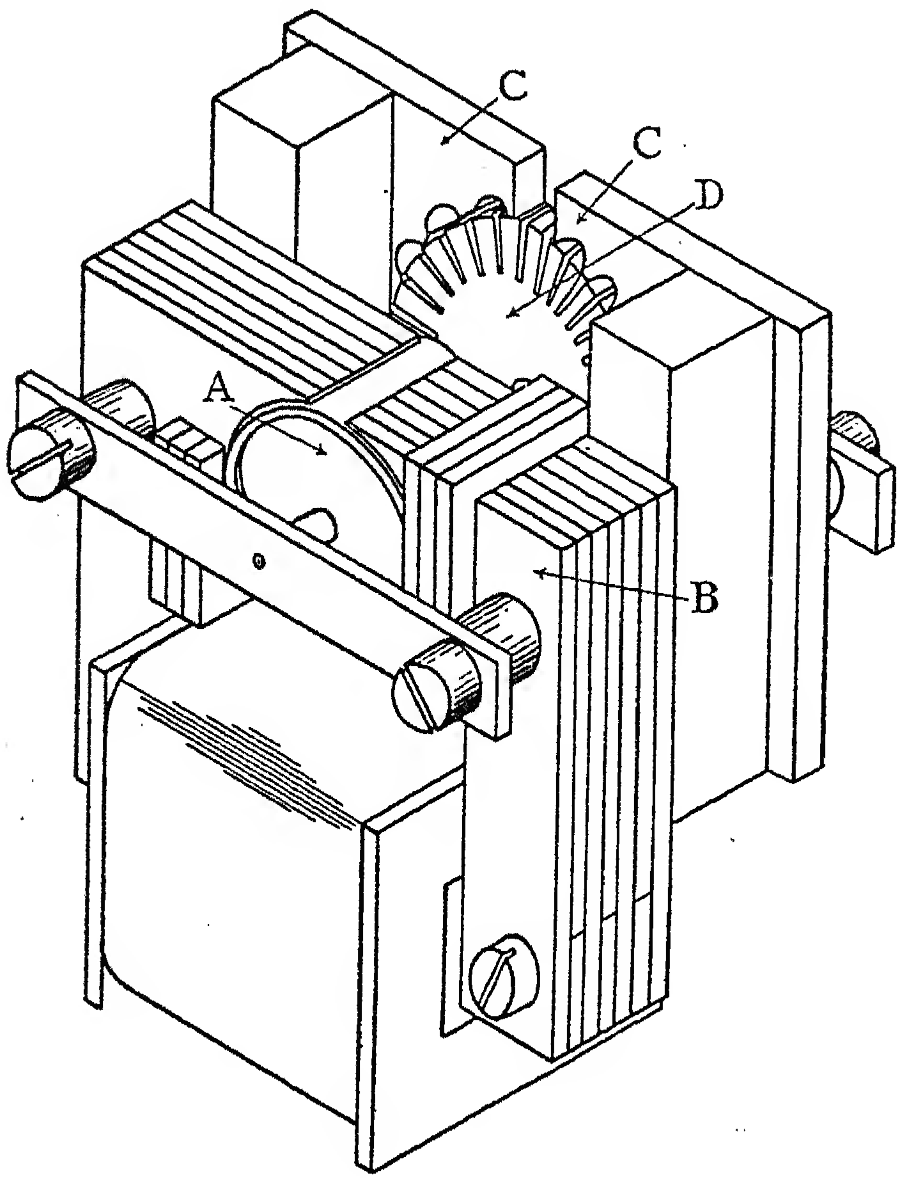


Fig. 8A.

Type IV. In the latter case, however, the fields were in series, while in this instance they are in parallel.

The field for the induction rotor A consists of a 2-pole construction B, each pole of which is divided into two equal parts, and one half of each pole is shaded in the well-known manner to produce a rotating Ferraris field (see Fig. 8A). Shunted from this system a purely alternating field is disposed, having a number of polar projections CC, from which it follows that those from one pole will be the same polarity, and those from the other of opposite polarity.

Mounted in the rotating-field system is an induction rotor A of the squirrel-cage type, and in the shunted field CC is a rotor D which may be of (*a*) soft iron, (*b*) permanent-magnet material, or (*c*) a permanent magnet; the poles are pitched in relation to the polar projections on its corresponding stator. The tops of the slots on the rotor D are filled with pieces of iron to prevent magnetic locking of the rotor at standstill. The induction rotor A is firmly keyed to the shaft, whilst the synchronous rotor is freely mounted thereon and coupled to the induction rotor through a spiral spring; the latter is not discernible in the illustration. Owing to the fact that the induction element has a 2-pole structure, and as the synchronous element is multi-polar, the synchronous speed of the former will be many times greater than that of the latter, thus allowing a safe value of running-in torque.

The operation of this motor is more or less identical

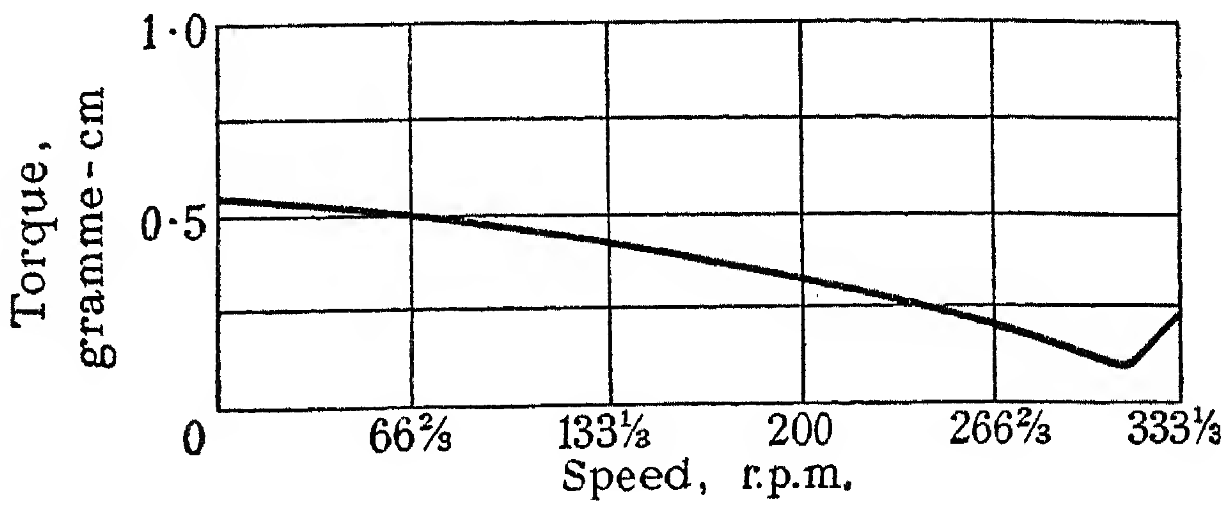


Fig. 8B.—333⅓ r.p.m., 230 volts, 50 cycles.

with that of Type IV, which obviates the necessity for further detailed explanation.

Results are given in Table 8 and Fig 8B.

(6) Description of, and Experiments on, Type IX.

The description of this motor includes the general experimental research conducted in its development, and shows many of the characteristics of the various stages in the design. The motor operates on the hysteresis principle and runs at low speed.

From what has previously been stated, the two known examples of motors having rotors of permanent-magnet and magnetic material are Types A1 and B1, and it was felt that some limit had been reached in design regarding the former type in obtaining a high starting torque and predetermined direction of rotation. Many experiments were conducted, with various shapes and settings of rotor and stator teeth, to cause the rotor always to start in a definite direction, and although a motor was devised to start the correct way it would occasionally rotate in the opposite direction; it was not

Table 8.

Results on Type VIII Motor at 333⅓ r.p.m., 230 volts, 50 cycles.

Volts	Watts	I^2R	Starting torque	Synchronous torque	Running-in torque	Work done	B.H.P. × 10^5	Efficiency
			g-cm	g-cm	g-cm	g-cm/min.		per cent
180	1·51	0·32	0·35	0·15	0·065	136	0·0298	0·014
210	2·08	0·44	0·5	0·19	0·09	188	0·0413	0·014
230	2·53	0·54	0·57	0·22	0·1	209	0·0458	0·013
250	3·0	0·65	0·69	0·25	0·12	251	0·0549	0·014
280	3·81	0·83	0·9	0·35	0·18	377	0·0827	0·016

possible, from a commercial manufacturing point of view, to set the poles or stator teeth with sufficient accuracy to be certain always of a correct start.

Looking farther, the hysteresis type seemed to offer potentialities although, probably owing to the absence of the modern magnet steels, it had not been possible previously to construct a motor working on this principle running at a low speed. The theory of this motor was fully dealt with by Steinmetz 34 years ago, and a close study of his research revealed that, by choosing the correct material for the rotor, exceptionally good results could be obtained, provided the stator system could be constructed with reasonable magnetic efficiency. The stator system must be of the rotating-field type, i.e. at any particular instant, considering the phase of the fluxes in the poles going in a particular direction round the stator, the flux must be, approximately, zero, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, zero, etc.; any material then having a hysteretic coefficient and free to rotate under the influence of this field will have a pronounced synchronous running torque, and under certain conditions a reasonable starting torque can be achieved (the rotor may be in the form of either a thin disc or a rim). Fig. 9A illustrates one form of this construction.

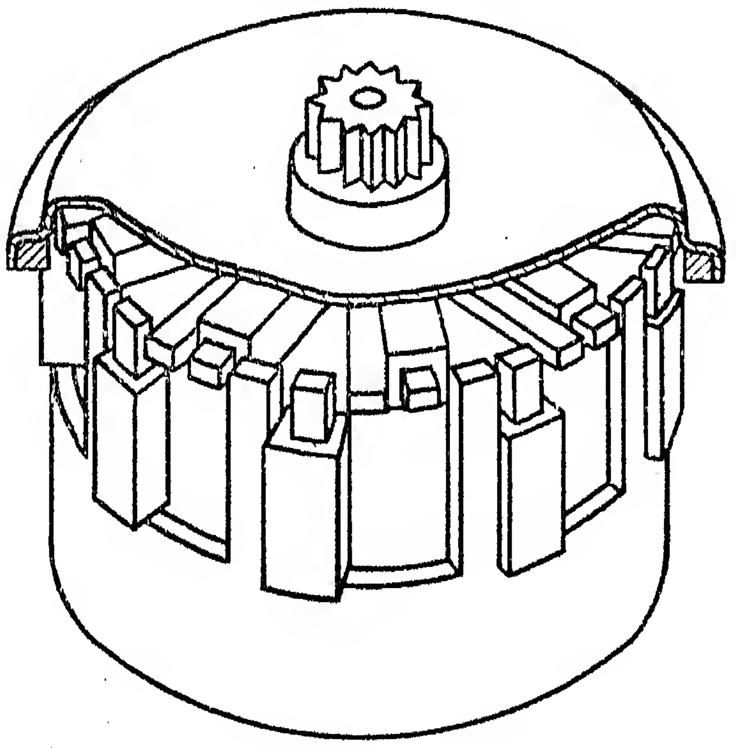


FIG. 9A.

Steinmetz* states that the synchronous torque obtained with such a system is numerically equal to the work done in hysteresis per cm^3 per cycle, and, as is well known, the expression for work done in hysteresis per cm^3 per cycle is $W = \eta B^{1\cdot6}$, where η is the hysteretic coefficient and B is the maximum flux density in the rotor.

For a preliminary consideration, the work done, and consequently the synchronous torque, will be proportional to the hysteretic coefficient. The hysteretic coefficient for hardened cast steel (1·2 per cent carbon) is given as 0·028, and for tungsten steel 0·055, from which, disregarding the value of flux density in the ring, and whether it varied in the two cases or not, the synchronous torque produced by a tungsten ring should be roughly double that produced by a hardened cast-steel ring; experiments proved this to be the case.

After considerable search it was learned that the hysteretic coefficients of cobalt steel, nickel-aluminium steel, and 82 per cent silver alloy, were not available, and it was necessary for the authors to construct the hysteresis loops; these have since been confirmed by another source.* Fig. 9B gives these experimental results, but those on the silver alloy have not yet been concluded. The area of the loop in the correct units gives the work done in hysteresis per cycle per cm^3 of material, and from the expression $W = \eta B^{1\cdot6}$ the value of η in each case was calculated and found to be 0·145 in the case of cobalt steel, and 0·390 in the case of nickel-aluminium steel.

The assessment of the factors governing starting torque presented certain difficulties, because accurate measurement of quantities of so low an order as were prevailing in these motors was not an easy task. With a small and interconnected stator magnetic circuit, flux leakage played a very important part, and calculations of the actual working flux on the ring could not include any really accurate conception of the leakage. Experiments were conducted to measure the flux density at the stator pole-tips; search coils were wound on each pole-tip and connected in series, and the total e.m.f. was measured by null methods from which the flux was calculated. In addition to these tests the calculation still had to be made of leakage, but as this consisted only of leakage between the pole under consideration and adjacent poles, errors likely to be included could be assumed to be small and the figure obtained to be fairly accurate. At the normal working ampere-turns of the motor, namely 170, the flux density at the pole-tip was 850 lines per cm^2, and this figure had to be the starting point from which any further conceptions could be based. So far as could be seen, the starting torque should be proportional to the actual flux density in the ring, since this would be the only variable factor with which to contend; the working flux available at the stator poles, the length of air-gap, the length of iron circuit in the stator, and the length of iron circuit on the rotor, would be the same, irrespective of the material of the rotor. Accordingly, since it has been stated that the starting torque must be proportional to the actual density in the ring, it must in turn be proportional to the reluctance of the material for the particular density at which the system was working.

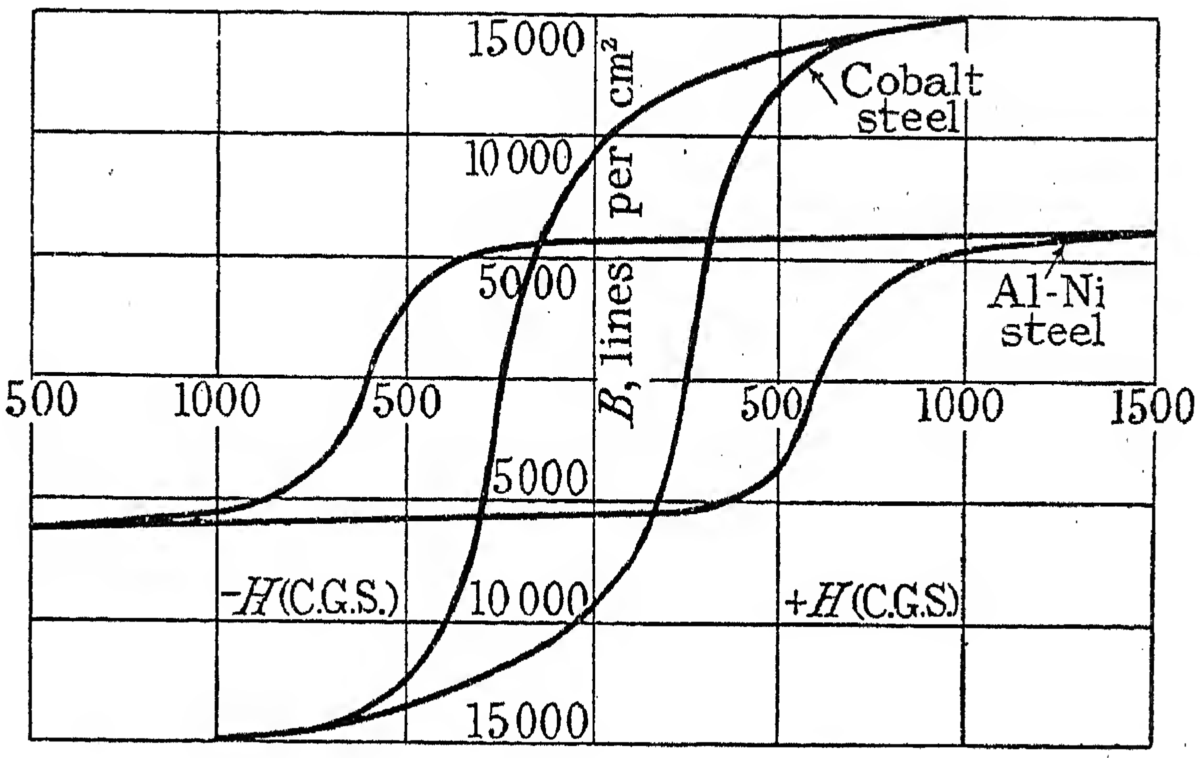


FIG. 9B.

The relation between the length of the air-gap and the length of the iron circuit of the rotor was of such

* "Alternating Current Phenomena," 1908 edition, p. 336.

* The English Steel Corporation.

a value that the permeability of the rotor material was responsible for a considerable effect on the starting torque of these motors. To take an actual case, the following figures were evident. The path of the flux starting at a stator pole-tip would be: across the air-gap, through the rotor, and back through the air-gap to the stator pole-tip located 180 electrical degrees away. Stating this in another manner, the flux will go from a pole-tip having, say, N polarity at any instant, and eventually return to the adjacent pole having S polarity at the same instant. This governs the length of the pole induced in the rotor, and referring to Fig. 9A, where an 8-pole-pair system is illustrated, the length of pole induced in the rotor would be 1/16th of the circumference, which in the case of the motor under consideration is 0·76 cm; the length of the air-gap is 0·028 cm, and since there are two of these in series the total length of the gap would be 0·056 cm. Now the reluctance of the previously stated path from the pole-tip of one polarity to the pole-tip of the other is

$$S = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2}$$

neglecting the reluctance of the return circuit: l_1 is the total length of the air-gap, A_1 the area of the air-gap, A_2 the cross-sectional area of the rotor, μ_1 the permeability of the air-gap, l_2 the length of the iron circuit of the rotor, and μ_2 the permeability of the material of the rotor at the working density. Substituting the known values in the above expression, we obtain

$$S = \frac{0\cdot056}{0\cdot0277} + \frac{0\cdot76}{0\cdot025} \times \frac{1}{\mu_2}$$
$$= 2\cdot02 + 30 \times \frac{1}{\mu_2}$$

For the working value of H the permeability of tungsten steel was 500, and of cobalt steel 50, approximately. Substituting these values in the expression above, we obtain

$$\begin{aligned} S \text{ (tungsten steel)} &= 2\cdot02 + 30 \times \tfrac{1}{500} \\ &= 2\cdot02 + 0\cdot06 \\ &= 2\cdot08 \\ S \text{ (cobalt steel)} &= 2\cdot02 + 30 \times \tfrac{1}{50} \\ &= 2\cdot02 + 0\cdot6 \\ &= 2\cdot62 \end{aligned}$$

The density being inversely proportional to the reluctance, the density in the two cases was

$$\begin{aligned} B \text{ (tungsten steel)} &\propto 0\cdot48 \\ B \text{ (cobalt steel)} &\propto 0\cdot38 \end{aligned}$$

that is, the ratio of density in the cobalt steel to that in tungsten steel is 79 per cent, from which the starting torque of the cobalt-steel rotor would be 79 per cent of that of the tungsten steel.

In the verification of this result by experiment, it must be borne in mind that the experimental figures must be taken from the motor whose component parts give the most consistent and efficient results, and, as will be seen later when reviewing the actual experimental figures, these specific results were obtained with the rotor comprising an annular rim of cross-section 1/16 in. × 1/16 in. Actually, giving this verification for the same ampere-turns in each case, the starting torque of a 35 per cent cobalt-steel ring was 2·5 gramme-cm, and the corresponding figure for tungsten steel was 3·5 gramme-cm.

The preceding remarks and figures are given to indicate to some extent the principle of the hysteresis motor, and before enlarging on that it is now proposed to give the various stages which eventually led to the design of motor illustrated in Fig. 9A.

So far as the authors were aware the only self-starting synchronous motor operating on the hysteresis principle was the one known as the Warren, running at 3 000 r.p.m. when connected to a 50-cycle supply.

A motor was constructed having 12 pole-pairs, and a disc of tungsten steel, made skeleton in form to minimize mass, was mounted in jewel bearings. This gave a starting torque of 0·25 gramme-cm with 4·5 watts input; to obtain a starting torque of 1 gramme-cm it became necessary to supply 13 watts to the stator. Making a rotor consisting of a thin rim of tungsten steel, supported by an aluminium former somewhat as fitted in Fig. 9A, it was possible to obtain a starting torque of 1 gramme-cm with 6 watts input.

Considerable work was then performed towards improving the stator magnetic system, and eventually the motor illustrated in Fig. 9A was evolved. This shows an 8-pole-pair arrangement giving a synchronous speed of 375 r.p.m. when connected to a 50-cycle supply. With a hardened cast-steel rotor a starting torque of 1·2 gramme-cm and a synchronous torque of 0·53 gramme-cm were obtained with a watt input of 1, and when the watts were increased to 2·7 a starting torque of 4·0 gramme-cm was obtained. At this figure of 2·7 watts the rotor ran with a definite slip, i.e. it would not keep in step with the frequency. This point is dealt with later in the text.

The temperature-rise of this motor when operating with 2·7 watts was 40 deg. C., and in an endeavour to reduce the heating effect the losses of the motor were separated. The total watts of 2·7 comprised 1 watt copper loss (I^2R), 1·16 eddy loss, and 0·54 hysteresis loss; by making a radial saw-cut through the iron circuit the eddy losses were reduced from 1·16 to 0·56, which reduced the total watts from 2·7 to 2·1. The hard cast-steel rotor gave a starting torque of 4·0 gramme-cm, but could not attain synchronous speed.

The reason for this rotor having no synchronous torque with inputs greater than 1·5 watts is explained by the high permeability of its material and the increasing values of leakage flux at the higher inputs, causing much wider and indefinable rotor poles to be formed.

The foregoing experiments were conducted with a rotor of cross-section 1/16 in. × 1/16 in. Experiments were now made using the same material but with a different cross-section. As was to be expected, reducing the cross-section by 33⅓ per cent did not appreciably alter the synchronous characteristics, but actually lowered the starting values. Increasing the cross-section by the same amount still further reduced the starting characteristics, indubitably due to a combination of lower working flux density and greater mass, but the motor ran in step with the frequency with up to

4·7 watts input, although the value of the synchronous torque steadily decreased from 1·7 gramme-cm at 1·5 watts input to 0·5 gramme-cm at 4·7 watts input.

The inference was, then, that the best results would be obtained with the cross-section 1/16 in. × 1/16 in., but of magnetic material possessing greater hysteretic constant than cast steel in order to give greater synchronous torque. As the constant for tungsten steel was known, a series of rotors was made up in this material; for the same size of ring a synchronous torque approximately double that of cast steel was obtained. The permeability of tungsten steel is approximately equal to that of cast steel for the particular flux density at which it was worked, and, as expected, the starting torques were nearly the same. No real advantage was found by using rings of other dimensions.

At this stage the potentialities of 35 per cent cobalt steel, nickel-aluminium steel alloy and silver* magnetic alloy with their respective coercivities of 240, 600, and 2 000, were investigated.

The synchronous torque obtained in the case of cobalt steel showed a very pronounced increase above that for either tungsten steel or cast steel, although the starting torque suffered in consequence. It was found that the size of ring which gave the best results in the case of tungsten steel was also the most efficient in cobalt steel.

Although the results obtained with 35 per cent cobalt-steel rings were considered satisfactory for general purposes, in view of the greatly increased magnitude of the hysteresis coefficient for Al-Ni steel a ring was made up in this material. The machining qualities of Al-Ni steel at present are not good, and considerable difficulty was experienced in producing a ring of the requisite dimensions. When mounted in its stator it was found that for the same watts input as the cobalt steel it would not start, but on being spun to synchronous speed by hand it continued to rotate synchronously with a torque greater than that of cobalt—for the same watts input.

The fact that it was not self-starting except with a very large watts input, whilst not substantiating, pointed towards the theory that the starting qualities are proportional to the permeability of the material, and it will be realized that the permeability of Al-Ni steel for the variations of flux at which the experiments were conducted would be very low.

This factor, together with the difference in coercive force, also influenced the synchronous torque, but by increasing the magnetizing ampere-turns a synchronous torque of 16·5 gramme-cm was obtained.

* H. H. POTTER: *Philosophical Magazine*, 1931, vol. 12 p. 255.

With the motor operating under these conditions it is doubtful whether it can be considered a true hysteresis motor, as it still was not possible to make the motor self-start, and the synchronous running obtained appeared to be due to permanent poles induced in the rotor before its normal running commenced.

The deduction which was made then was that the greatest starting torque was obtained with cast steel of maximum permeability, and the greatest synchronous torque with Al-Ni steel of maximum hysteresis coefficient. A motor was therefore constructed having two rotors, one of cast steel to give it starting properties, and one of cobalt steel for the synchronizing action, and although the added weight and increased pole area had a diminishing effect on the starting torque the design was very promising.

The measurement of synchronous torque on these motors called for great care and patience, for it transpired that a minute excess of load over the true synchronous torque of the motor caused the rotor to slip a very slight amount, whilst the application of a greater load caused the slip to be proportional up to a point; beyond this, the rotor rapidly came to a standstill. The actual magnitude of slip with the very slight increase of load mentioned was in the neighbourhood of 1/80th of 1 per cent, equivalent to an error of 1 second in $2\frac{1}{3}$ hours, or 1 minute in 6 days; the maximum slip measurable before the motor actually stalled was 3 per cent, but the fact that this was transitory made measurement difficult and results rather unstable.

The only explanation that the authors can give to this rather unexpected point is that with the imposition of excess load the rotor falls behind the flux by an amount equal to a few degrees, the actual amount depending upon the magnitude of the extra load, and a slow and continuous drag of the poles round the rotor follows. As the magnitude of the excess load increases, the drag increases, with a consequent increase in slip and reduction in speed, until that point is reached when the stator poles have no control at all and the rotor stops.

(7) TYPE IX AS RELATED TO STEINMETZ'S THEORY.

Considering the hysteresis motor as a whole with respect to Steinmetz's theory, it was stated that the torque would be numerically equal to the work done

TABLE 9.

Results on Type IX Motor at 375 r.p.m., 230 volts, 50 cycles.

Volts	Watts	I^2R	Starting torque	Synchronous torque	Running-in torque	Work done	B.H.P. × 10^5	Efficiency
			g-cm	g-cm	g-cm	g-cm/min		per cent
180	1·1	0·37	1·1	3·3	1·0	2 360	0·52	0·350
210	1·5	0·54	1·9	4·7	1·75	4 150	0·91	0·450
230	1·8	0·64	2·5	5·4	2·3	5 400	1·19	0·49
250	2·2	0·78	3·2	5·6	3·0	7 070	1·55	0·52
280	2·8	1·0	4·4	5·7	4·15	9 700	2·12	0·56

in hysteresis per cycle per cm^3, and from this the synchronous torque derived from different rotor materials would be proportional to the hysteretic coefficient of the particular material. The difficulty in corroborating this theory was to decide upon which basis to start. To assume constant magnetizing ampere-turns in each case was incorrect, because, owing to the different B/H curves of the different materials, with the same ampere-turns a less value of induced $B_{max.}$ was obtained in a cobalt-steel rotor than was obtained, say, in the case of a tungsten-steel rotor. Taking this a stage farther, the comparison might have been made with equal maximum flux density in the various rotors, but again this would have been incorrect, since density does not represent the amount of energy available for the production of torque; coercive force must be incorporated.

The deduction was, then, that the maximum synchronous torque obtainable with a certain rotor, irrespective of the magnitude of the magnetizing ampere-turns, was the value which must be taken in order to compare the different alloys and as a basis for theory substantiation.

In this way the only method possible is utilized for compensating for the varying characteristics of the different materials used, with particular reference to the degree of magnetization necessary in the various steels and alloys. Accordingly, in the case of cast steel, tungsten steel, cobalt steel, and Al-Ni steel, for increasing values of the magnetizing force the maximum torque was measured at which the motor still rotated in synchronism.

Table 9A.

Material	Hysteretic coefficient	Synchronous torque
Cast steel	0·025	0·8
Tungsten steel	0·058	1·7
35 per cent cobalt steel ..	0·145	5·7
Al-Ni steel	0·390	16·5

For the purpose of easy comparison, these results are given in Table 9A, together with the hysteretic coefficient for each material.

It will be evident from these results that the synchronous torque is proportional to the hysteretic coefficient, for the deviation can be assumed to be due to inconsistency in results, unavoidable in the measurement of torques of magnitudes of this order.

An interesting point in connection with synchronous torque arose in the derivation of the results given in Table 9A. Taking, for example, the case of cobalt steel, when the magnetizing ampere-turns of the stator were increased beyond the value giving a torque of 5·7 gramme-cm, the torque of the motor decreased and eventually, with nearly 200 per cent excess ampere-turns, the motor reached a point when it no longer rotated synchronously. As in the case of cast steel, this is explained by the increased value of leakage flux causing much wider and indefinable poles to be formed.

A degree of magnetization 20 per cent greater than that necessary to give rise to the torque of 5·7 gramme-cm was attained, and then the maximum torque was measured at the lower magnetization value; it was found to have increased from 5·7 to 13·5 gramme-cm. In other words, the additional torque was produced by the "cogging" action of the highly-magnetized poles induced, in much the same way that the synchronous motor Type II operates. The energy available in each incremental magnet of the rotor was, however, insufficient to maintain this condition indefinitely, and after the motor had been run for several hours at the lower magnetization value mentioned, with many current interruptions, the torque resumed the former value of 5·7 gramme-cm.

Similar results were obtained with Al-Ni steel.

(8) Consideration of Starting Torque of Type IX.

It has been shown, in the case of tungsten and cobalt steels, that the starting torque is proportionate to the permeability of the rotor material at the particular value of ampere-turns ruling.

Table 9B.

Material	Proportionate density	Starting torque
Cast steel	0·5	3·6
Tungsten steel	0·48	3·5
35 per cent cobalt steel ..	0·38	2·5
Al-Ni steel	0·25	0·62

Applying the same reasoning to Al-Ni steel as was employed in the case of cobalt steel, the reluctance of the path was found to be nearly double that of tungsten steel, from which only half the flux density in the ring was obtained. It would appear, then, that the starting torque of Al-Ni steel should be approximately one-half that of tungsten steel, but experiment proved this to be incorrect, for whereas a starting torque of 0·62 gramme-cm was obtained, the rotor would not run up to synchronous speed.

Reference to Table 9B indicates that, with the exception of Al-Ni steel, the starting torque is proportional to the maximum flux density attained in the ring.

The magnetization curves of cast steel and tungsten steel do not vary sufficiently, at the point at which the rotor is operating, to influence to any degree the density in the rings in the two cases, and the starting torque of motors whose rotors are made from these materials is nearly the same in both cases.

(9) Type IX as an Industrial Accessory.

Analysing this motor as an industrial accessory, particularly with reference to time-measuring apparatus, the results obtained at present are rather encouraging. The best results, theoretically, should be obtained with an Al-Ni steel rotor in combination with an additional rotor such as cast steel to give starting properties; the

machining difficulties of the former, however, militate against its adoption commercially. It will be seen that at this stage the experimental research on motors with Al-Ni steel and silver-alloy rotors is not complete and is left over for further investigation. The cobalt-steel rotor gives very satisfactory operation with a single rotor, and this construction is promising. Further improvements in the methods of lagging the stator poles should increase the values of starting torque.

A further direction in which the research work is not concluded is in the 2-rotor system, having one rotor of high-permeability and the other of low-permeability steel.

At the same time, it is felt that having obtained a design, Fig. 9A, embodying a single cobalt-steel rotor, directional in its rotation, rotor mass 5·6 grammes, overall size $1\frac{3}{8}$ in. diameter × 1 in. deep, with the results given in Table 9, a step forward has been made.

The speed/torque curve is shown in Fig. 9C; the hook

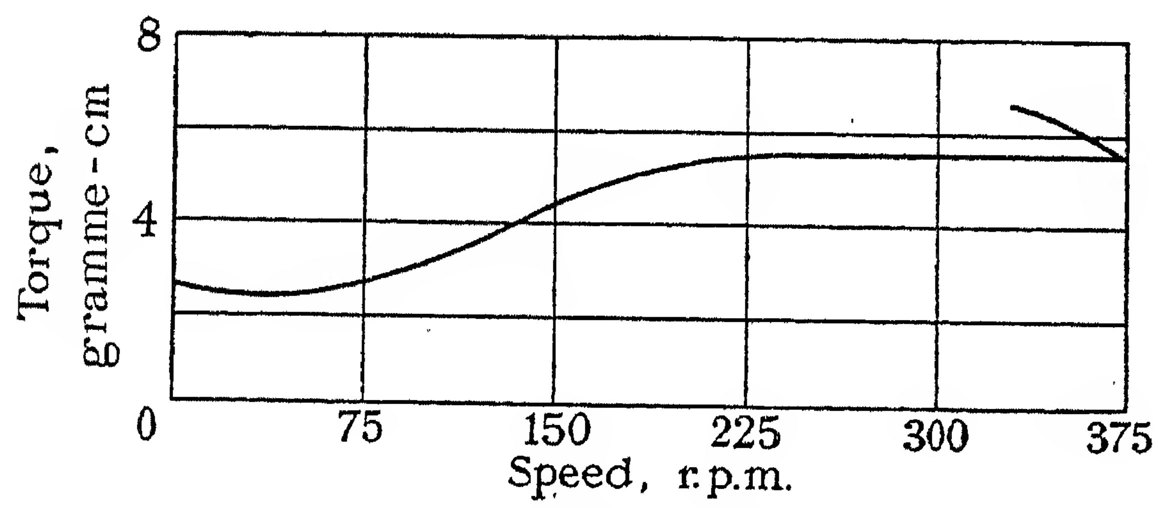


FIG. 9C.—375 r.p.m., 230 volts, 50 cycles.

at the top of the curve is obtained by loading the motor above synchronous torque, and the fall in speed is plotted against load.

(10) PERMANENT MAGNETS IN SELF-STARTING SYNCHRONOUS TIME MOTORS.

The permanency of magnets in self-starting synchronous motors has been considered by the authors, and they are of the opinion that such magnets are unquestionably reliable when correctly designed.

As previously mentioned, the Type B motors run on the hysteresis principle, and they use, and require, permanent-magnet material for the rotor. The permanent magnetism in the rotor is formed and maintained by the stator system, and the fact that the rotor partly loses its permanent magnetism when the supply is switched off does not affect the starting or running condition when the supply is resumed again.

In the Type A motors the rotor magnet is highly magnetized by external means, and it is important, both for starting and synchronous torques, that the permanent magnet should remain permanent.

Consider the permanent magnet constituting the rotor in the Type II motor, as illustrated in Fig. 2A. It is magnetized to saturation point whilst fitted to a specially-shaped keeper having a closed magnetic circuit, resulting in a remanent density of approximately 9 000 lines per cm^2.

It is then subjected to a very considerable reduction by removing the keeper, and the theory of the self-demagnetization is as follows:—

The poles formed by removing the keeper exert a self-demagnetizing effect according to the expression*

$$H_D = \frac{4\pi J}{\lambda} \cdot \frac{l_g A_m}{l_m A_g}$$

where
l_g = length of gap,
l_m = length of magnet,
A_m = area of magnet,
A_g = area of gap,
J = intensity of magnetization,
λ = dispersion coefficient.

The actual values are obtained by Ascoli's† construction when

$$\frac{H_D}{4\pi J} = \tan\alpha,\ \alpha \text{ being equal to } \frac{1}{\lambda} \cdot \frac{l_g A_m}{l_m A_g}$$

Inserting in the expression for α the actual figures corresponding to the magnet under consideration, viz.

$$l_g = 1\cdot 59 \text{ cm},$$
$$l_m = 2\cdot 86 \text{ cm},$$
$$A_g = 0\cdot 072 \text{ cm}^2,$$
$$A_m = 0\cdot 108 \text{ cm}^2,$$
$$\lambda = 2 \text{ (assumed)},$$

it is found that $\tan\alpha$ is 0·4.

Applying this value of the demagnetizing coefficient to the curve of the material in question, the value of B falls to 650, and this is indicated at $B_{rem.2}$ in Fig. 10(a).

With the rotor in this condition, it is placed in the motor stator where the length of the gap is now 0·076 cm, and applying the same mathematics as before, but with this new value of l_g, the value of $\tan\alpha$ is 0·02. Again employing Ascoli's construction, the reduced value of demagnetizing force is determined, but the density, instead of retracing its original path, follows a subsidiary curve giving a value equal to 1 650, indicated at $B_{rem.3}$ in Fig. 10(a). The subsidiary curve was obtained experimentally from standard test bars of the cobalt steel, after the point $B_{rem.2}$ of value 650 had been measured, the curve being constructed from the extreme point on the left, $B_{rem.2}$, to a point on the right corresponding to $B_{rem.3}$.

When an alternating current is applied to the stator magnetizing coil, the magnet is subjected to the demagnetizing and magnetizing action of the stator poles, and the value of this is derived in the following manner.

Assuming the rotor to be stationary (which gives the maximum demagnetizing case) the total lines from each stator pole, calculated from the magnetizing ampere-turns of the core and assuming no leakage, equal 600, but as the construction of the stator allows of leakage paths in many directions, a very small proportion of the total flux passes through the permanent magnet, and it is not possible to make any calculations as to the value of the actual working flux. It has therefore been determined by experimental means, and to this end a search coil was wound on each side of the 6 limbs of the permanent magnet, and the whole connected in series; the magnet had previously been magnetized and put in the stator in the ordinary way so as to retain its normal

* S. P. THOMPSON: *Journal I.E.E.*, 1913, vol. 50, p. 80.
† S. P. THOMPSON: *loc. cit.*

permeability at the working density. With the normal ampere-turns on the core, obtained by the application of 230 volts to the stator coil, the average e.m.f. produced was 160 millivolts, measured by null methods; this value corresponded to $\Phi_{max.} = 14 \cdot 7$ lines, and $B_{max.}$ is therefore 115 lines per cm². Applying this latter figure to the *B*/*H* magnetization curve of the material, it was found that the value of *H* to produce 115 lines per cm² was 9 gauss, and this was consequently the magnitude of the demagnetizing action of the stator poles on the permanent magnet.

Referring to Fig. 10(*a*) and to the enlargement shown in Fig. 10(*b*), the flux-density value of 1 600 is indicated at B, and the magnitude of the magnetizing force of

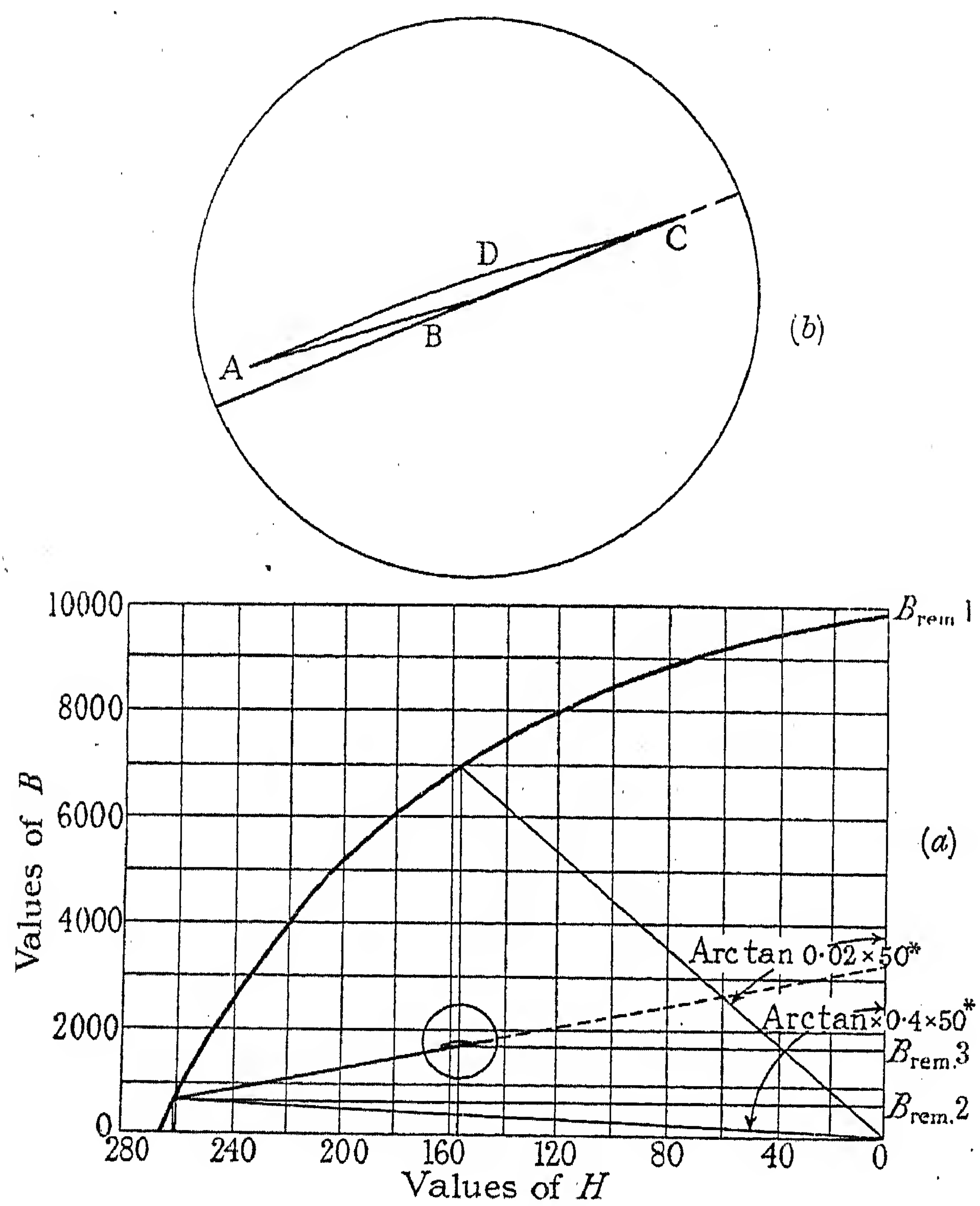


Fig. 10.

* The factor 50 compensates for the scale of the curve, i.e. $\dfrac{\text{Scale for } H}{\text{Scale for } B} = \dfrac{1}{50}$.

9 gauss is given by the horizontal component of the distance AB. The effect of the magnetizing force of the a.c. field is for the remanent density to move first from B to C, and, on the cycle reversing, from C to A through any point D: from A at the next part of the cycle it will revert to B, and the cycle will be repeated.

There is a small hysteresis loop formed from A to C which continues as long as the alternating current is applied with the rotor held stationary. The loop does not prove that the magnet will remain permanent, but as the point A is well within the limit of the demagnetizing curve it indicates that the magnet under consideration will remain permanent against the demagnetizing forces operating. As the rotor is allowed to revolve up to synchronous speed this loop disappears and the remanent density of the magnet remains constant at B. In a batch of magnets tested a slight demagnetization is measured which varies between 0 and 2 per cent in different magnets, and is probably due to some slight disturbance of the magnetic poles during manufacture.

Ageing.

A number of tests have been conducted with a view to obtaining information on the performance of the magnets over a period of time. The magnets were arranged to operate under two distinct and separate conditions; in the one case they were free to rotate in their stators, and in the other they were permanently locked in a stationary position in the stators. Table 10 gives the results of a test taken at normal voltage, and Table 11 at excess voltage.

Table 10.

(Average results) Condition of magnet	Flux density in magnet	
	Free to rotate	Locked in stationary position
(*a*) Fully magnetized, then keeper removed	660	—
(*b*) Fitted in stator	1 675	1 675
(*c*) Stator energized for few seconds	1 650	1 650
(*d*) Stator energized for 1 month	1 650	1 650
(*e*) Stator energized for 1 year	1 650	—

Table 11.

(Average results) Condition of magnet	Percentage flux density in magnet	
	Free to rotate	Locked in stationary position
(*a*) Normal voltage applied to stator	100	100
(*b*) 25 per cent excess voltage for 1 week	100	100
(*c*) 50 per cent excess voltage for 1 week	99	99
(*d*) 100 per cent excess voltage for 1 week	93	93
(*e*) Normal voltage for further 1 week	93	93

(11) The Design of Rotor Bearings.

Although this phase of the self-starting synchronous motor is really a mechanical engineering problem, it is linked with electrical considerations, and very particular care must be exercised in the design. The construction must be such as will ensure the following conditions being maintained:—

(1) Attention to lubrication will not be required for a number of years, meaning that a maximum oil reservoir must be embodied.

(2) The ratio of bearing length to diameter must be high to reduce wear.

(3) The clearances in the bearing should be small to minimize noise.

(4) Where the bearing is situated in the stator core, the amount of iron remaining must be sufficiently high to retain a reasonable flux density there.

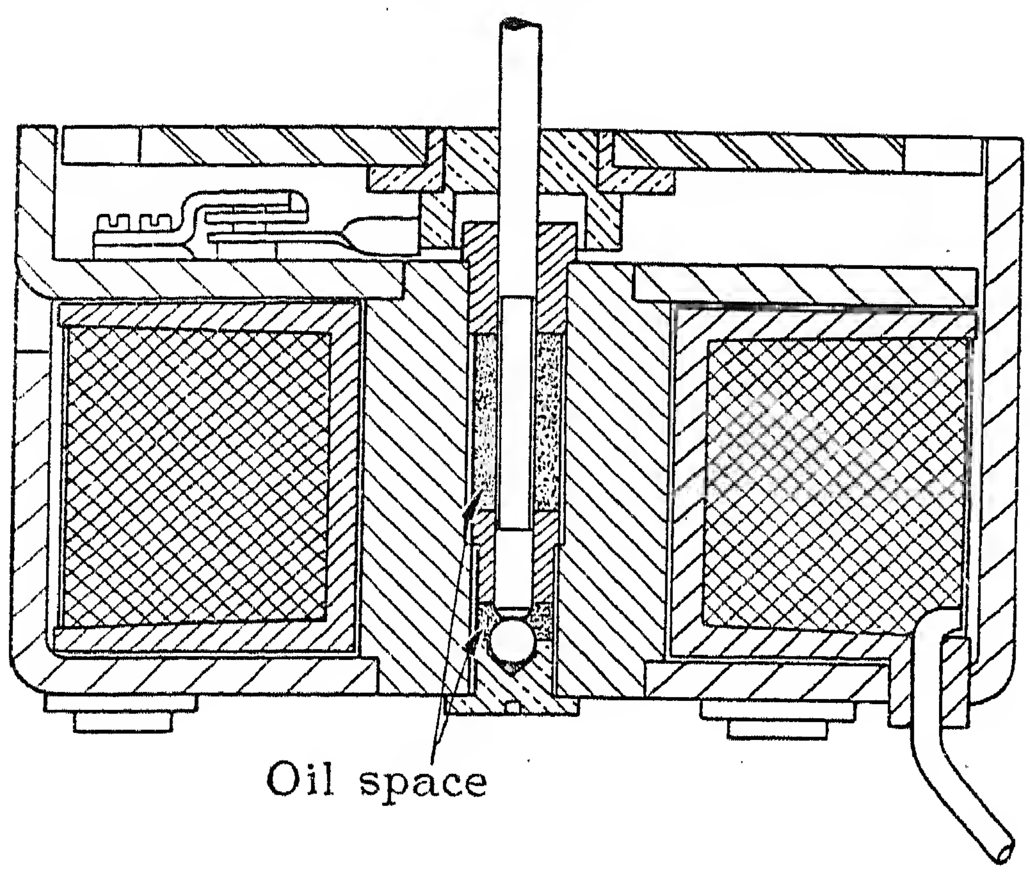


FIG. 11.

It is felt that the method adopted in the high-speed type of motor is sufficiently well known to enable a detailed description to be omitted, and it suffices to state that the rotor bearing and gear train are enclosed in a sealed chamber containing oil.

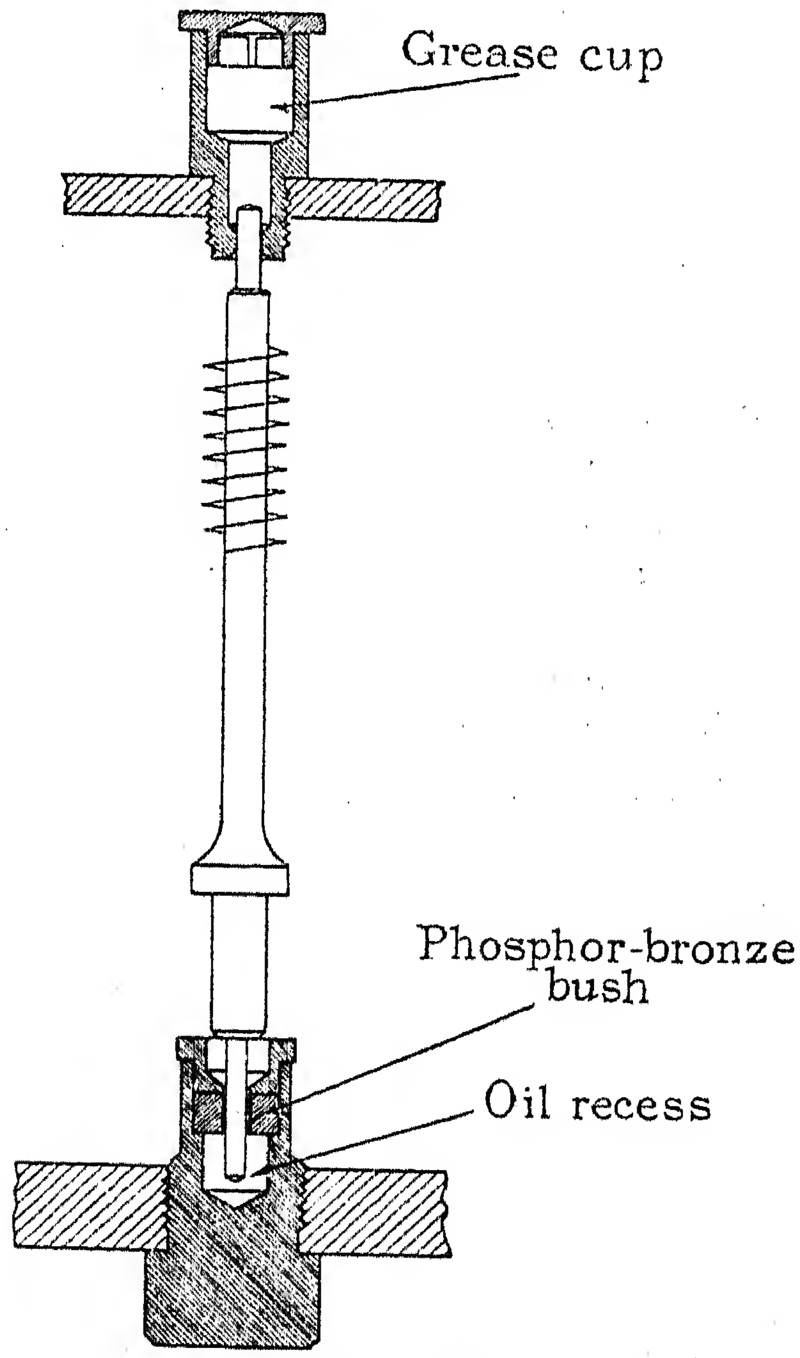


FIG. 12.

Fig. 11 illustrates a bearing used in a low-speed motor, and it will be seen that it is located inside the stator core; ample provision for oil storage is made and bearing proportions are adequate.

In a typical example of a higher-speed motor, namely one running at 1 000 r.p.m., Fig. 12 shows the construction adopted, which will be seen to be of quite a different nature from that of the previous case. Here, owing to the somewhat heavier rotor, a bearing is provided at both ends, and each is arranged to contain an oil reservoir. As stated in the description of this motor, the rotor is arranged to run with the shaft vertical, and fluctuations in voltage and load cause an up-and-down motion of the rotor, thereby distributing the oil over the bearing surfaces.

(12) DESCRIPTION OF THE TESTING APPARATUS.

Measurement of Starting Torque.

The measurement of starting torque for the various types of motors, with the exception of the impulse starter type, presents no difficulties, and is performed by the torque meter. This accessory, which is well known in the meter and instrument laboratory, operates on the spring dynamometer principle.

Were one to attempt to measure the starting torque of the impulse type motor by this meter, it would be found impossible to obtain stable results as, owing to the vibrating nature of the starting, the meter pointer would in turn vibrate, and it would be impossible to operate the spring. Accordingly, a method of interpolation was used by the authors, which it was felt was sufficiently accurate for general laboratory work. In this instance, referring to Fig. 13, two friction surfaces were applied to the motor under test, with provision for loading up as indicated, and values of the load were obtained when the motor would just not start. A series of figures was obtained for the impulse motor, Type II, for various values of input watts, and a similar series was obtained for any one of the induction synchronous motors. A curve of starting torque plotted against input watts had previously been taken of the latter motor, obtained by means of the torque meter, and it was then possible to translate the frictional load in grammes, say, into torque in gramme-centimetres by comparing the one with the other (for the same motor, of course) at the same value of input watts. The next step was to sketch a curve showing the relation between frictional load and torque, from which, when testing the impulse motor, the frictional load could immediately be written in terms of gramme-centimetres torque.

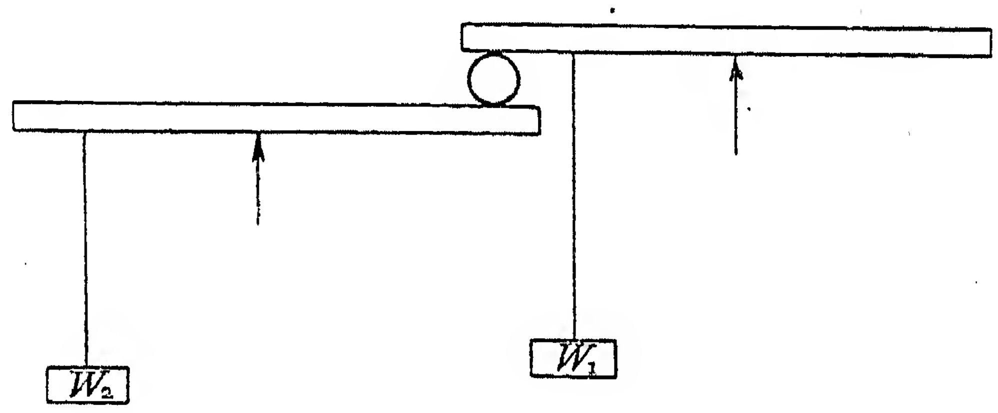


FIG. 13.

Measurement of Synchronous Torque.

In all types of motors, this may be performed by the well-known dynamometer method, which needs no description here.

Measurement of Running-in Torque.

For the measurement of running-in torque of motors of this size, special apparatus is necessary, consisting of a special spring dynamometer, a variable-speed motor, and a rotoscope. The variable-speed motor used was a series-wound 50-watt commutator motor, and was connected to the dynamometer shaft, to the other end of which the motor to be tested was connected through the medium of a calibrated spring: rigidly secured to the driving motor was a circular dial having four identical scales each occupying 90 degrees. Fig. 14 shows the gear used.

During the test the tested motor attempts to run at synchronous speed, but is held to any desired speed by the driving motor; the deflection of the spring is therefore a measure of the torque of the tested motor, and by varying the driving speed the torque at any speed can be determined. The dial rotates with the motors,

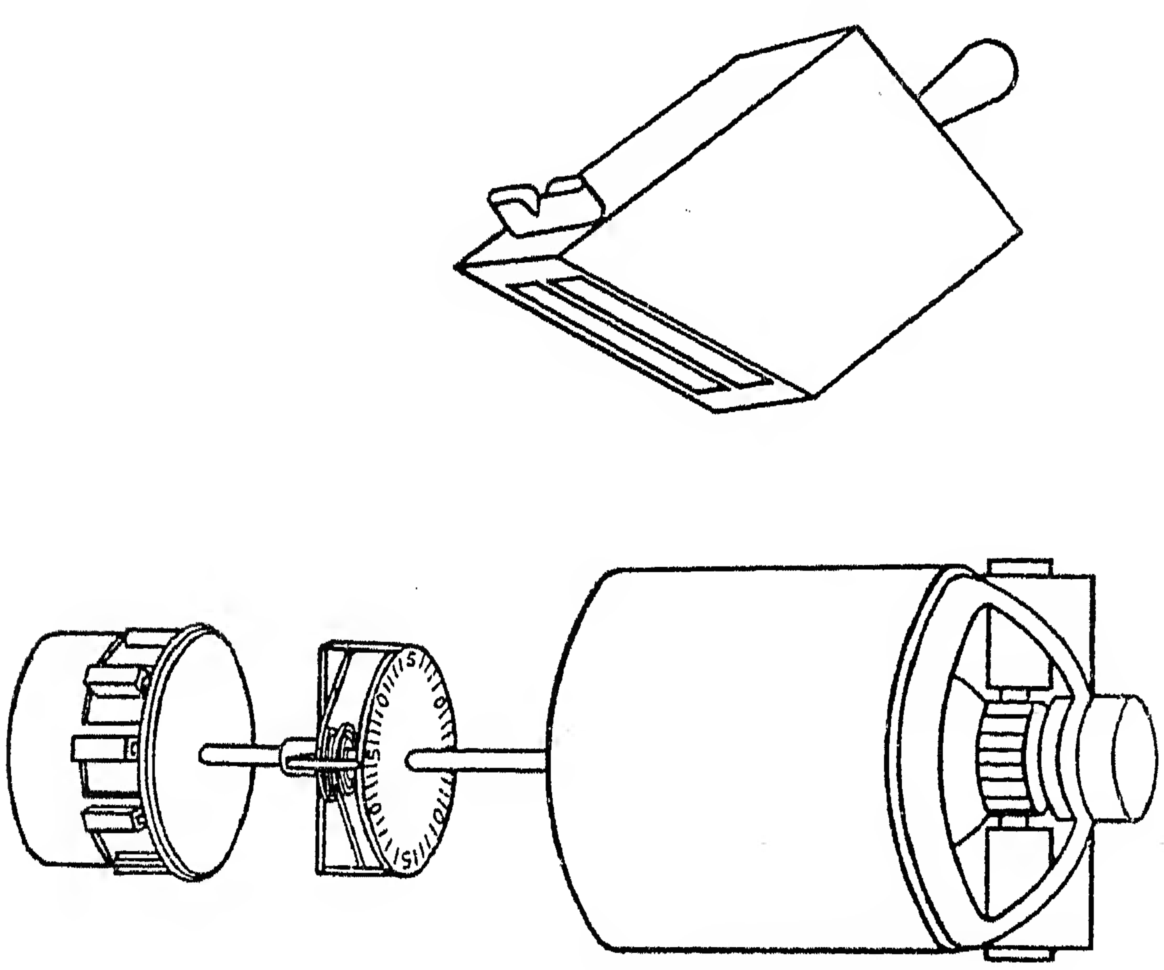


FIG. 14.

and the rotoscope is necessary to observe the spring deflection; the four scales of the dial are provided to enable the rotoscope to run at a comparatively high speed and thereby reduce flicker; at the same time the four pointers act as a mechanically balanced unit. As an alternative to the rotoscope, a stroboscope may be employed, consisting of a mercury lamp as used in television, operated by means of the usual " blinking neon " circuit, except that a valve is inserted in place of the usual variable resistance to give more accurate speed control of the lamp flashes. As a point of interest, thyratron valves were originally used as the stroboscope, but insufficient light intensity to read the moving scales prevented their adoption.

In testing the hysteresis motor, Type IX, for running-in torque, results in certain instances could not be obtained using this method; this applied particularly to the hard-steel alloys used for the rotor rim, for at speeds well below synchronism the rotor ran very erratically, making reading of the spring deflection impracticable. This was overcome by constructing a different test gear which utilized a variable-speed driving motor as before but was coupled to the motor under test through a clutch. Energizing the driving motor and the motor under test, the latter was held to any desired speed by the former, and a frictional load was applied to the motor under test. The maximum value of this load was determined by experiment, which, on the clutch being let out, would allow the motor under test to attain synchronous speed, and this value, translated into torque, gave the running-in torque for the particular speed: it was repeated for various speeds and the complete curve obtained.

(13) Some Applications of Small Self-Starting Synchronous Motors.

With the grid giving time-controlled frequency over practically the whole of the country, the synchronous motor may be used wherever accurate time-indicating is required, and already its use has been seen in time switches. In this instance the synchronous motor supersedes the spring-driven clock movement previously embodied, and obviates the necessity for periodical winding, thereby effecting a considerable economy to supply authorities. With this form of time-keeping, in combination with a mechanical reserve to tide over supply interruptions, time switches once installed may be left unattended for a matter of years.

Another increasing demand for these motors has been in house-service meters where the fixed-charge amount is collected in addition to the unit charge, the motor registering time through a train of gearing, and co-operating with the meter-element train to open the prepayment switch.

For laboratory purposes an electric stop-watch has been produced embodying a synchronous motor, and when used on time-controlled frequencies for the measurement of periods of time of short duration an accuracy of up to 1/20th sec. is easily obtained. The operation of this instrument is precisely similar to that of the well-known laboratory stop-watch, an electrical contact serving to set the pointer in rotation and removal of the contact arresting it. To overcome starting errors the motor is arranged to run continuously, and the function of the electrical contacts is to let in a clutch through which the pointer is connected.

Any form of relay requiring a delay action could embody a synchronous motor, and some have actually been constructed to give a delay period up to 30 minutes. This type of relay is used in workshops where automatic timing of repetition operations is desired. The dial is set to the time required, say 15 minutes, and at the commencement of an operation the synchronous motor driving the relay is energized. At the end of the pre-determined interval of 15 minutes, the relay closes an electrical contact, which may be arranged to illuminate a signal lamp or sound an alarm. The device is usually made automatic, whereby it is self-resetting, ready to repeat the cycle.

For electric clocks of large dial sizes where a fair amount of mechanical energy is required, and situated in not easily accessible places, these self-starting motors may be used in place of the usual hand-starting type.

Synchronous motors could be used with advantage for clocks on board ship, for they can be run from a special alternator with combined mechanical and electrical means for regulating the frequency; the frequency

could be adjusted for advancing and retarding the clocks when the ship is moving in an easterly or westerly direction respectively.

In any instruments where the recording of quantities, such as kilowatt demand, barometric pressure, steam pressure, etc., is required, the small motors are adapted without difficulty to provide the timing mechanism.

The well-known spring-wound hour meter could be superseded by one having a synchronous-motor drive, to register the total time that a circuit has been closed, or to register the number of hours that any particular machine has been in use.

In the works with which the authors are associated, these motors have been used for very many applications, and one which comes to mind is the automatic filling of d.c.-meter mercury baths. Again, in the motion study experiments a synchronous-motor timing device has been designed which, in conjunction with a cinematograph camera, times works' operations to 1/100 of a sec.

Television has already claimed the synchronous motor, and experiments utilizing the self-starting type for scanning are being conducted.

Developments are being made to use the synchronous motor as a primary factor in the timing of watch escapements by stroboscopic methods, and promise is indicated of a considerable economy in the length of time required for the observations and regulation.

ACKNOWLEDGMENTS.

The authors desire to express their thanks to Messrs. Ferranti, Ltd., for permission to publish the paper, to several members of the Ferranti organization for the assistance they have rendered, and to many manufacturers of synchronous motors for the loan of models, drawings, and lantern slides.

DISCUSSION BEFORE THE METER AND INSTRUMENT SECTION, 1ST MARCH, 1935.

Mr. F. E. J. Ockenden: Taking the paper as a whole, the authors have divided the motors considered into nine types; these, however, in my opinion might be divided into two broad classes—those which, from an engineer's point of view, are an engineering job, and those, on the other hand, which are not. The latter no doubt fulfil a useful role, but I consider the former to be of more general interest. Of the motors mentioned, Types V, VI, and IX, fulfil best the engineering conditions.

In the paper, Type VI is described as a hysteresis motor, and it is run up to speed by a set of bars rather like an ordinary induction motor; but I think the machine would be more correctly described as a variable-reactance motor; that is to say, at synchronous speed, the rotating armature poles, in passing the stator pole-faces, induce pulsations in the reactance of the motor which have the effect of so distorting the hysteresis loop of the machine taken as a whole as to include the external energy delivered at the shaft as well as the electrical energy (if any) consumed in the iron. The entire absence of true hysteresis loss, however, would not in any way interfere with the mechanical performance of the machine.

Coming now to Type V, I am not very sure about the curve shown in Fig. 5. In the first place, Type V is a true hysteresis motor, and, as Steinmetz has shown, the hysteretic torque is independent of the speed. There is therefore no reason why the starting torque should vary between zero speed and full speed. On the other hand, owing to the clearly defined poles which are part of the rotating unit, there is a locking action when the bar in the stator goes right across the main fields, and this is relied on subsequently to maintain the synchronous speed under load. It represents either an attraction or a repulsion, depending on whether the bar is approaching the main field or receding from it, and I think that a straight line at a torque of about 0·5 g-cm on which an oscillation is superimposed representing the oscillating torque due to the main bar would more accurately show the curve. I expect that Fig. 5 represents an average curve taken on the main shaft, which is running at only 1/3 000th of the speed of the high-speed shaft, and therefore such fluctuations are averaged out. I cannot agree with the authors' straight-line relation for the falling-off in speed as the maximum load is exceeded, because I know from experience that the speed falls in a series of sub-harmonics of the main speed.

I also cannot agree with the authors' assumption that the running-in torque is the weakest link in the chain and that the performance of the motor must be based on this quantity. This may possibly be the case, however, for a low-speed motor tightly coupled to its load. Both calculation and measurement lead me to believe the acceleration of the disc in the Type V motor to be of the order of 7 000 radians per sec. per sec., which means that it will run into step in the course of only $2\frac{1}{2}$ revolutions. The slack in the gearing between the high-speed shaft and the slow external shaft far exceeds $2\frac{1}{2}$ revolutions of the former, and the result is that the rotor always has a chance to run up to speed before the load comes on. We may therefore reasonably assume that the synchronous torque developed by these motors is always the running torque, in which case it would be fair to multiply the brake horse-power given in Table 5 by a factor of about 3.

The tests made by the authors on this class of motor were carried out on one having tungsten-steel discs. Motors with cobalt-steel discs have been lately introduced. The delay in introducing such discs arose not because their possible virtues were not recognized, but simply because 0·01-in. cobalt-steel sheet had not previously been obtainable. The improvement in running torque when this material is employed, however, is well marked.

Type IX, the authors' latest model, is of great interest, and obviously represents the outcome of a very great deal of work. In it they have taken advantage of the very high retentivity of the latest magnetic materials, particularly the recently-developed nickel-aluminium steel. This steel, with its particularly high coercive force, permits the induction of a number of set poles in the rotor rim, but no definite pole-locking device is provided to prevent tendency to pole drift. The authors admit that, if the maximum safe load is slightly exceeded, a very slow drift in the position of the poles on the continuous rim takes place; it is not certain that this does

not take place at all times. I feel that one or two holes or slots, rather analogous to the anti-creep holes in a meter disc, could be placed with advantage on the rim to prevent the poles from drifting past. It may be of interest to note that a form of Warren motor made in Germany incorporates such a device. The disc employed, instead of consisting of a rim with a main bar across it, is formed of a slightly cupped disc with a pair of diametrically opposite holes drilled in it, the latter acting as a lock to prevent the main pole from drifting past.

Mr. R. C. Graseby: With regard to the spring-controlled reverse-running pawls of Type II, it is rather a difficult matter to conceive that these light springs will stand up to the job, particularly at 200 r.p.m. Has any trouble due to the springs been experienced? What is the weight of the rotor of Type II? The armature appears to have no outboard bearing, and with an overhang of that magnitude there may be difficulties with the bearing. The field design also is not so good as that in Type III, because it produces end-thrust, whereas the Type III field does not: the latter gives a chance of longer life for the bearings. I notice that Type II has a frequency range of only $\pm$ 10 per cent, whereas Type III has a useful frequency range of 20 cycles per sec. and will run at any voltage between 150 and 250 volts. I should like to know what the voltage is on which Type II will start satisfactorily. The extended frequency range is a very useful feature. A large number of Type III motors have been supplied to Newcastle, where the frequency is either 40 or 50 cycles per sec., and the same motor is required to run on both frequencies. The rotor of Type II apparently has to be redesigned for different frequencies, whereas Type III only requires a slight weighting for 25 cycles per sec.: otherwise it is not altered at all.

I cannot quite agree with the authors' figures for starting torques. Many motors of Type III have to start up against the full torque of the mainspring of a 3-day storage clock, showing that the starting torque is very much higher than that given in the paper. With regard to the motor vibrating before it starts, there is no necessity for this to happen, although it may do so owing to the type of non-reversing clutch which is used.

In Table 10 the authors give no figures for periods greater than a year; I have had two stators installed on a normal service voltage (230 volts) for $2\frac{1}{4}$ years—one 15 per cent cobalt and the other 35 per cent—and neither has yet shown any appreciable change.

The dynamometer test described cannot be satisfactory at other than synchronous speed, because the type of motor dealt with in the paper is either stopped or wanting to run at synchronous speed; it must be very difficult indeed to get any reliable results from such a test.

Mr. L. J. Matthews: Among the "requirements" set out on page 380 is the statement "Synchronous torque should be high"; but the authors give no recommendation as to its actual value. I think the highest figures included in the paper are about 19 g-cm; I should be interested to know whether the authors are visualizing still higher values. I agree that one requires a considerable reserve of torque to meet increase of friction, but there are applications where a moderate torque gives a reasonable reserve. For example, in some types of 2-part tariff meters the effort required of the motor is extremely small, through quite a considerable train of reduction gearing, and there would have to be something seriously wrong with that train of gearing to use up the reserve in a moderate-torque motor. For such applications I suggest it might be more popular with the supply industry to concentrate on increasing the efficiency by reducing the watts input, which constitutes a dead loss.

Requirement (vi) says that the speed of the rotor should not be too low or too high. What speed have the authors in mind? I think that the speed should be settled in relation to the application for which the motor is intended, bearing in mind other factors such as the torque, and the amount and cost of reduction gear. If there are any other considerations which enable the authors to decide what is a suitable speed, I should like to know of them.

I agree with them that the question of bearing design is a most important one, and I think that during the next few years it will perhaps be brought home to the users of some of the earlier-type clocks and other synchronous-motor-driven devices which are now on the market. In a large number of recent tests on bearings composed of different materials, some of the best results were given by bearings the journals of which were composed of synthetic sapphire, operating in conjunction with adequate oil recesses; somewhat similar to the bearings of a meter, but of course providing the necessary length in regard to the diameter of the journals. Another point which may be mentioned is that it is almost as important to choose the right kind of oil for a small synchronous motor as for a meter, because that oil has to be relied on to maintain its qualities for a number of years.

There is one point in connection with Class A1 motors to which I should like to refer. In testing one of these which apparently was identical with that shown in Fig. 11, it was noticed on many occasions when starting up, that, instead of running into definite synchronism, the motor quickly developed very rapid hunting, slightly below and above synchronism, and continued indefinitely in that state. Can the authors suggest any reason for this effect? Is it due to the deliberate lack of symmetry between the stator and the rotor poles?

The authors do not sufficiently stress the fact that, in addition to having very small clearances, it is essential that the rotor should be most exactly balanced. A very slight lack of balance in conjunction with a clearance in the bearing as small as 0·001 in. is sufficient very often to set up a great amount of noise.

Lieut.-Col. K. Edgcumbe: A point of particular interest to me is the way in which the authors have attempted to take the Warren motor and, so to speak, turn it inside out. The Warren motor has normally two inward pointing poles; the authors have turned these poles outwards, put the rotor outside, and increased the number of poles to 16 in order to obtain a low speed.

The proper criterion for comparing a high-speed motor with a low-speed motor is the weight of similarly running wheels. One might think that the most important bearing would be that of the highest-speed wheel; but experience shows that if there is any wear in a train of wheels running from 3 000 down to 1 r.p.m. it does not occur at the high-speed, but always at the low-speed end. The reason is that

at the high-speed end the weight is so small that the film of oil is not broken, and there is practically no friction; whereas at the low-speed end there is considerable torque and a resultant grinding effect. I should like to mention the relative weights of corresponding rotating parts (all running at the same speed of approximately 200 r.p.m.) from three motors as made a year or 18 months ago. One is a high-speed (3 000 r.p.m.) motor and the other two are low-speed motors. The weight of the wheel from the high-speed motor is about 1/50th of that of either of those from the low-speed motors, while the bearing pressure—and with it the wear—in the high-speed motor is negligibly small compared with that of the other two.

The torque/weight ratio, to which the authors refer, is not the only important factor in this connection. Instrument makers found many years ago that they got very much better results with a given torque/weight ratio if they reduced the weight. The formula which the instrument maker prefers to use in working out the torque/weight ratio is based not on the weight but on the 1·5th power of the weight, and if we apply this relation we find that the high-speed motor scores all along the line.

I am a little nervous of the bearings shown in Figs. 11 and 12. Their success depends on whether enough oil can be got into them, a matter about which the authors do not say much. The authors mention jewelled bearings, but I doubt whether they are a practical proposition. It may be of interest to say that the experiment has been tried by Warren of using fibre plates for the ordinary journal bearings, and it has been extremely successful. Fibre bearings are used to a very large extent in the United States, and they wear extremely well. They have one rather unexpected disadvantage: they are distinctly more noisy than metal bearings, which tells against their use in clocks.

Mr. E. E. Sharp: I do not understand why the authors say that the Type I motor, which is of the shaded-pole type, is of the same class as Types II and III, which are both of them magnetic types without shaded poles. I feel that the description "impulse starter" which they apply to Types II and III may confuse us with what some people call the "non-self-starting" type. Seeing that all the rest of the motors come under some other heading, we might call Types II and III "synchronous motors."

The authors have made quite elaborate tests to prove what I thought we all knew, namely, that by taking a running start one is more likely to get some work done than by starting up against a dead load. They have also made some fairly elaborate experiments to show that by applying the load gradually one gives the motor a chance of getting up speed and also doing a little more work. While that involved a lot of work on their part, I hardly think it was necessary!

They mention that they can make Type II hunt for about 1 second: is this hunting period intentional? It may be due to the fact, mentioned in the Summary, that they do not profess to have obtained necessarily the best results from their tests owing to the variations due to inconsistencies of manufacture.

There is another point with regard to Type II about which I am not clear; the authors call it a "vibrating" motor, and there is a good deal in the information which they put before us which seems to bear out that contention: but they mention that it sometimes jumps straight away into full speed; is this a fault in design?

The authors mention in regard to Type III that the rebound of the cam tends to produce rotation: I think that here they have rather mistaken an accidental mechanical feature. The cam is a ball cam, and it is obvious that sometimes that ball may be wedged in a position which prevents any backward run, or at the other extreme it may get a fair run back before it comes to a stop. The rebound of the cam has nothing to do with the production of rotation.

Referring to the hysteresis type of motor, the authors mention that the permanent magnetism is formed and maintained by the stator system; I do not quite understand that. Surely one cannot get permanent magnetism with an a.c. flasher.

If Type II has been so successful, why have the authors gone on so laboriously with Type IX, which does not seem to have any advantages over Type II? It has a much smaller synchronous locking effect, and not a great deal of extra starting torque. They tell us, moreover, that the starting torque is not very important, because at the beginning the effort of the motor to start is taken up by the backlash in the gearing.

One of the figures which the authors give is the efficiency; so long as the watts input is not very high, we need not worry very much about efficiencies which are as low as they are bound to be in this class of instrument. At the spindle which has work to do—I am not thinking so much about clocks in this connection, because they present a fairly easy problem—one should make sure that there is plenty of power and that it will remain in step. The question of remaining in step is in my experience a very serious one. The lock-in on frequency in several of the motors mentioned in the paper is insufficient; variations in voltage, or an extra load, cause such motors to get out of step. A synchronous motor, while it is doing its work, should be either exactly in step or should stop altogether; there should be no halfway house. With voltage-drops such as one gets on public supplies at times of heavy load, many synchronous clocks and other instruments stop or get out of step. What is the range of voltage, and what are the starting voltages, of the motors which the authors put forward?

Mr. V. J. S. Russell: Dealing with Type III, I do not think it is generally appreciated that the output is a function not only of the input in watts but also of the permanent magnetic strength, which, as we all know, can be varied not only by the type of magnetic material —whether it is tungsten or one of the various grades of cobalt or nickel-aluminium—but also by the size and mass of the rotor. The authors give a torque of about 15 g-cm; I should like to suggest that, with an input of about 1·5 watts, a torque considerably in excess of this figure can be obtained.

Mr. S. Hunt: In the ordinary cheap watch there is a very light movement in the balance wheel, and after a little while the end stones become worn. In a ship's chronometer, on the other hand, where the mass of the balance wheel is thousands of times greater and the speed much less, the wear is negligible. We learn in the watch-making industry that anything of a light nature moving

very quickly is unstable, while anything heavy (and properly made) moving at a low speed is much more reliable. I should work on these lines if I had to decide whether to use a low-speed or a high-speed mechanism.

Mr. H. P. Bramwell: An examination of the speed/torque curves for the types of motors described by the authors shows that their performances when loaded slightly beyond synchronous torque may be divided into three classes. Class 1 includes Types V and IX, which depend on magnetic hysteresis for their operation, and which develop a slip when synchronous torque is exceeded. This slip is apparently proportional to the degree of overload, within certain limits. Class 2 includes Types VI, VII, and VIII, in which the running-in torque at low speeds is higher than the synchronous torque. An overload on a motor of this class would cause a sudden drop in speed to a point on the speed/torque curve at which the running-in torque was high enough to maintain rotation. Class 3 includes Types I, II, III, and IV, in which synchronous torque is maximum torque, and which would therefore stop if overloaded. Under conditions of overload, motors in Class 1, i.e. Types V and IX, are at a disadvantage compared with those which definitely stop, as it is better that records should be lost altogether than that they should be open to suspicion. Motors in Class 2 certainly do not stop when slightly overloaded, but the drop in speed is so marked that it should easily be detected.

In using synchronous motors, mostly of the non-self-starting type, for stroboscopic testing, I have noticed a kind of hunting, when running at synchronous speed, which shows up as a vibration on the stroboscopic figure. I am interested to know whether the authors have any information as to the relative steadiness, under these conditions, of the several types of self-starting motors which they have tested.

When experimenting on a Type IV motor I found that if the induction element is held clear from the permanent magnet the latter does not lock in one position, but develops a violent oscillation of amplitude equal to about one pole pitch. Can the authors give any reason for this?

Mr. E. S. Ritter: I should like to suggest to the authors that in their tables, e.g. Table 9, the outputs might have been given in microwatts instead of in brake horse-power.

Mr. R. P. Bossom: With regard to requirement (ix), page 380, "The rotor should be of low mass," is it not possible to differentiate between the two series of motors? In the case of those which depend on "induction" starting, a low mass is almost impossible of attainment, or at any rate it is undesirable; whereas, with the impulse type, low mass is necessary in order that the motor may jump quickly into synchronous speed. The requirement of low mass is not applicable to every type of synchronous motor.

I should like to refer to the question of whether the drive should be of the pinion or the worm type. In Type VI a worm drive is adopted, and this feature allows the rotor to develop quite a large starting torque and therefore get away with the load.

Owing to the flotation of the rotor, Type VI does not run on the bearings at all, and therefore the amount of wear is very small. On the high-speed portion the wear is negligible.

The inputs, in watts, of the various types are given in the paper; it would be desirable if the volt-ampere inputs could be given as well. I agree with a previous speaker that the output is not really related to the watts, which are largely dissipated in the form of iron losses.

Prof. J. T. MacGregor-Morris: I am engaged on certain work in which it is essential for us to know how much the frequency of the current, which is controlled by the "grid," varies from second to second. Fig. 1B suggests that the frequency will keep constant to within 0·1 cycle per sec. for long periods, and yet I am told by some that the frequency varies far more than 1 part in 500, and I should like to know what the authors have to say about this.

Secondly, I notice that the efficiencies of many of the motors described in the paper are only 0·3 to 0·5 per cent, i.e. only 1/200th of the energy supplied is delivered as useful work. Where is the rest going? Is it desirable to have this loss?

My third and last point is with regard to the method of testing the outputs of these small machines. Some years ago I was trying to obtain the horse-power of an induction motor throughout the whole of its speed range. Now if one loads such a motor with a band brake, owing to the form of the speed/torque curve one can only get a few points at very low speeds and near synchronous speed, as the motor and brake are unstable at other parts of the curve. We therefore made use of the simple device of using a fan brake, thus making the torque proportional to the square of the speed and the horse-power proportional to the cube of the speed. This enabled us to run the motor at many intermediate speeds which were otherwise unobtainable. Would the same method be useful in the testing of small motors such as those dealt with in the paper? My fan brake was simply a rectangular piece of wood clamped on to the shaft, sawn square in section, 2 in. wide, and tapered at the ends. I had a number which I could clamp on to the shaft, varying from 4 in. up to 12 in. in length. This is an extraordinarily convenient way of absorbing powers from 0·1 h.p. up to 5 h.p. The fan brake gets noisy at high speeds, of course, but the precision of measurement is high. From the dimensions of the brake, and measurements of the constants for a given radial length of the blades, one can determine the horse-power with a precision of 1 per cent. Even greater accuracy can be achieved if one takes account of the barometer. In testing small synchronous motors by this method it would be necessary to have a much smaller brake, perhaps only a matchstick, to take the place of my block of wood, and by mounting the matchstick in an enclosure which could be exhausted if necessary the load could be changed by varying the atmospheric pressure, instead of by changing the length.

Mr. E. A. Watson (*communicated*): The paper is of great value in that it for the first time gives a clear description of the various types of small synchronous motors which have been developed and which are on the market. Broadly, these may be divided into the plain single-phase type, which is inherently not self-starting but which may be made to self-start by utilizing the

oscillation principle, and the split-phase type, which is inherently self-starting and which is directional in its rotation. In the plain single-phase type the rotation is not inherently directional and some mechanical means has to be provided in order to ensure that when the motor starts it shall rotate in the correct sense. As the paper points out, the plain single-phase type suffers from the disadvantage of a relatively low running-in torque and also from the mechanical complication of whatever device is adopted to ensure rotation in the correct direction; the low running-in torque is not, however, of importance if a spring coupling is provided between the rotor and the pinion. In a large number of cases a spring coupling is necessary in order to prevent noise in the gearing, as the rotor torque of a plain single-phase motor is intermittent and liable to give rise to gear chatter if a rigid coupling is employed. A spring drive not only eliminates the possibility of gear rattle, but at the same time enables a motor of this type to start up against practically the full synchronous torque. While at first sight the necessity of adopting some mechanical means for defining the direction of rotation appears open to objection, actual experience over a considerable length of time has shown that the small unidirectional clutches employed for this purpose do not give trouble in actual service.

The second type of motor, described in the paper as a hysteresis motor (Class B1), is attractive for the reason that its direction of rotation is inherently defined and also for the reason that as the torque is produced by a 2-phase field there is much less torque variation, and the necessity for a spring drive is avoided. The construction of the motor is, however, inherently more expensive than in the simple single-phase type, and for this reason it is probably only applicable to certain classes of work. Viewed from first principles it would appear definitely uneconomic to use a rotor of permanent-magnet material, which is only magnetized by the working flux of the motor. For a given stator flux, or given stator ampere-turns, the synchronous torque which the motor can give is probably very nearly proportional to what may be termed "the effective coercive force" of the rotor material, i.e. the coercive force which would be measured if the material were magnetized to the point corresponding to the stator flux. It is well known that to obtain the full coercive force of the magnet steel it is necessary to magnetize with a magnetomotive force some 5 or 6 times the coercive force itself, and the coercive force measured on a piece of initially unmagnetized material, magnetized with a relatively small m.m.f., is very much less than the coercive force obtained if the material is magnetized initially for saturation. Further, it would seem that for very small cycles of magnetization, and in particular for cycles between m.m.f. values small compared with the loop of the material when saturated, the steel behaves in almost a true cyclical manner, with very small hysteresis and with low apparent coercive force. This appears to be borne out by the results given in the paper for the case when aluminium-nickel steel was employed in the hysteresis motor of Type IX, and it would seem that if the maximum possible power at synchronous speed is required from a motor fitted with a permanent-magnet rotor the magnet must be initially magnetized by other means than the m.m.f. given by the stator itself.

I am interested in the reference in the paper to the silver magnetic alloy with a coercive force of 2 000 C.G.S. units, and I am sorry to find that no figures for this alloy, and no data as to the results which the authors were able to obtain with it, are given. As far as I can ascertain from the published data given for this material the value of 2 000 for the coercive force is on the basis $4\pi I = 0$ and not $H = 0$, and as the remanence of the material is extremely low as compared with that of an ordinary steel alloy the coercive force on the basis $H = 0$ is no greater than, if as great as, that of the nickel-aluminium type. It would seem too that it is the value of the coercive force on the basis of $H = 0$ that is really effective in determining whether the material will give good results or not, and in view of this I should hardly have expected the authors to have obtained with this alloy results better than, if as good as, those obtained with the alloys enumerated in the paper. I shall be very interested indeed to have confirmation of this, and also to know the results which they were actually able to observe.

One other point in the paper to which I would refer is in connection with bearings for these small motors. Apparently the authors have confined themselves to the employment of metallic bearing bushes, and I should like to ask whether they have not found the non-metallic type of bush, and particularly the bakelized-fabric bushes, to have many advantages. These bushes seem to run particularly well under conditions of defective lubrication, and they have been found to stand up satisfactorily on ignition contact-breakers under conditions where metal bushes give very short service indeed.

Messrs. W. Holmes and **E. Grundy** (*in reply*): In reply to Mr. Ockenden, the speed/torque curve of the type V motor was determined by means of the special spring dynamometer described on page 398, and consequently the torque measured is the actual torque at each speed. We are fully aware of Steinmetz's publication in which he shows that the hysteretic torque is independent of the speed, but the actual torque between starting and attaining synchronous torque will not be a function merely of the hysteretic torque but a function of the algebraic sum of the hysteretic torque and that torque due to the synchronous locking effect at several sub-multiples of true synchronous speed; the latter torque may be positive or negative, depending upon whether an attraction or repulsion is taking place. The curve, as a matter of fact, represents the average torque developed on the rotor shaft, but the tests were conducted on a shaft running at 160 r.p.m. With regard to the curve for the falling-off in speed as the maximum load is exceeded, we agree that the speed will fall off in a series of sub-harmonics of the main speed. Some difficulty was experienced in obtaining the published curve, and the one given represents again the average values.

For theoretical considerations the running-in torque is the weakest link in the chain, because all self-starting synchronous time motors must pass through this condition before attaining synchronous running, and for purposes of comparison the performance of the motors must be given as taken on their high-speed shaft. We

agree with Mr. Ockenden that in the case of the type V motor the rotor attains synchronism in the course of only $2\frac{1}{2}$ revolutions, with the result that the rotor will always run up to speed before the load comes on, since the backlash in the gearing between the high-speed shaft and the slow external shaft far exceeds $2\frac{1}{2}$ revolutions of the former. At the same time the frictional torque of the rotor bearings and gear-train bearings must be considered.

From our experience with the type IX motor better results for synchronous torque are to be expected from the use of cobalt-steel discs in place of the tungsten-steel discs from which the tests on the type V motor were taken; there will be, however, a consequent diminution in starting torque.

Although advantage was not taken of a definite pole-locking device to prevent tendencies to pole drift in the type IX motor, it was carefully considered and found to have disadvantages in other directions. It is interesting to note that a form of Warren motor made in Germany incorporates such a device and has given very satisfactory results.

The description of the type VI motor given by Mr. Ockenden is correct, but we think he is mistaken when he says that it is described in the paper as operating on the hysteresis principle.

Mr. Graseby raises the question of the reliability of the spring control reverse pawl used with the type II motor. This was the subject of very careful design, and when it is considered that the maximum angular displacement of the inner coil of the spring for each revolution of the rotor is only 4° and the load on the cam due to the spring is of very microscopic dimensions, the fact that the system has proved itself entirely satisfactory in practice is apparent.

It appears that Mr. Graseby does not like overhung bearings. This type is used universally with many advantages in ships and aeroplanes, and particularly in these small motors an outstanding feature is the way they lend themselves to the provision of an oil reservoir. The majority of the motors described in the paper have overhung bearings and there is no doubt that when carefully designed they give good results.

In the type II motor the radial component of the magnetic pull is balanced round the whole circumference and there is only a very minute downward axial component due to the relative displacement in the axial plane of the rotor and stator poles. It is advisable, however, as Mr. Graseby indicates, not to fit the pinions too far away from the rotor magnet.

With regard to the frequency range, it is well known on the types II and III motors that for the maximum starting and synchronous torques the same rotor is not suitable for frequencies other than the one for which it is designed. Where the system depends upon an oscillation for its starting qualities, in the case of a rotor designed for 50 cycles it only becomes necessary to increase the weight by means of, for example, a brass disc in order to make it operate on 25 cycles, but it will be seen that its starting torque will suffer in consequence. In order to make the type II rotor suitable for frequencies other than 50 cycles per second, taking the case of 25 cycles again, the rotor weight is increased by manufacturing the rotor from thicker permanent-magnet material by means of which the most efficient system is obtained.

The dynamometer described on page 398 was not used in connection with the testing of the types II and III motors, because it is known that they will not run at any speed other than synchronous speed. The starting torque was obtained from the method shown in Fig. 13.

Regarding Mr. Graseby's statement that the type III motors in many cases have to start up against the full torque of the mainspring of the 3-day storage clock, we would say that the starting torque given is that at the rotor shaft, and that the actual torque developed at the main-spring arbor will depend on the mechanical advantage due to the gearing between the motor and the spring.

In reply to Mr. Matthews, we consider that for the majority of uses of self-starting synchronous time motors a synchronous torque of 4 or 5 g-cm is satisfactory at a motor speed of 200–400 r.p.m. In the case of some types of 2-part tariff meters, as Mr. Matthews says, the motor is in fact running on no load. The type II motor at normal voltage has a total power consumption of 1·5 watts and separating these it is found that the copper loss is 0·48 and the iron loss 1·02 watts. Generally it will be found that the iron circuit with its many leakage paths constitutes the higher source of loss and it is difficult to visualize any substantial increase of efficiency by reducing the watts input. The maximum speed of the rotor should be considered from the viewpoint of wear, and the minimum speed from the viewpoint of work done per minute, that is, with a motor having a moderate torque of say 2 g-cm the speed should be chosen to give a reasonable output. No theoretical considerations for the correct speed adopted can be given; it is a matter of experience and knowledge of the particular application of the motor.

We have not considered the use of synthetic sapphire journal bearings for use at speeds other than meter speeds. The question of using the correct oil for a small synchronous motor has occupied the attention of chemists for some time, and a mixture of colloidal graphite and fine-quality mineral oil has in some cases been adopted.

Hunting at starting on the type II motor is provided for in the design in order to increase the starting effort, and it is due to the deliberate lack of symmetry between the stator and rotor poles. This hunting takes place for about 1 sec. and then the motor jumps into synchronism. It cannot continue indefinitely in the oscillatory state unless the stator system has been damaged. The type II motor is balanced dynamically and we appreciate the remarks made by Mr. Matthews regarding bearing clearances.

The remarks made by Col. Edgcumbe regarding the high-speed motor with its oil film in the high-speed shaft are very interesting. The question of torque/weight ratio does not present quite the same problem in the case of the synchronous motor as it does in the case of the indicating instrument. The former runs at a constant speed, whilst the instrument has a continually changing indication. After further consideration we feel that as applied to synchronous motors the formula to be used in working the torque/weight ratio should be

based on the weight, not on the 1·5th power of the weight.

With regard to the bearings to be seen in Figs. 11 and 12, the latter illustrates a motor which has been used for a great many years, and so far as we know has proved satisfactory. In Fig. 11, in addition to our comparatively long experience, it has been used for many years with success in the United States. Jewel bearings were mentioned by us on page 392 in connection with the development of the type IX motor, and they were not advocated for general use.

Mr. Sharp is rather confused with the mechanics involved in the special tests we made and described on page 383 to improve the starting torque of the type II motor. It is not possible to obtain more power from a motor by giving it a running start. The only method of obtaining work from the stored energy in a rotating body is by reducing its speed, and this is not suggested in any of the descriptions given. The motors dealt with have rotors so light that the kinetic energy is extremely small. The kinetic energy of the type III motor taken at synchronous speed is 2·2 cm-g and this is equivalent to a torque of 10 g-cm maintained for 0·011 sec. or approximately half a cycle. The effect, therefore, of any use of this stored energy is infinitesimal, and in the test figures given no advantage is claimed for it.

Reference to page 382 will show that the type II motor is made to hunt intentionally for about 1 sec. at the instant of switching on, and the reason for this is to give the motor a running-in torque of nearly half synchronous torque against load. Both type II and III motors depend on oscillation for their starting qualities, and whether they start immediately or oscillate slightly depends at what part of the current wave the supply is switched on.

The statement in regard to type III that the rebound of the cam tends to produce rotation was given to complete the description of the starting of this motor, and we agree that it may be an accidental feature; it was obtained from the published information of the manufacturer of the motor in question.

Referring to the hysteresis type of motor it is realized that a high degree of magnetic intensity could not be obtained with an a.c. flasher, but in any steel there will remain remanent magnetism after it has been subjected to an a.c. field. In the case of the hysteresis motor type IX the rotor does become magnetized in this way, and its induced poles are maintained at the correct polarity due to the fact that when the rotor revolves at synchronous speed the polarity of the induced poles on the rotor come round in phase with the exciting poles on the stator in turn.

The efficiences are given for comparative purposes. In the tables given in the paper the actual figures of torques are given for a voltage range $\pm$ 20 per cent, which will cover any possible fluctuations in supply voltage due to heavy loads.

The point Mr. Russell raises is not quite clear, because from Table 3 with an input of 1·75 watts the torque of the type III motor is given as 14·4 g-cm. The rotor is made from cobalt steel and there is no doubt that greater torque could be obtained with a nickel-aluminium rotor or with a cobalt-steel rotor of larger diameter.

Mr. Hunt's remarks of his experience of wear in bearings are very interesting and we agree generally that good workmanship and long life of the bearings are closely allied.

We are in entire agreement with the early remarks of Mr. Bramwell with reference to the classification of the types of motors, and the advantages and disadvantages in each case. It should be remembered that generally these motors are only loaded to a small percentage of their capacity and there is very little possibility that slipping or stalling would ever occur in use. With regard to a slight hunting taking place when running at synchronous speed, it is felt that this will occur on all types and, although we have not actually tabulated the results as to the varying amounts in each case, it varies insufficiently to warrant a discrimination being made. When the induction element is taken away from the type IV motor, the permanent-magnet part acts in a similar way to the types II and III, and it will, therefore, oscillate violently as the a.c. wave goes through its cycle. Due to the fact that this magnet has no asymmetry the amplitude of the vibration will not be greater than a pole pitch and consequently the rotor will not revolve.

In reply to Mr. Ritter, the output of the motors might have been given in microwatts instead of in brake horsepower, but it is a fairly simple calculation to obtain this from the efficiency figures given and it was felt that to many engineers the output in mechanical units would be more interesting.

Replying to Mr. Bossom, the requirement that the rotor should be of low mass was inserted from the angle not of design but rather of bearing wear, and was given for the user's interest.

Prof. MacGregor-Morris refers to Fig. 1B which does not show any rapid momentary fluctuations of frequency; it is a reproduction on a small scale of an actual chart from a recording frequency meter taken by a large supply company; we have not studied the variation from second to second.

The principal loss in these motors was in the iron circuit and, due to the closely-interconnected poles, leakage plays a very important part and it appears from mechanical considerations and also from the point of view of overall dimensions that this loss is not easily overcome.

The remarks with regard to the obtaining of the horsepower of an induction motor throughout the whole of its speed range by means of a fan brake were very interesting. We realized that it was impossible to obtain a speed/torque curve on these small motors with a hand brake, and consequently the special spring dynamometer described on page 398 was evolved. At the same time it is doubtful whether with the spring dynamometer, or indeed with the fan-brake method, it is possible to obtain a horse-power figure with a precision of 1 per cent for these very small motors.

The remarks of Mr. Watson concerning the type II motor and the spring coupling device described on page 383 are very interesting indeed and it is gratifying to know that they are in entire agreement with those expressed in the paper. We feel that the hysteresis motor, type IX, is attractive for the reason that its direction

of rotation is inherently defined and also for the reason that the torque is more uniform. The construction will perhaps be a little more expensive than that of type II, although an item will be saved by the fact that it is only necessary to provide a thin rim of magnetic material instead of the expensive permanent-magnet rotor. The experiments proved that the synchronous torque which the motor could give was very nearly proportional to the effective coercive force, that is, the coercive force which would be measured if the material were magnetized to the point corresponding to the stator flux.

We agree that in order to obtain the maximum possible power at synchronous speed from a motor fitted with a permanent-magnet rotor, the magnet must be initially magnetized by other means than the m.m.f. given by the stator itself. In the case of the type IX motor, where experiments were tried of magnetizing the rotor to a high degree of intensity by existing means before fitting in the stator, a very pronounced increase in the synchronous torque was obtained, but the constants of the magnet poles induced were insufficient to maintain this higher magnetic intensity. In any event when the rotor is polarized by external means prior to being fitted in its stator, locking effect at starting takes place and means have to be provided to enable the motor to start up against it.

We must confess that a further investigation of the silver magnetic alloy with a coercive force of 2 000 c.g.s. units has proved this figure to be incorrect. As Mr. Watson states, the value of 2 000 for the coercive force is on the basis of $4\pi I = 0$ and not $H = 0$. There is no mention in the paper of any results being obtained with the silver magnetic alloy, and as a matter of fact our laboratories were very disappointed with the tests they conducted on the alloy.

With reference to the bearings, we have been rather nervous of utilizing the non-metallic bushes, due in the main to an apprehensive feeling that they might " move " with temperature and barometric variations. It is very interesting to notice that they have been used extensively on ignition contact breakers and have given much better service than have the metal bushes.

VIBRATION IN OVERHEAD CONDUCTORS.*

By Theodore Varney, B.Sc., Associate Member.

(*Paper first received* 26*th September*, 1934, *and in final form* 12*th January*, 1935.)

Summary.

The paper describes laboratory tests which give the static stresses in the various strands of conductors when subjected to certain tensions. These results are compared with those obtained from long-time vibration tests upon similar conductors while subjected to corresponding total tensions.

It is concluded that unless the sharp bending at the support is relieved, vibration will cause fracture in a relatively short time. Decreasing the lay or the tension within economic limits will not alone insure against breakage.

While the general principles discussed may be applied to any metallic conductor, the tests described were made upon steel-reinforced aluminium (S.C.A.) cable.

Reference is made to available devices for the suppression of vibration; these obviously prevent breakage by removing the cause. The paper offers a diagnosis of the vibration trouble, and describes available remedies which experience has proved to be thoroughly efficient and reliable.

If a piece of wire be stressed to a point where it receives a permanent elongation and if the load be gradually reduced to zero, the contraction curve will be substantially a straight line parallel to the tangent to the original curve at the origin. If the sample be again stressed, its elongation curve will coincide with its contraction curve until the original stress is reached. If the stress be increased beyond this point and again reduced to zero, a third straight line will result parallel to the first two. This process can be carried on practically to the ultimate strength of the sample.

If tension be applied to a stranded conductor, its complete modulus increases as the strands slip, but after they "bed down" it remains constant until they take permanent sets, after which it decreases. Upon contraction, the stress will eventually leave the aluminium portion, and some slipping between strands may occur. Subsequently the contraction curve follows the steel modulus.

As long as the stress in any strand is uniform throughout the sample, no slipping between strands occurs for that portion of the stress-strain curve which follows the modulus of the complete conductor. If, however, any local increase of stress occurs at any point in a strand, as by reason of a nick or injury, or by some condition which produces a localized increase in the tension in the strand, it will elongate locally and, if the length of the increased stressed portion is short enough, a typical square "fatigue" fracture will occur. The characteristic appearance of such a fracture has sometimes led to the erroneous expression of "crystallization."

* The Papers Committee invite written communications, for consideration with a view to publication, on papers published in the *Journal* without being read at a meeting. Communications (except those from abroad) should reach the Secretary of the Institution not later than one month after publication of the paper to which they relate.

Fig. 1 represents the sine-wave loop in a vibrating conductor. In the span away from a support there is no bending at a node. The maximum bending occurs at the middle of such a loop, and its radius is given by the expression

$$R = \frac{2l^2}{\pi^2 a}$$

There is no record of any vibration breakage in S.C.A. conductors at such places.

At a support, however, the conditions are different and the conductor is forced to bend an amount somewhere between zero and the angle ϕ above and below the neutral position.

If adjacent spans were vibrating in exact synchronism, a perfectly balanced clamp would noticeably reduce the

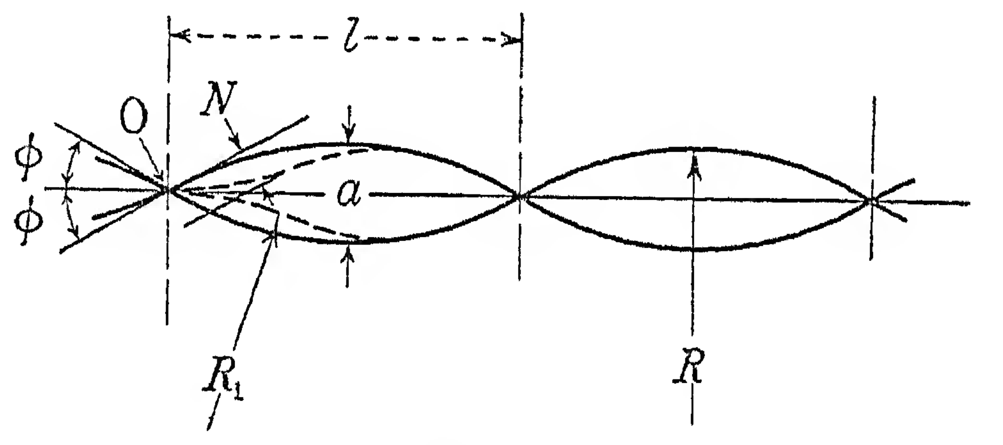


Fig. 1.

bending, but this condition seldom prevails for any length of time.

The angle ϕ is given by the expression

$$\tan \phi = \frac{\pi a}{2l}$$

and if the clamp were rigid and had square jaws, the radius of the bend at the clamp would be given by the expression

$$R_1 = \frac{\mathbf{I}M}{K}$$

where $\mathbf{I}$ is the effective moment of inertia of the conductor section, M the effective modulus of elasticity of the conductor, and K the bending moment applied to the conductor at the edge of the clamp. If the strands freely slipped relatively to each other, $\mathbf{I}$ would be the sum of the individual moments of inertia of the strands each about its own centre of gravity. If the strands did not slip at all, $\mathbf{I}$ would be the moment of the whole conductor about its axis. Actually, $\mathbf{I}$ has some value between these extremes and quite indeterminate by calculation. If the reaction of the clamp which produces the bend in the conductor were removed, the force P represented by the tension in the conductor would pass

through O. Therefore the bending moment K produced on the conductor at the clamp is the product of P and the offset of the bend, N.

Since it is known that R is a safe value, it may be assumed that if **I** can be sufficiently increased by some means, R_1 may be made equal to R and then the value of N, and hence of K, could be approximately calculated.

The problem is, however, so incapable of exact calculation that cut-and-try methods were resorted to. First a bundle of straight aluminium rods was clamped about the cable and twisted around it, being held at the outer ends by small clamps. With conductors of large size the "scuffing" effect at the ends of the rods was found objectionable, and the expedient was hit upon of tapering the rods and twisting them with the lay of the cable.

Since the measure of the tendency to produce fracture is the sum of the static stress in each strand and the fluctuating bending stress, it is apparent that the

TABLE 1.

Temperature	Total tension	Stress (lb. per sq. in.)					Remarks
		Outer Al layer	Middle Al layer	Inner Al layer	Average Al portion	Steel portion	
	lb.						
60° F.	3 500	3 845	2 285	2 800	2 975	19 340	First loading
	4 500	4 890	3 300	3 815	4 000	23 730	
	5 000	5 480	3 805	4 325	4 535	25 710	
	7 000	7 950	5 665	6 360	6 660	33 620	
	9 000	10 290	7 695	8 395	8 795	41 530	
	10 000	11 465	8 625	9 415	9 835	45 705	
	14 000	15 135	12 855	13 230	13 755	65 045	
	15 000	16 055	13 870	13 995	14 640	70 320	
	18 000	17 795	16 745	16 030	16 890	90 535	
	19 250	18 215	16 915	16 285	17 320	103 720	
	18 000	17 795	16 745	16 030	16 890	90 535	After loading to 18 000 lb.
	15 000	14 090	13 615	13 230	13 645	79 170	
	14 000	12 850	12 515	12 215	12 525	75 590	
	10 000	7 785	8 540	8 140	8 155	60 870	
	9 000	6 740	7 275	7 380	7 130	56 695	
	7 000	4 270	5 330	5 090	4 895	49 445	
	5 000	1 675	3 385	3 180	2 745	41 970	
	4 500	1 275	2 705	2 800	2 260	39 555	
	3 500	0	1 655	1 780	1 145	36 040	
0° F.	3 500	4 670	3 045	3 435	3 715	13 405	First loading
	4 500	5 845	3 890	4 580	4 770	17 580	
	5 000	6 305	4 480	5 090	5 290	19 775	
	7 000	8 580	6 340	7 250	7 390	28 125	
	9 000	10 865	8 370	9 540	9 590	35 820	
	10 000	11 845	9 470	10 430	10 580	40 435	
	14 000	15 770	13 530	13 995	14 430	59 770	
	15 000	16 625	14 290	15 015	15 310	65 485	
	18 000	18 175	16 575	17 050	17 265	87 900	

DETAILS OF CONDUCTORS.

Conductor	Stranding	Area of steel	Areas of Al layers			Total area of conductor	Lay ratios of layers				Diameter of conductor (d)	Weight per foot of conductor (w)	Ultimate strength of conductor
			Inner	Middle	Outer		Steel	Inner Al	Middle Al	Outer Al			
795 000 circ. mils S.C.A.	54/7 × 0·1214 in.	0·0806 sq. in.	0·1392 sq. in.	0·2094 sq. in.	0·2791 sq. in.	0·7083 sq. in.	37·5	18·0	15·3	11·1	1·093 in.	1·02 lb.	28 500 lb.

TABLE 2.

Weather loading			1 200-foot span		Remarks
Temperature	Ice (radial)	Wind	Sag	Tension	
	in.	lb/sq. ft.	ft.	lb.	
0° F.	1	8	53·8	13 825	First loading
0° F.	$\frac{1}{2}$	8	49·4	9 000	
32° F.	$\frac{1}{2}$	0	49·4	7 270	
− 20° F.	0	0	43·4	4 300	
0° F.	0	0	44·5	4 230	
32° F.	0	0	46·3	4 060	
60° F.	0	0	47·9	3 950	
120° F.	0	0	50·8	3 670	
0° F.	$\frac{1}{2}$	8	49·4	9 000	After loading to 9 000 lb.
32° F.	$\frac{1}{2}$	0	50·0	7 200	
− 20° F.	0	0	44·5	4 200	
0° F.	0	0	45·6	4 090	
32° F.	0	0	47·4	3 950	
60° F.	0	0	49·1	3 810	
120° F.	0	0	52·0	3 600	

DETAILS OF CONDUCTORS.

Conductor	Stranding	Area of steel	Areas of Al layers			Total area of conductor	Lay ratios of layers				Diameter of conductor (d)	Weight per foot of conductor (w)	Ultimate strength of conductor
			Inner	Middle	Outer		Steel	Inner Al	Middle Al	Outer Al			
795 000 circ. mils S.C.A.	54/7 × 0·1214 in.	0·0806 sq. in.	0·1392 sq. in.	0·2094 sq. in.	0·2791 sq. in.	0·7083 sq. in.	37·5	18·0	15·3	11·1	1·093 in.	1·02 lb.	28 500 lb.

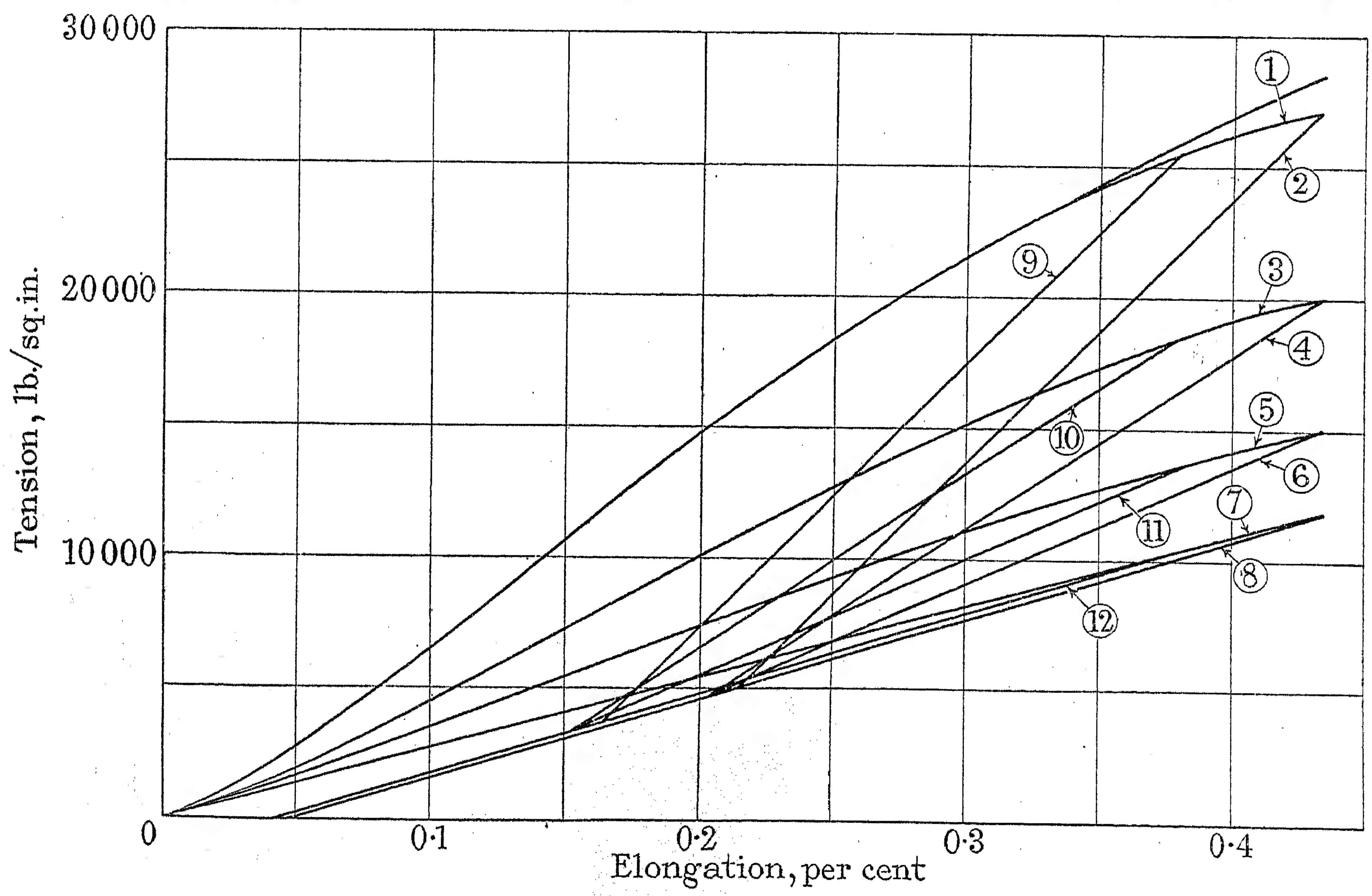


FIG. 2.

endurance limit of the conductor is some combined function of the tension P and the angle ϕ.

A series of static tests were first made upon a sample of 795 000 circular mils S.C.A. conductor having the characteristics given beneath Tables 1, 2, and 3. The increments of load were slowly applied, and the maximum in each case was held for 1 hour. Tests of this character have been described by G. W. Stickley.*

Curves 9, 10, 11, and 12, are drawn parallel with curves 2, 4, 6, and 8, respectively, at the elongation corresponding to a load of 18 000 lb. on curve 1.

It is obvious from Fig. 2 that if a new piece of conductor be installed with sags and tensions computed from the final modulus—as given by curve 9—the sags after the conductor has experienced its maximum load will be greater than those at which it has been installed.

TABLE 3.

P	P as percentage of ultimate	l	a	ϕ	f	v	Square clamp	Bell-mouth clamp	Armour rods		Remarks
							Cycles	Cycles	Cycles	Hours	
5 000	17·5	17	$1\frac{1}{8}$	0·495°	11·7	3·9	75 000 000	75 200 000	500 000 000	11 800	First loading. Load held 1 hour before vibration was begun
7 000	24·5				14·0	4·7	34 000 000	16 211 000	500 000 000	10 000	
10 000	35·0				16·4	5·5	5 580 000	6 242 000	119 000 000	2 030	
15 000	52·5				20·3	6·8	1 203 000	2 078 000	40 538 000	550	
18 000	63·0				22·3	7·5	433 000	1 148 000	24 000 000	295	
5 000	17·5	8·5	$\frac{9}{16}$	0·515°	23·4	7·8	—	91 580 000	534 000 000	6 400	
7 000	24·5				28·0	9·4	13 145 000	11 670 000	84 000 000	840	
10 000	35·0				32·8	11·0	6 585 000	4 616 000	76 000 000	625	
15 000	52·5				40·6	13·6	—	2 697 000	—	—	
10 000	35·0	8·5	$\frac{3}{8}$	0·330°	32·8	11·0	—	228 000 000	—	—	
15 000	52·5				40·6	13·6	—	52 000 000	500 000 000	3 420	
18 000	63·0				44·6	15·0	—	—	122 000 000	760	
15 000	52·5	8·5	$\frac{3}{16}$	0·165°	40·6	13·6	—	500 000 000	—	—	
18 000	63·0				44·6	15·0	—	500 000 000	—	—	
7 000	24·5	17	$1\frac{1}{8}$	0·495°	14·0	4·7	—	33 825 000	—	—	After loading to 18 000 lb.

DETAILS OF CONDUCTORS.

Conductor	Stranding	Area of steel	Areas of Al layers			Total area of conductor	Lay ratios of layers				Diameter of conductor (d)	Weight per foot of conductor (w)	Ultimate strength of conductor
			Inner	Middle	Outer		Steel	Inner Al	Middle Al	Outer Al			
795 000 circ. mils S.C.A.	54/7 × 0·1214 in.	0·0806 sq. in.	0·1392 sq. in.	0·2094 sq. in.	0·2791 sq. in.	0·7083 sq. in.	37·5	18·0	15·3	11·1	1·093 in.	1·02 lb.	28 500 lb.

First, a complete sample of cable was tested up to a maximum load of 19 250 lb. with an elongation of 0·434 per cent. The load was gradually reduced to zero. These results are shown respectively as curves 1 and 2 of Fig. 2. Next, a test was made with the outer aluminium layer removed, with results—shown in curves 3 and 4—out to the same final elongation. The third test was made with the two outer aluminium layers removed—as shown in curves 5 and 6—and finally the steel core was tested—as shown in curves 7 and 8.

* See Reference (1).

If, however, the conductor be installed with the smaller sags as calculated by the initial modulus from curve 1, the final sags after maximum load in service will be the correct ones as determined by the final modulus of curve 9.

Fig. 4 gives sags and tensions used in the construction of an important 220 000-volt line in Canada. The design basis is a ruling span of 1 200 ft.; a loading of 9 000 lb. with $\frac{1}{2}$ in. ice and 8 lb. per sq. ft. wind pressure on ice-coated conductor at 0° F., but with a check tension curve with 1 in. of ice on the conductor with

8 lb. per sq. ft. wind at 0° F. Under the latter condition the tension in the ruling span is 13 825 lb.

The curves of Fig. 4 were made with the final modulus after loading to 9 000 lb.

The maximum extraordinary load of 13 825 lb. is about 48 per cent of the ultimate strength of the conductor, whereas the tension without ice, which represents the normal condition and that under which vibration occurs, is not over 16 per cent of the ultimate tension. The point this paper seeks to show is that, under the conditions described, destructive vibration can, and has been known to, occur unless some means are employed to prevent it, and by the application of available means even tighter stringing is safe.

A point which has troubled many is the effect of temperature on a bimetallic conductor. The coefficient of expansion of aluminium is 0·0000128 per deg. F., and that of steel is 0·0000064. Suppose the conductor to be stranded at 60° F., and then suppose a length of the cable to be subjected to stress at 0° F. The effect of this change can be graphically determined by sliding the steel and aluminium portions of Fig. 2 to the left through distances corresponding respectively to their coefficient of expansion multiplied by the temperature-drop; or sliding them to the right for a temperature-rise. Fig. 3 (0° F.) was obtained from Fig. 2 (60° F.) by sliding the aluminium portion of curves 1 to 6 inclusive to the left a distance of 60 × 0·00128 per cent elongation, and the steel portion a distance of 60 × 0·00064 per cent. The stresses are shown in Table 1.

A reliable graphic method of calculating sags and tensions for S.C.A. conductors subjected to varying temperatures, ice, and wind loads, has been in use in America and elsewhere for the past 7 years.* It is based upon the use of charts similar to Figs. 2 and 3 made on transparent paper. These are superimposed upon another drawn to the same scale upon which the percentage changes of catenary arc length are plotted as abscissæ, and changes of sag and tension as ordinates for any particular span length.

By its use it has been possible to calculate the unstressed length of large S.C.A. cable for a number of very long river crossings. During successive summers and winters since installation, the sags have been found to agree closely with the calculations. One of the spans in question is the Kanawaki crossing over the St. Lawrence River of the Montreal Light, Heat, and Power Consolidated. Among others are the following: Boston Edison Illuminating Co., Fore River crossing, Massachusetts; Baton Rouge Electric Co., Mississippi River crossing at Baton Rouge, Mississippi; Arkansas Power and Light Co., Mississippi River crossings at Memphis, Tennessee, and at Greenville, Mississippi; New York Power and Light Co., Hudson River crossing.

The aerodynamic effect of wind on a cylindrical body has been investigated by Messrs. Relf and Ower.†

By means of clock-driven recorders attached to the conductors, continuous night and day records have been obtained which show that many lines that had never been seen to vibrate do so for a considerable proportion of the time, especially at night. On account of wind fluctuation, the vibration practically always contains "beats." When these are present the amplitudes are

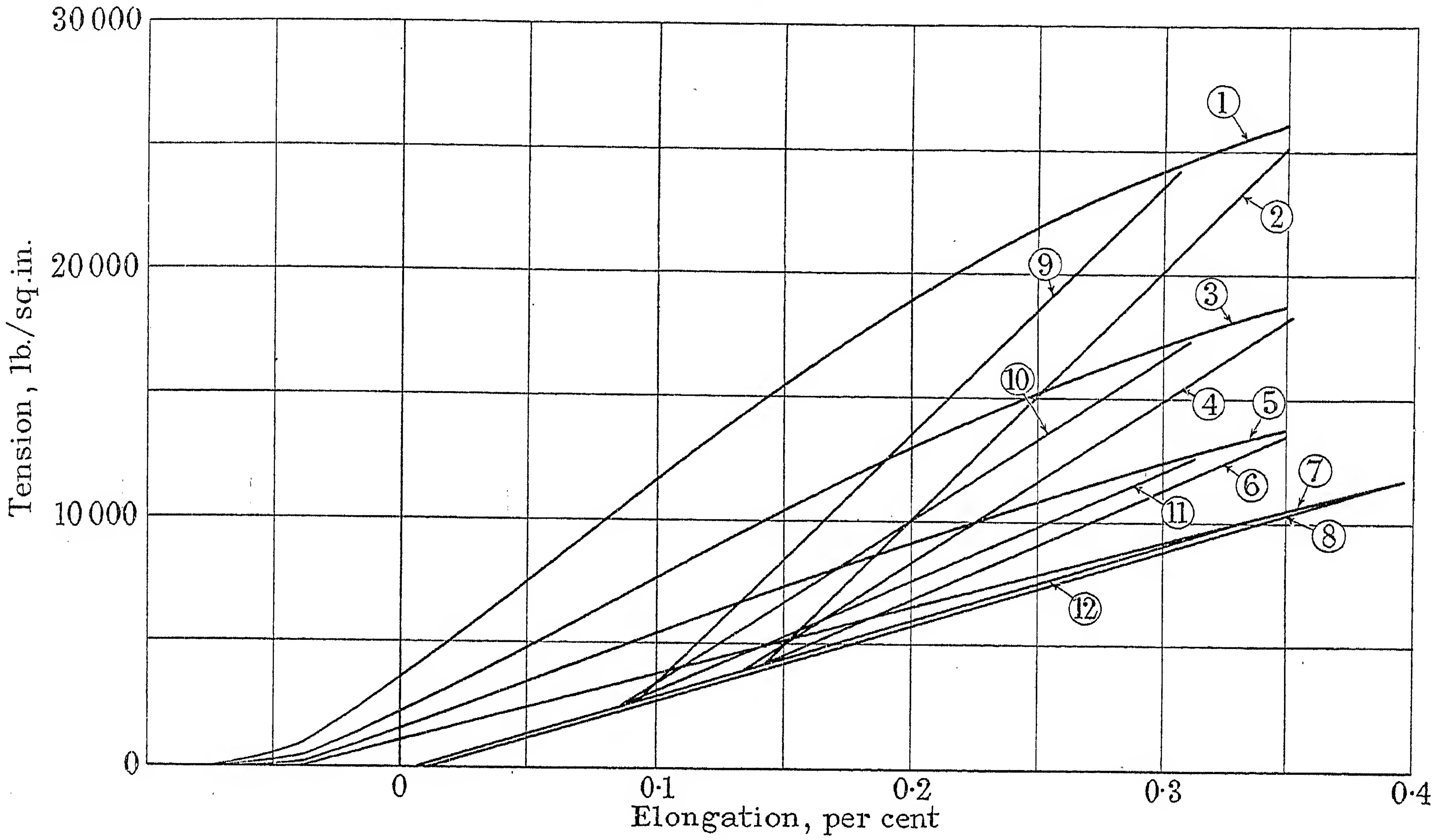


Fig. 3.

* See Reference (6).

† *Ibid.*, (2).

greater. A variety of typical charts have been shown by the author* and by Messrs. Monroe and Templin.†

In order to obtain some definite connection between the endurance limit of a conductor and the combined static and bending stresses produced by vibration, a long series of experiments was begun about 7 years ago. Heavy concrete blocks were built within a building where the temperature could be kept uniform. They were arranged to take a 120-ft. sample of conductor, and means were provided by levers and weights to maintain tensions up to about 20 000 lb. Means were also provided to maintain constant resonant vibration in the conductor for long periods of time by means of small electric motors. The conductor at one end of the span was held by a light pivoted clamp and at the other it passed through a test clamp which held it snugly to prevent any lateral motion, but was anchored behind this clamp, of which three forms were used (see Figs. 5–8 inclusive).

It was at first thought that the snapping of a strand would be noticeable by the dropping of the scale beam. This was found not to be true, and it was difficult to detect broken strands by any observable increase in the length of the conductor. The clamps and armour rods were opened carefully at intervals during the tests, and the conductor was examined for broken strands. While it was difficult to determine the exact time of breakage of strands in inside layers, yet after many tests under varying conditions the appearance of the first break in

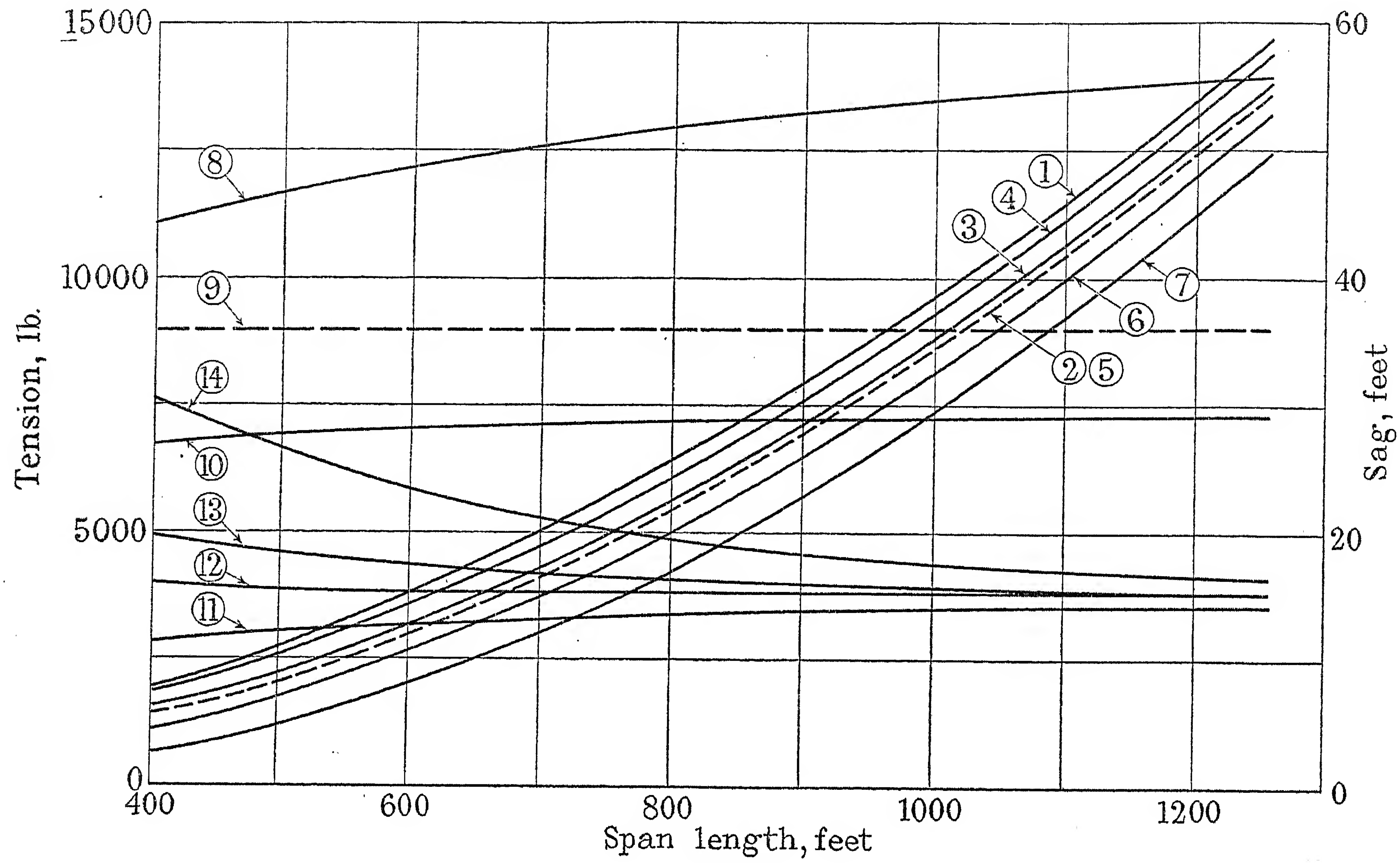


FIG. 4.

Conductor: 795 000 circ. mils; $\frac{54 \times 0{\cdot}1214 \text{ in. alum.}}{7 \times 0{\cdot}1214 \text{ in. steel}}$ S.C.A.

Maximum loading conditions:—
Tension .. 9 000 lb.
Load .. ½ in. ice plus 8 lb. wind at 0° F.

Curve Number		Ice	Wind	Temperature
		in.	lb./sq. ft.	°F.
1	Resultant sag with	1	8	0
2	Resultant sag with	½	8	0
3	Vertical sag with	½	0	32
4	Vertical sag with	0	0	120
5	Vertical sag with	0	0	60
6	Vertical sag with	0	0	32
7	Vertical sag with	0	0	−20
8	Tension with	1	8	0
9	Tension with	½	8	0
10	Tension with	½	0	32
11	Tension with	0	0	120
12	Tension with	0	0	60
13	Tension with	0	0	32
14	Tension with	0	0	−20

* See References (3) and (4). † *Ibid.*, (5).

the outside layer served as a very good means of comparison.

The Relf and Ower formula may be written

$$f = \frac{0 \cdot 271 v_w}{d}$$

where f is the resonant vibration frequency, v_w is the wind velocity in miles per hour, and d is the diameter of the conductor in feet. When the tension in the conductor is constant, i.e. when the amplitude of the loop is comparatively small, the velocity v of the transverse wave is given by the expression

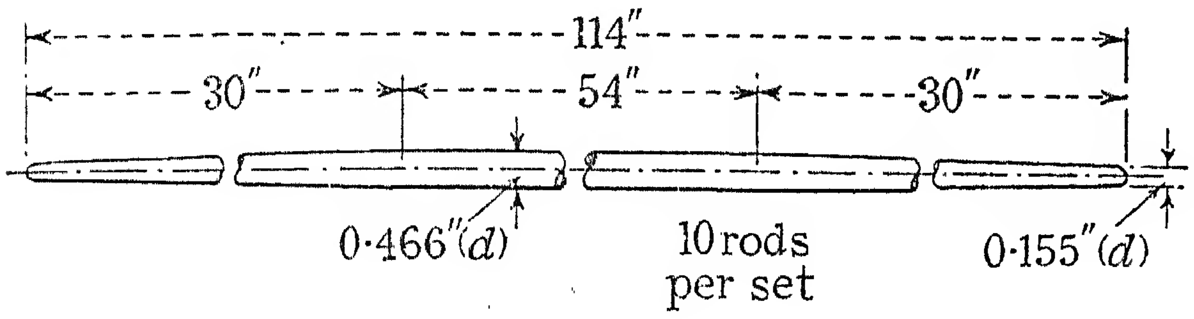


Fig. 5.

$$v = \sqrt{\left(\frac{Pg}{w}\right)}$$

where P is the total tension in pounds, g is the acceleration due to gravity, and w is the weight per foot of the conductor. Also

$$v = 2lf$$

Table 3 gives a condensed compilation of a long series of tests covering several years.

By comparing Table 3 with Table 1 for corresponding tensions, the endurance limit of this particular conductor is quite definitely determined.

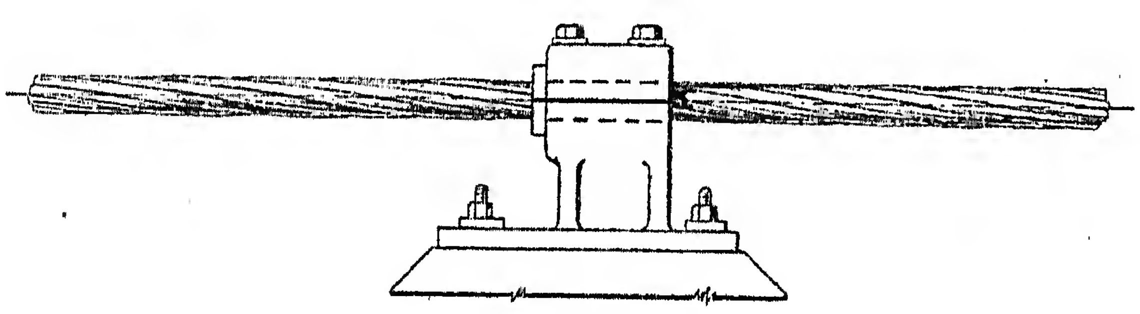


Fig. 6.

Tables 4 and 5 give comparative data for 397 500 circular mils 30/7 strand S.C.A., and 3/0, 7-strand S.C.A. cable.

Pre-stretching before shipment has been found impracticable because the process of reeling and unreeling again in the field displaces the lay of the strands so that the effect is lost. Pre-stretching in the field, especially with large conductors, is rather expensive and hazardous to both the conductor and the supports.

Mr. G. H. Stockbridge of California directed his efforts toward a device which went to the root of the problem by damping the conductor vibration. His work, supplemented in later years by that of Messrs. M. E. Noyes, R. A. Monroe, and R. L. Templin,* has resulted in a very simple and efficient device known as the Stockbridge damper. It is shown in its latest form in Fig. 9. Fig. 10 illustrates a pair of simultaneous continuous clock-driven charts showing the vibrations in 795 000 circular mils S.C.A. cable with and without dampers. The accompanying wind velocity and direction record shows the wide range of effectiveness of these dampers.

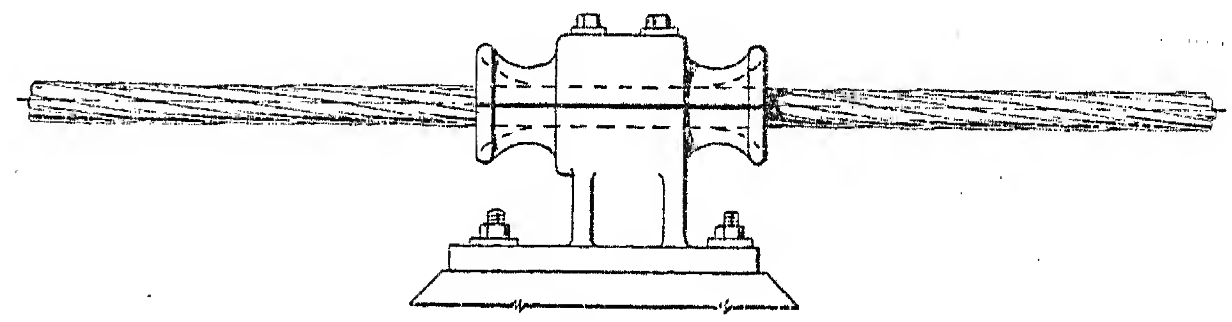


Fig. 7.

Bate† has given an expression which is generally accepted for the maximum vertical oscillatory force in pounds per linear foot of conductor, as follows:—

$$F = 0 \cdot 0025 v_w^2 d$$

where v_w represents wind velocity in miles per hour, and d is the diameter in feet. The author has seen no experimental verification of the accuracy of the numerical constant of this formula, and some wind-tunnel tests would be of great interest.

Apparently the formula is dimensionally correct, and, if it be assumed that the wind blows uniformly over a long span, the eddy frequency of the wind would often be in resonance with one of the many natural periods of the conductor.

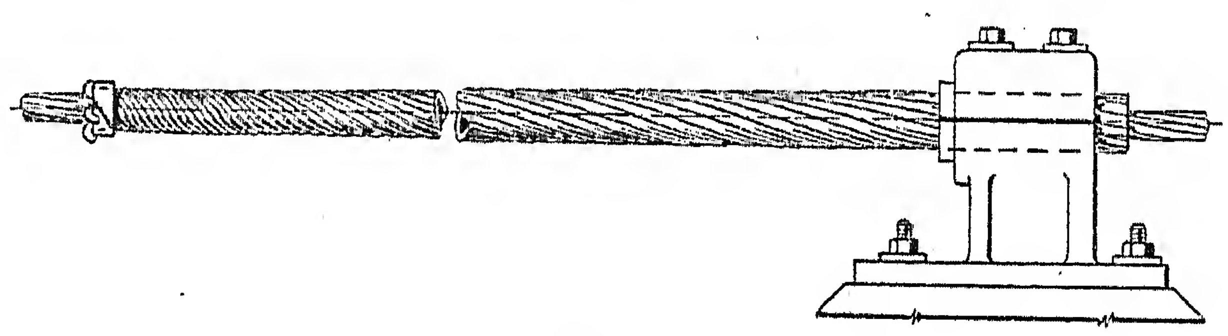


Fig. 8.

The oscillatory forces on all the longitudinal elements of the conductor would not, at first, be in synchronism, but after a time these forces get into step and the conductor vibrates in the form of a standing wave composed of stationary nodes and loops. Such a wave is the resultant of two travelling waves moving in opposite directions. Thus, at the middle of one loop the maximum instantaneous unit force is upward and corresponds

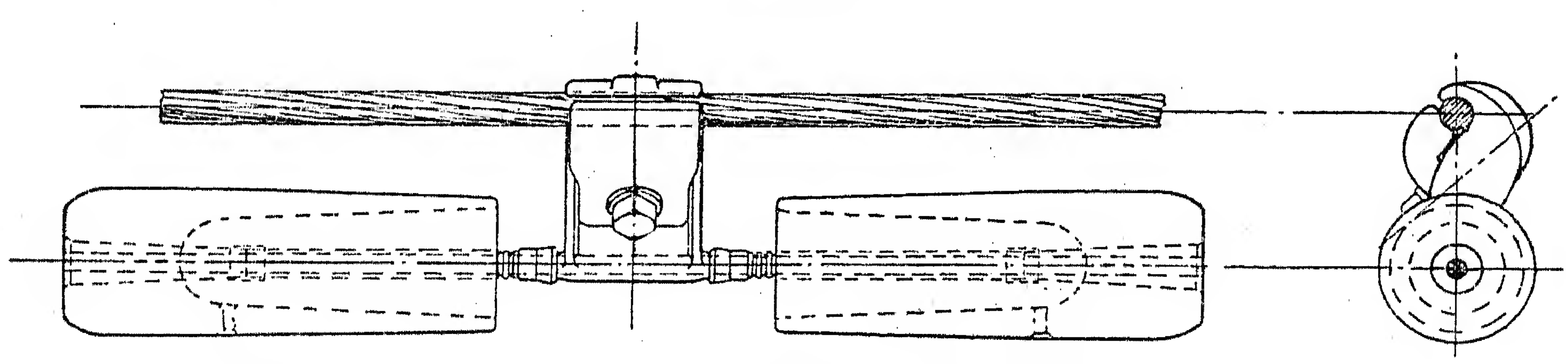


Fig. 9.

* See Reference (5).

† *Ibid.*, (7).

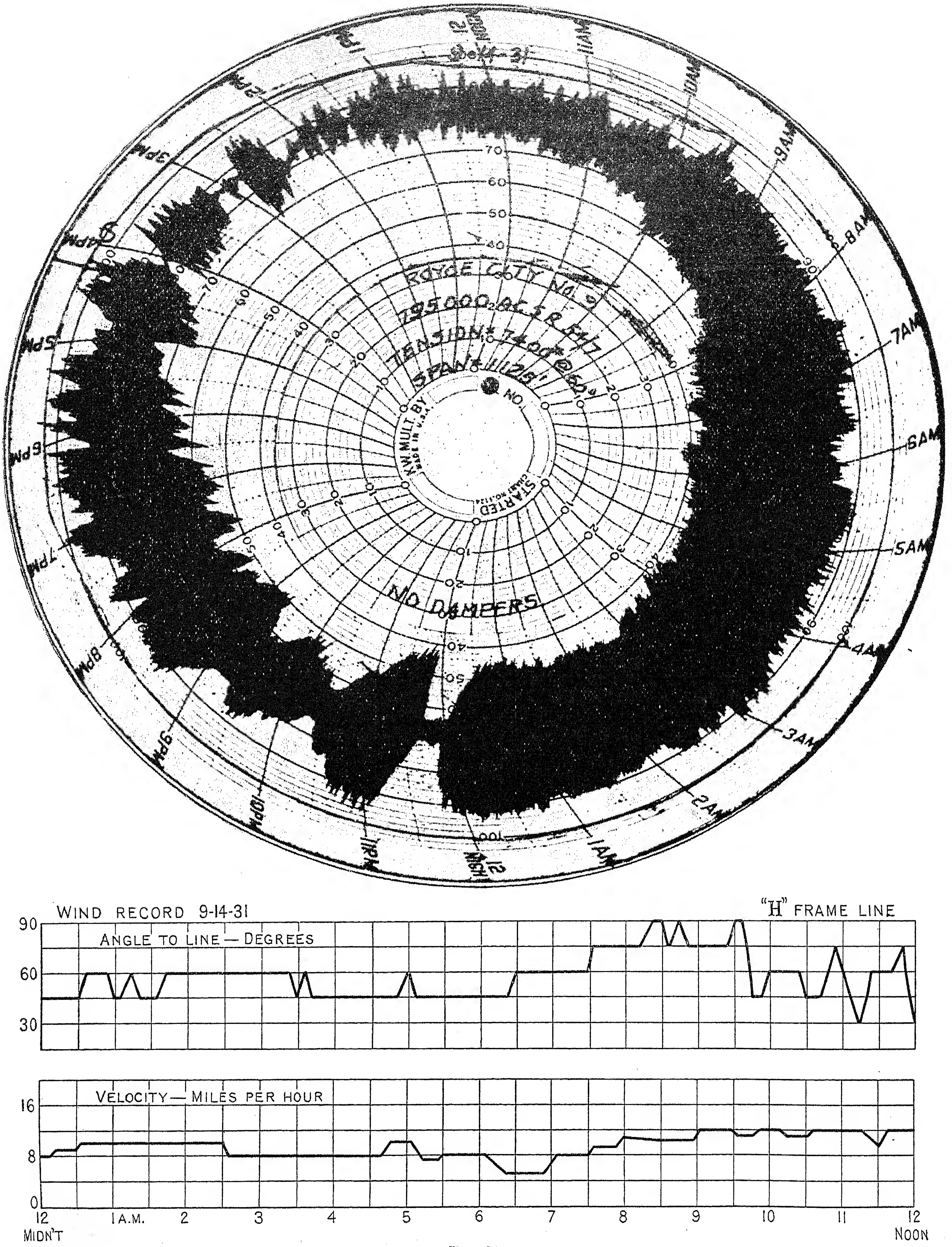

NO DAMPERS
STARTED
K.W. MULT. BY
WIND RECORD 9-14-31
"H" FRAME LINE
ANGLE TO LINE — DEGREES
VELOCITY — MILES PER HOUR
12 MIDN'T
1 A.M.
12 NOON

FIG. 10.

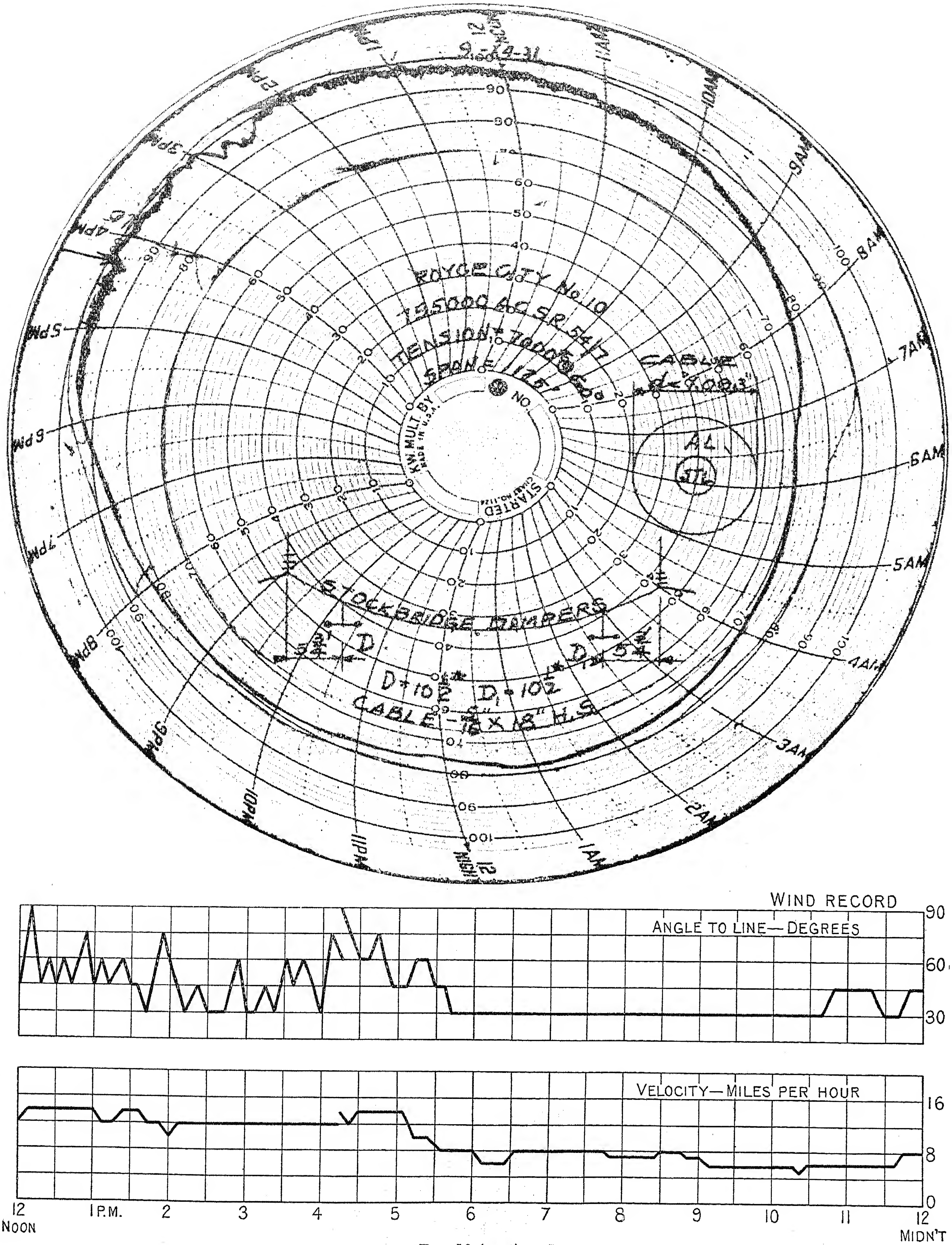


FIG. 10 (*continued*).

to the value F given above. At the same instant the force at the middle of the adjacent loop has an equal value downward. At the node the oscillatory resultant force is zero.

Robertson* has shown that the energy per cycle imparted by a harmonic force F to a mass m oscillating with a maximum displacement X is given by the expression

$$\pi F X \sin \beta$$

where β is the angle between the force and displacement vectors.

Referring to the case of a vibrating conductor loop and applying the above expression to the harmonic motion of a differential section of a conductor, the energy may be written

$$\pi F_w dx a' \sin \frac{\pi x}{l} \sin \beta$$

where a' ($= \frac{1}{2}a$) is the half amplitude of the loop at its centre. Integrated over the entire loop this expression reduces to

$$2F_w a' l \sin \beta$$

At resonance, β is 90° and the expression becomes

$$2F_w a' l$$

* See Reference (8).

TABLE 4.

Test temperature	Total tension	Stress (lb. per sq. in.)		Remarks
		Average Al portion	Steel portion	
	lb.			
74° F.	4 000 5 000 7 000	4 900 6 200 9 000	35 000 42 500 57 500	First loading; 397 500 circ. mils; 30/7; lay ratio: 29·0, 20·0, 13·4
70° F.	2 220 3 330 4 000 4 440	9 800 13 900 15 700 16 700	42 000 70 000 88 000 102 000	First loading; 3/0–6/1; lay ratio 19·0
60° F.	1 000 3 000 5 900	2 500 13 500 14 900	20 000 97 000 126 000	First loading; 4/0–6/1; lay ratio 15·3
60° F.	1 000 3 000 5 900	3 200 15 100 16 700	18 700 88 000 115 000	First loading; 4/0–6/1; lay ratio 19·3
60° F.	1 000 3 000 5 900	4 000 15 500 17 000	17 000 82 000 116 500	First loading; 4/0–6/1; lay ratio 24·2

DETAILS OF CONDUCTORS.

Conductor	Stranding	Area of steel	Areas of Al layers			Total area of conductor	Lay ratios of layers				Diameter of conductor (d)	Weight per foot of conductor (w)	Ultimate strength of conductor
			Inner	Middle	Outer		Steel	Inner Al	Middle Al	Outer Al			
397 500 circ. mils	30/7 × 0·1151 in.	0·073 sq. in.	0·125 sq. in.	—	0·187 sq. in.	0·385 sq. in.	29·0	20·0	—	13·4	0·806 in.	0·6195 lb.	19 970 lb.
3/0 S.C.A.	6/1 × 0·1672 in.	0·0221 sq. in.	—	—	0·1316 sq. in.	0·1537 sq. in.	—	—	—	19·0	0·502 in.	0·230 lb.	6 535 lb.
4/0 S.C.A.	6/1 × 0·1878 in.	0·0276 sq. in.	—	—	0·1663 sq. in.	0·1939 sq. in.	—	—	—	Various	0·563 in.	0·290 lb.	8 255 lb.

The absorbed energy is given by the expression

$$\pi a_2 \omega_2 m (a')^2 l$$

where a_2 is the friction constant of the conductor, and m is its mass per unit length.

The constant a_2 in the above expression is difficult to determine for each particular case because it is dependent upon the tension at the time as well as upon the previous history of the conductor as to tension, previous condition of handling, and other variable factors. For a 30/7 × 0·125 in. S.C.A. cable described in Bate's experiments (see Reference 7) the value of a_2 varies from 0·451 to 0·000638, depending upon tension, loop length, and amplitude. ω_2 is the angular velocity of the conductor, which is the same as that of the wind at resonance, and is represented by

$$2\pi f_2$$

As the conductor vibrates freely in response to the wind, the input in each loop is balanced by the friction loss in that loop and no bending occurs at node points except at the ends of the span at the supports.

Under this balanced condition the half amplitude at the centre of a loop is represented by the expression

$$a = 0\cdot000935 v_w \frac{d^2}{a_2 m}$$

If any damping device eliminates vibration throughout any part of the span when attached at some individual point, it must absorb practically all the energy of vibration imparted to that section of the span by the wind because, if it is performing efficiently, the motion of the conductor is extremely small and the value of a_2 is also small. In addition, the damper must oppose the oscillatory force of the wind required to tune the damper up or down until the period of its forced vibration corresponds to that of the conductor and the wind. It is convenient and customary to supply two dampers per span—one at each end. Since a damper must never be at a nodal point, it has been found practicable to locate each at a point from the support $\frac{3}{4}$ of the length of the shortest loop likely to occur in service. For all sizes of conductors generally used in transmission lines, the shortest loop corresponds to a maximum wind velocity of 12 m.p.h. or less.

Since each damper acts at a single point, K ft. from the support, it is necessary to determine the equivalent maximum, vertical, oscillatory force of the wind over

TABLE 5.

P	P as percentage of ultimate	l	a	ϕ	F	v	Square clamp	Bell-mouth clamp	Armour rods		Remarks
							Cycles	Cycles	Cycles	Hours	
4 000	20·0	8·5	$\frac{9}{16}$	0·515°	27·0	6·7	45 316 000	106 210 000	655 000 000	6 750	First loading; 397 500 circ. mils; 30/7
5 000	25·0				30·0	7·5	20 995 000	26 380 000	197 800 000	1 820	
7 000	35·0				36·0	9·0	4 070 000	11 299 000	49 910 000	381	
2 220	34·0	8·5	$\frac{9}{16}$	0·515°	22·5	3·5	5 975 000	51 820 000	—	—	First loading; 3/0–6/1
3 330	51·0				27·5	4·3	—	12 681 000	500 000 000	4 950	
4 000	62·0				30·0	4·7	—	—	45 000 000	418	
4 440	68·0				31·5	4·9	—	5 031 000	67 486 000	590	

DETAILS OF CONDUCTORS.

Conductor	Stranding	Area of steel	Areas of Al layers			Total area of conductor	Lay ratios of layers				Diameter of conductor (d)	Weight per foot of conductor (w)	Ultimate strength of conductor
			Inner	Middle	Outer		Steel	Inner Al	Middle Al	Outer Al			
397 500 circ. mils	30/7 × 0·1151 in.	0·073 sq. in.	0·125 sq. in.	—	0·187 sq. in.	0·385 sq. in.	29·0	20·0	—	13·4	0·806 in.	0·6195 lb.	19 970 lb.
3/0 S.C.A.	6/1 × 0·1672 in.	0·0221 sq. in.	—	—	0·1316 sq. in.	0·1537 sq. in.	—	—	—	19·0	0·502 in.	0·230 lb.	6 535 lb.
4/0 S.C.A.	6/1 × 0·1878 in.	0·0276 sq. in.	—	—	0·1663 sq. in.	0·1939 sq. in.	—	—	—	Various	0·563 in.	0·290 lb.	8 255 lb.

the half span, concentrated at that point, also the equivalent mass of the half span at the same point. These quantities may be determined as follows.

The energy of vibration of a unit length of the conductor is represented by

$$\tfrac{1}{2}\omega_2^2 y^2 m$$

where m is the mass per unit length of conductor and y is the half amplitude at distance x from a nodal point.

The total energy of vibration of one loop is

$$\int_0^l \tfrac{1}{2}\omega_2^2 y^2 m\,dx$$

which reduces to

$$\tfrac{1}{4}\omega_2^2 (a')^2 ml \left(\text{since } y = a' \sin \frac{\pi x}{l}\right)$$

where a' is the half amplitude at the centre of the loop.

The total energy of vibration of the half span is

$$\tfrac{1}{8}\omega_2^2 (a')^2 mlN$$

where N is the total number of loops in the whole span.

If y_2 represents the half amplitude of the point of attachment of the damper,

$$y_2 = a' \sin \frac{\pi K}{l}$$

The equivalent total energy of vibration of the half span, concentrated at the damper is

$$\tfrac{1}{2}\omega_2^2 (a')^2 \sin^2 \frac{\pi K}{l} m_2$$

where m_2 is the equivalent mass of the half span. Therefore

$$\tfrac{1}{2}\omega_2^2 (a')^2 \sin^2 \frac{\pi K}{l} m_2 = \tfrac{1}{8}\omega_2^2 (a')^2 mlN,$$

which reduces to

$$m_2 = \frac{mS}{4 \sin^2 (\pi K/l)}$$

where S = total length of span.

In the half span the total wind energy is

$$2F_w a' l \sin \beta (N/2)$$

The corresponding energy at the damper is

$$\pi F a' \sin \frac{\pi K}{L} \sin \beta$$

where F is the concentrated, equivalent, maximum, vertical, oscillatory wind force over the half span.

Hence

$$F = \frac{2F_w a' l \sin \beta (N/2)}{\pi a' \sin (\pi K/L) \sin \beta} = \frac{F_w S}{\pi \sin (\pi K/L)}$$

The conductor span may thus be resolved into a concentrated mass m_2 suspended by a resilient means from a rigid support, the resilient suspension containing a friction constant a_2 and a period of vibration ω_2 in resonance with the wind eddy frequency and, therefore, equal to ω which in turn has a value previously explained. The concentrated force of the wind is F, as explained above.

If m_1, a_1, and ω_1, represent respectively the mass, friction constant, and natural period of the damper (which factors can be determined in the laboratory), the entire problem is resolved into a compound elastic system acted upon by a single harmonic force F whose period is ω.

By the application of d'Alembert's principle, the value of y_2 and y_1 can be determined, where y_2 is the half amplitude of the motion of the conductor at point K, and y_1 is the half amplitude of the motion of the centre of gravity of the damper weight.*

There is a different type of conductor vibration which, although of more rare occurrence than resonant vibration, has been frequently observed in various localities. It consists of a low-period vertical oscillation of the conductors, accompanied by extraordinarily large amplitudes—in some cases as much as 15–20 ft. They have usually occurred when there is an accumulation of ice or snow on the conductors, combined with a wind of a certain character. In long spans several loops have been observed, while in short spans the conductor moves up and down in a single loop with larger amplitude at mid-span. In most cases of this particular form of vibration, which has come to be called in America "dancing," motion of the supports in rhythm with the conductors has been observed. When the supports of short-span lines are of wood or light steel construction, "dancing" has been observed where there was no sleet accumulation on the conductors. Usually in these cases the wind blows along the line, swaying the poles and setting up in the entire system rhythmical gyrations which may extend for long distances.

A discussion of "dancing" is outside the scope of this paper.

Conclusion.

The experience of years has shown that both armour rods and Stockbridge dampers are effective in preventing damage to conductors from resonant vibration. Armour rods reduce bending stresses at supports to safe limits, and reduce the amplitude of resonant vibration, but they do not entirely suppress vibration throughout the span. Since this type of vibration never damages the conductor in the span away from the supports, this point is not important. Armour rods have proved effective in reducing the damage to the conductor itself from insulator flash-overs caused by lightning and other high-potential surges. They have also proved an efficient means of repairing conductors which had not been previously protected against vibration damage and have had strands broken at the supports. It has been found unnecessary to cut and splice the conductor even though a considerable number of strands had been broken. This is due to the very heavy binding effect of the spiral armour rods.

Stockbridge dampers go to the root of the matter by preventing resonant vibration from building-up. When

* See Reference (9).

properly designed and applied, they afford an efficient safeguard for the majority of vibration conditions encountered in transmission-line construction.

As time goes on and the records become more comprehensive, it is apparent that aerial conductors of all materials vibrate in the wind. While the amplitudes of the smaller conductors were at first thought negligible, recent investigation of certain old lines has disclosed vibration breakages. Evidence shows that steel and copper conductors, especially the hollow-core types, vibrate as well as S.C.A. conductors, and if not equipped with suitable protection their useful careers are likely to be short.

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THE THEORY, PERFORMANCE, AND CALCULATIONS, OF A POLYPHASE CAPACITOR-TYPE MOTOR.*

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SUMMARY.

A description is given of the polyphase capacitor-type motor, and its theory for various connections of the compensating winding is explained with the help of vector, circuit, and locus diagrams. Performance curves of 100-h.p. and 5-h.p. motors are given and experimental locus diagrams of the latter are compared with those given by theory. Formulæ for predetermining the complete performance of the motor with a slide rule are given, and calculated performance curves are compared with test-results.

(1) INTRODUCTION.

A type of polyphase compensated induction motor† has recently been put upon the market under the trade name Lokaveay (pronounced "low kVA") in which compensation is obtained with a static condenser and an additional stator winding as in the single-phase capacitor motor; it may be given the name "polyphase capacitor motor." The interesting features of the motor are: (*a*) It has no commutator, brushes, and rotating parts in the phase-advancing device; (*b*) ordinarily, it requires no other starter than a change-over switch, and the starting current is much less than in the ordinary induction motor provided with a star-delta or auto-transformer starter; (*c*) the combination of the additional winding and the condenser is cheaper and more effective than the same condenser connected across the terminals of an ordinary induction motor through an equivalent auto-transformer; and (*d*) compensation results in a decrease of the primary current without any increase in the secondary current or the secondary losses. The last feature, it will be recalled, is not true of compensated commutator induction motors and of ordinary induction motors employing a phase advancer or a static condenser.

(2) DESCRIPTION OF POLYPHASE CAPACITOR MOTOR.

Fig. 1 is a diagrammatic sketch of the stator and rotor of the polyphase capacitor motor. The rotor may be of the squirrel-cage or the slip-ring type—the choice being guided by the same considerations as in the case of the ordinary induction motor. p_1P_1, p_2P_2, p_3P_3 are the phases of the primary which is located on the stator. c_1C_1, c_2C_2, c_3C_3 are the phases of what will be referred to as the "compensating winding" which is located in the same slots as, and is coaxial with, the primary. It has generally more turns than the primary and is of much thinner wire. The terminals of the primary and the compensating winding are connected to a throw-over switch as shown in the diagram. A static condenser of the paper-tinfoil type is connected across the terminals c_1, c_2, c_3. The condenser is generally connected in delta but for theoretical considerations it has been replaced by its star-equivalent, i.e. one having a capacitance equal to three times that in the delta connection. The sense of the stator windings is such that, when the switch is in the "start" position, the current flowing through the corresponding phases of the two windings produces fluxes which add to each other. With these connections the starting current taken by the motor is very much smaller than what it would be if the compensating winding were out and the supply were switched on to the primary.

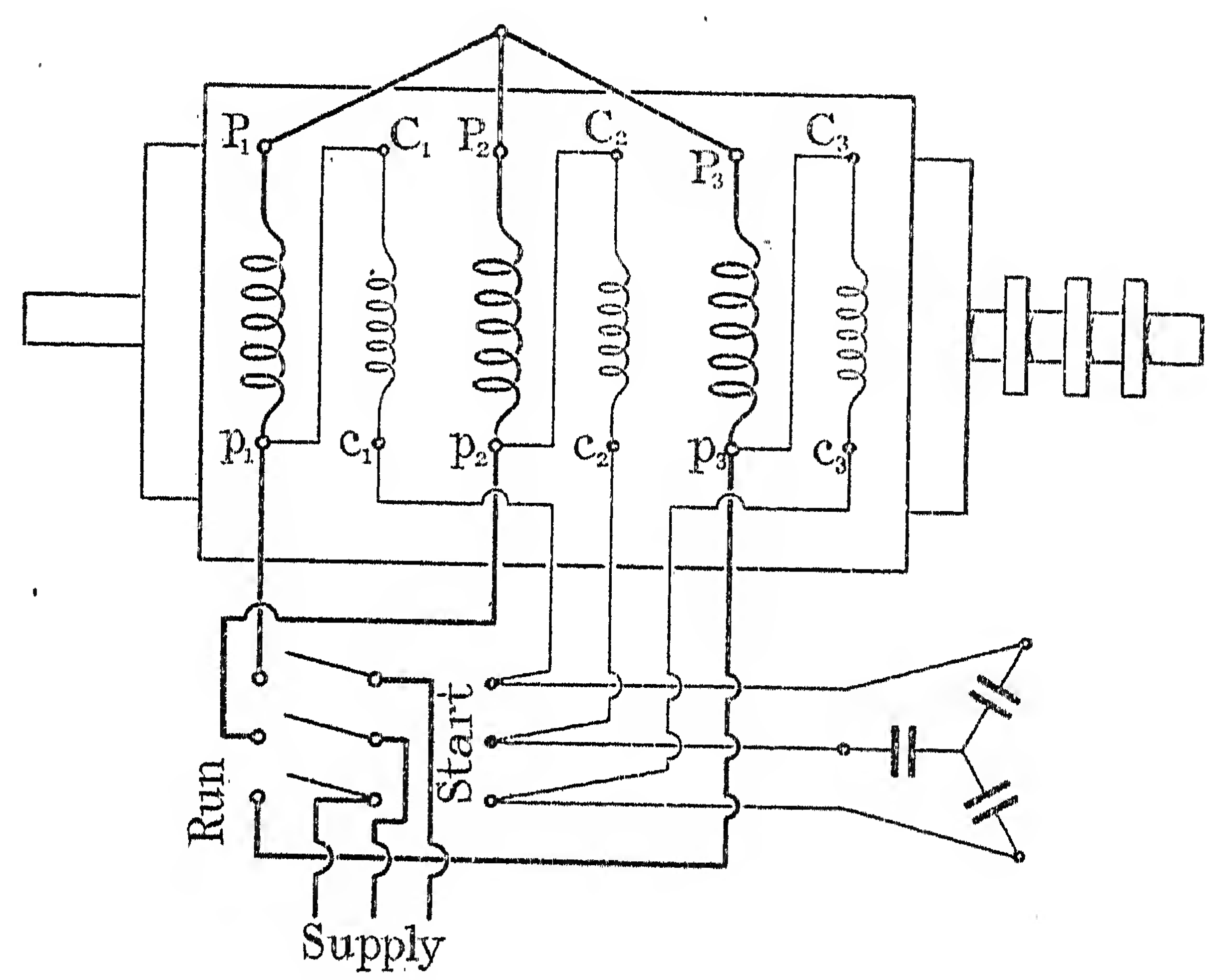

FIG. 1.—Diagram of the polyphase capacitor motor.

The compensating winding thus plays the part of a starter, and unless a high starting torque is required the motor can be switched directly on to the line. When the switch is in the "run" position, the primary and the compensating winding are connected in parallel across the supply. The latter performs the function which its name suggests and is dealt with later. There are various possible ways of connecting the compensating winding, but maximum compensation is obtained when it is connected as in Fig. 1. To show this and also to make a general study of the action of the compensating winding the following cases will be considered:—

Case 1. Compensating winding disconnected from the primary, starred by joining C_1, C_2, C_3, and closed on the condenser.

Case 2. Compensating winding connected as shown in Fig. 1.

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† British Patent No. 220652.

Case 3. Compensating winding connections the reverse of those shown in Fig. 1.

(3) BEHAVIOUR OF THE MOTOR WITH THE COMPENSATING WINDING STARRED AND CLOSED ON THE CONDENSER (CASE 1).

Suppose that the compensating winding is disconnected from the primary and the condenser, and that the primary is connected to the supply after short-circuiting the secondary. The motor will then behave like an

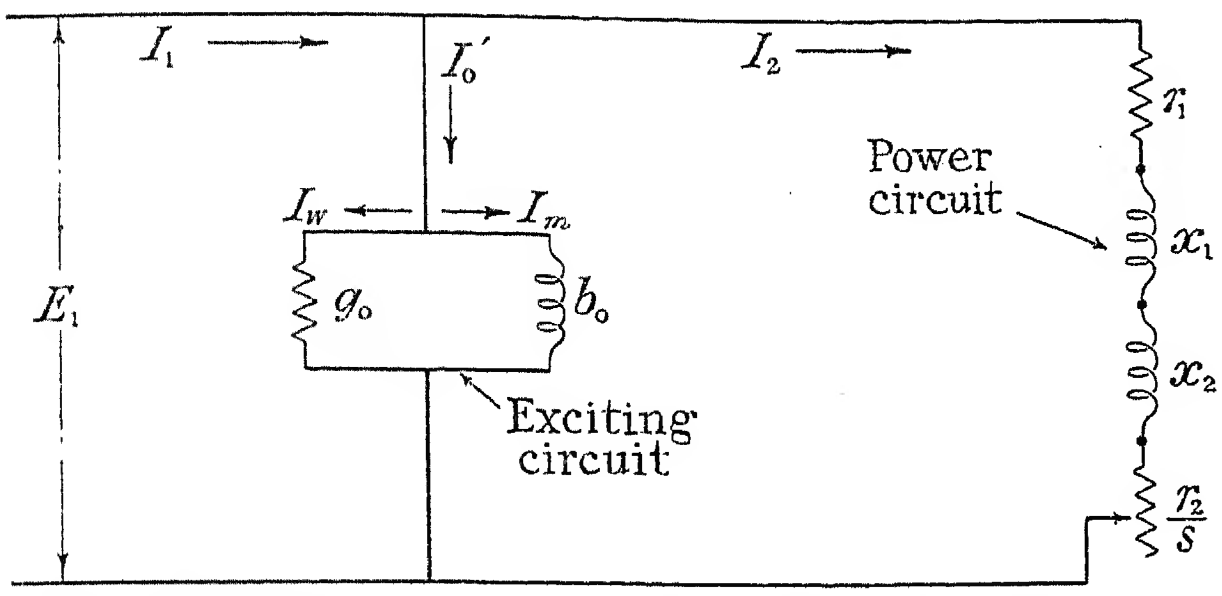

FIG. 2.—Equivalent circuit of the ordinary induction motor.

ordinary induction motor and may be represented by the familiar equivalent circuit of Fig. 2, in which the symbols have the usual significance. This circuit forms the basis of the theory of the polyphase capacitor motor, and its two components are referred to as exciting circuit and power circuit. If OE (Fig. 3) represents the direction of the primary impressed e.m.f., OA $= I_m = b_0E_1$, AN$_1$ $= I_w = g_0E_1$, then ON$_1$ $= I_0$ and the locus of Q$_1$, the extremity of the primary and the secondary current vectors, is a circle through N$_1$, whose centre q$_1$ is on the perpendicular from N$_1$ on OE and whose radius is P $= E_1/2(x_1 + x_2)$.

Let the quantities per phase relating to the polyphase capacitor motor in general be denoted as follows:—

Φ = air-gap flux of the motor when working as an ordinary induction motor;

E = voltage induced in the primary by the air-gap flux;

n_1 = number of turns of the primary;

= number of turns of the secondary;

cn_1 = number of turns of the compensating winding;

r_c = resistance of the compensating winding;

x_c = reactance of the compensating winding;

I_c = current in the compensating winding;

C = star-equivalent capacitance of the condenser if delta-connected, or capacitance per phase if star-connected;

γ = ratio of susceptance of the condenser to the exciting susceptance of the primary;

$= \dfrac{\omega C}{b_0}$;

$-I_k = \gamma I_m = j\omega CE_1$

= current taken by the condenser when it is directly connected to the line;

I_0' = exciting current in the primary of the capacitor motor to distinguish it from I_0—the exciting current of the ordinary induction motor;

I_m' = primary magnetizing current, to distinguish it from I_m;

I_w' = power component of the primary exciting current, to distinguish it from I_w;

I_{0L} = line current on open-circuit;

I_{mL} = wattless component of line current on open-circuit;

I_{wL} = power component of line current on open-circuit;

I_2 = secondary current on load;

I_1 = primary current on load;

I_L = line current on load.

Suppose that the compensating winding is disconnected from the condenser and is starred by connecting C$_1$, C$_2$, C$_3$; also that the secondary is open and full voltage is impressed on the primary. Then the primary takes a current I_0 and the voltage induced in any phase of the compensating winding, say C$_1$c$_1$, is $(-cE_1)$ acting from C$_1$ to c$_1$, assuming that voltage impressed on the corresponding phase of the primary is acting from p$_1$ to

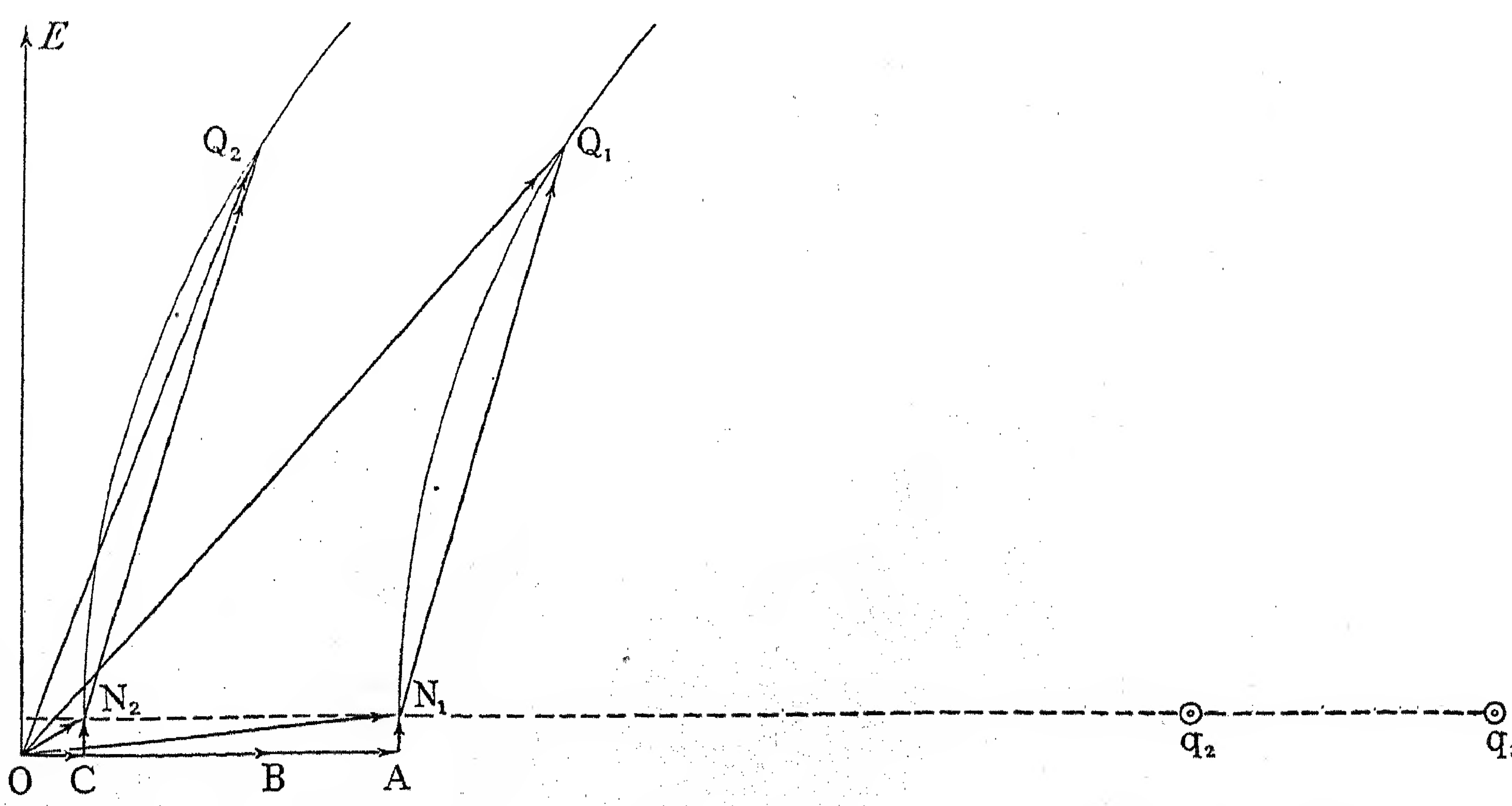

FIG. 3.—Vector diagram and current locus of the polyphase capacitor motor with compensating winding starred and closed on the condenser.

P_1. If the terminals c_1, c_2, c_3 are joined to the condenser, the current in the compensating winding is

$$I_c = \frac{-cE_1}{r_c + j\left(x_c - \frac{1}{\omega C}\right)}$$

Since r_c and x_c are small compared with $1/\omega C$,

$$I_c = -jc\omega CE_1 = c\gamma I_m = \text{BA (Fig. 3)}.$$

Ampere-turns of the compensating winding

$$= cn_1 \,.\, c\gamma I_m$$

Primary current to overcome these ampere-turns

$$= -c^2\gamma I_m$$

Magnetizing current drawn by the primary is therefore

$$I'_m = I_m - c^2\gamma I_m = -j(b_0 - c^2\omega C)E_1 = \text{OC (Fig. 3)}.$$

The iron losses are the same as in an ordinary induction motor, because the air-gap flux is the same. The primary magnetizing current being less than I_m, the primary copper loss is less but there are some additional losses, viz. copper loss due to the current in the compensating winding and the small dielectric losses in the condenser, but for simplicity these may be regarded as equal to the decrease in the primary copper loss, so that

Power component of the exciting current

$$I'_w = I_w = g_0E_1 = CN_2 \text{ (Fig. 3)}.$$

Primary exciting current

$$I'_0 = I_w + I_m - c^2\gamma I_m = g_0E_1 - j(b_0 - c^2\omega C)E_1$$

The compensating-winding current thus reduces the primary magnetizing current by $c^2\gamma I_m$. In other words, the primary behaves as if its exciting reactance were $(b_0 - c^2\omega C)$ instead of b_0. The exciting circuit of the motor may therefore be expressed as shown in Fig. 4.

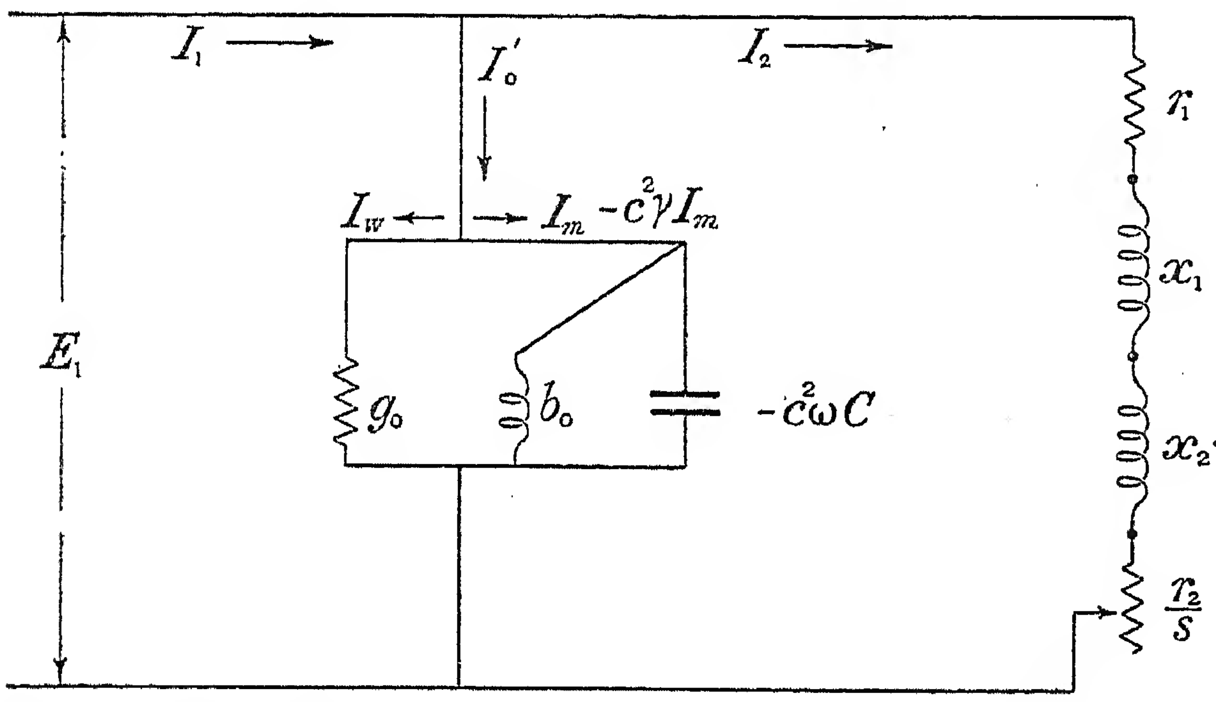

FIG. 4.—Equivalent circuit of the polyphase capacitor motor with compensating winding.

Let OQ_2 (Fig. 3) be the primary current at any load, then N_2Q_2 is the secondary current. As the leakage flux linked with the secondary is still proportional to n_1I_2 ampere-turns, as in the ordinary induction motor, the leakage-reactance drop in the secondary is sx_2I_2. The leakage flux linked with either the primary or the compensating winding is due to their combined ampere-turns, i.e. $n_1I_1 + n_1c^2\gamma I_m$. Therefore the leakage-reactance drop in the primary $= x_1(I_1 + c^2\gamma I_m) = x_1(I_2 + I'_0 + c^2\gamma I_m) = x_1(I_2 + I_0)$. Neglecting I_0 for simplicity, as is usually done in the case of the ordinary induction motor, the leakage-reactance drop in the primary is x_1I_2. It may similarly be seen that the resistance-drops in the primary and the secondary are the same as in the ordinary induction motor. Consequently the power circuit of the motor is the same as in the ordinary induction motor, and the complete equivalent circuit is as shown in Fig. 4. The locus of Q_2 is therefore a circle through N_2 of radius ρ and centre q_2, where q_2 is a point on the perpendicular from N_2 on OE. It will be observed that for a given input the primary current is less than in the ordinary induction motor, and reduction in both the line and primary currents, which are identical, takes place without any increase in the secondary current or the secondary losses.

(4) BEHAVIOUR OF THE MOTOR WITH NORMAL CONNECTIONS (CASE 2).

Suppose now that the connections are as in the normal motor shown in Fig. 1; the secondary and the star point of the condenser are open and the switch is in the "run" position. Then the exciting current taken by the primary is I_0, represented by ON_1 in Fig. 5. If the voltage impressed on the primary phase p_1P_1 is assumed to be acting from p_1 to P_1, the voltage induced in the corresponding phase of the compensating winding by the air-gap flux is acting from C_1 to c_1, i.e. in the same direction as the voltage E_1 impressed on that phase, and is therefore equal to cE_1. Hence the voltage between two terminals of the open star point of the condenser is equal to $(\sqrt{3}cE_1 + \sqrt{3}E_1)$, and if the star point is closed, a voltage $(cE_1 + E_1) = (c + 1)E_1$ will be impressed on each phase of the condenser, and

Current in the compensating winding

$$I_c = j(c + 1)\omega CE = -(c + 1)\gamma I_m = \text{OD (Fig. 5)}$$

The flux produced by the current in the compensating winding is in time phase with OD, neglecting hysteretic lag. Owing to the sense of the winding, however, its space phase is reversed and is coincident with DO and, consequently, the air-gap flux. To make this clear and also to point out a dangerous condition that may arise in this motor, suppose that the condenser is disconnected and the secondary is open; also that the primary and the compensating winding have equal turns and are exactly alike. On putting the switch in the "start" position a current equal to one-quarter of that produced by impressing the same voltage on either of the windings separately will flow into the stator windings, because they are wound in the same sense. Now let the switch be in the "run" position and let the compensating winding be starred by joining c_1, c_2, and c_3. Since the two windings are exactly alike the currents flowing in them will be equal and have the same phase relation to the impressed e.m.f. Hence if OE (Fig. 6) represents the impressed e.m.f. and OP the current in the primary, then PP′ = OP will be the current in the compensating winding and OP′ will be the total current taken by the two windings. Let Op represent the flux produced by

the primary in time phase with the current OP and let the direction of Op represent the space phase of the flux. Then Op also represents the magnitude and time phase of the flux produced by the compensating winding; but its space phase is the reverse of this, and is therefore

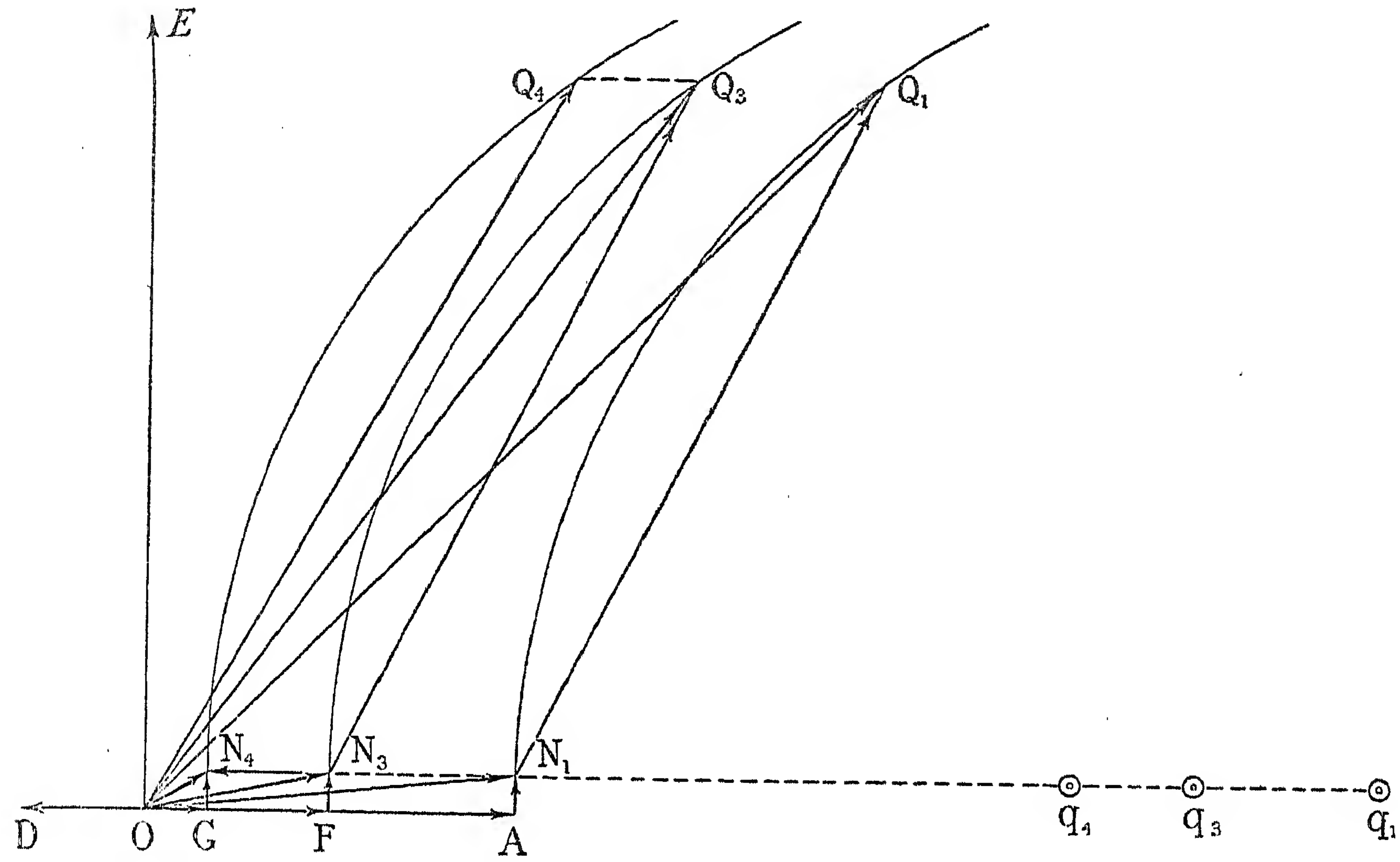

Fig. 5.—Vector and locus diagrams of the polyphase capacitor motor with compensating winding connected in the normal way.

Op′ = Op. The two fluxes therefore oppose one another and the only flux linked with each winding is its leakage flux. The result is that a large current flows in each winding. From this example it will also be seen that if the condenser is delta-connected and if one of its phases breaks down, the motor will take a very large current, but not if the condenser is star-connected.

Referring back to Fig. 5,

The ampere-turns of the compensating winding

$$= cn_1 \,.\, \mathrm{OD}$$
$$= n_1 c(c+1)\gamma I_m$$

Therefore, the magnetizing current required in the primary to supplement the flux of the compensating winding in order that the air-gap flux may be equal to Φ is

$$I'_m = I_m - c(c+1)\gamma I_m = \mathrm{OF} \text{ (Fig. 5)}$$

With the same reasoning as in Case 1, it will be seen that the power component of the exciting current is

$$I'_w = I_w = \mathrm{FN}_3 \text{ (Fig. 5)}$$

The exciting current drawn by the primary is therefore

$$I'_0 = I_w + I_m - c(c+1)\gamma I_m = g_0 E_1 - j\{b_0 - c(c+1)\omega C\}E_1 = \mathrm{ON}_3 \text{ (Fig. 5)}$$

The compensating winding current thus relieves the primary of a part of the magnetizing current equal to $c(c+1)\gamma I_m$, and as a result the primary behaves as if its exciting reactance were $\{b_0 - c(c+1)\omega C\}$ instead of b_0. The exciting circuit of the primary may therefore be expressed as shown in Fig. 7.

The line current on open-circuit is the resultant of the primary exciting current and the leading current OD drawn by the compensating winding from the line.

The wattless component of the line current is

$$I_{mL} = I_m - c(c+1)\gamma I_m - (c+1)\gamma I_m = I_m - (c+1)^2\gamma I_m = \mathrm{OG} \text{ (Fig. 5)}$$

The power component of the line current is

$$I_{wL} = I_w = \mathrm{GN}_4 \text{ (Fig. 5)}$$

The line current is therefore

$$I_{0L} = I_w + I_m - (c+1)^2\gamma I_m = \mathrm{ON}_4$$

Thus the compensating winding not only relieves the primary and the lines of a part of the primary magnetiz-

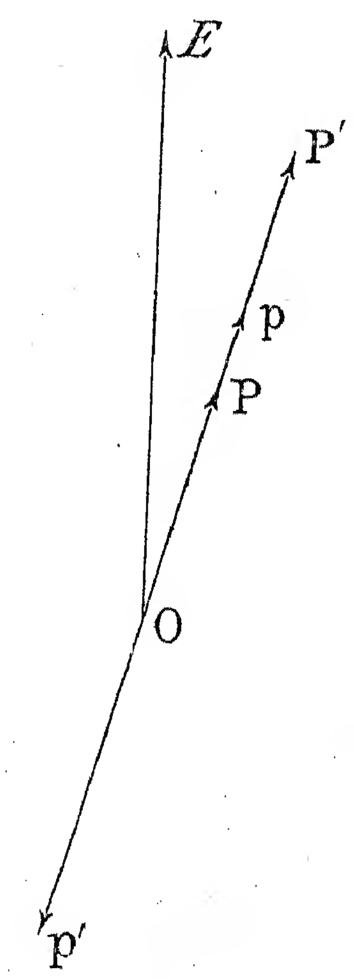

Fig. 6.—Diagram explaining the action of the compensating-winding ampere-turns.

ing current equal to $c(c+1)\gamma I_m$ but further relieves the lines of a wattless current equal to $(c+1)\gamma I_m$. Since the latter is equal to $(c+1)\omega CE_1$, it can be shown in the

equivalent circuit by connecting a condenser of susceptance $-(c+1)\omega C$ as indicated by dotted lines in Fig. 7.

It can be shown, as in Case 1, that the power circuit of the motor is the same as that of an ordinary induction motor, and the complete equivalent circuit is as shown in Fig. 7. Let OQ_3 (Fig. 5) be the primary current at any load, then N_3Q_3 is the secondary current and $OQ_4 = OQ_3 + Q_3Q_4$, where $Q_3Q_4 = OD$ is the line current. Q_3 and Q_4 lie respectively on circles of centres q_3 and q_4 passing through N_3 and N_4. For the same input, the primary and secondary currents of the ordinary induction motor would be OQ_1 and N_1Q_1 respectively, the latter being equal to N_3Q_3. Thus the wattless line current by $c^2\gamma I_m$, $(c+1)^2\gamma I_m$, or $(c-1)^2\gamma I_m$, according to whether the compensating winding is connected as in Case 1, 2, or 3. Also these reductions take place without any increase in the secondary current. Obviously the case in which the compensating winding is connected as shown in Fig. 1 is the best, and this case will therefore be considered in detail. In what follows "polyphase capacitor motor" will mean the motor with connections as in Fig. 1, and "ordinary induction motor" will mean the polyphase capacitor motor with compensating winding and condenser inactive.

(6) Approximations Used in the Theory.

In considering the theory of the polyphase capacitor

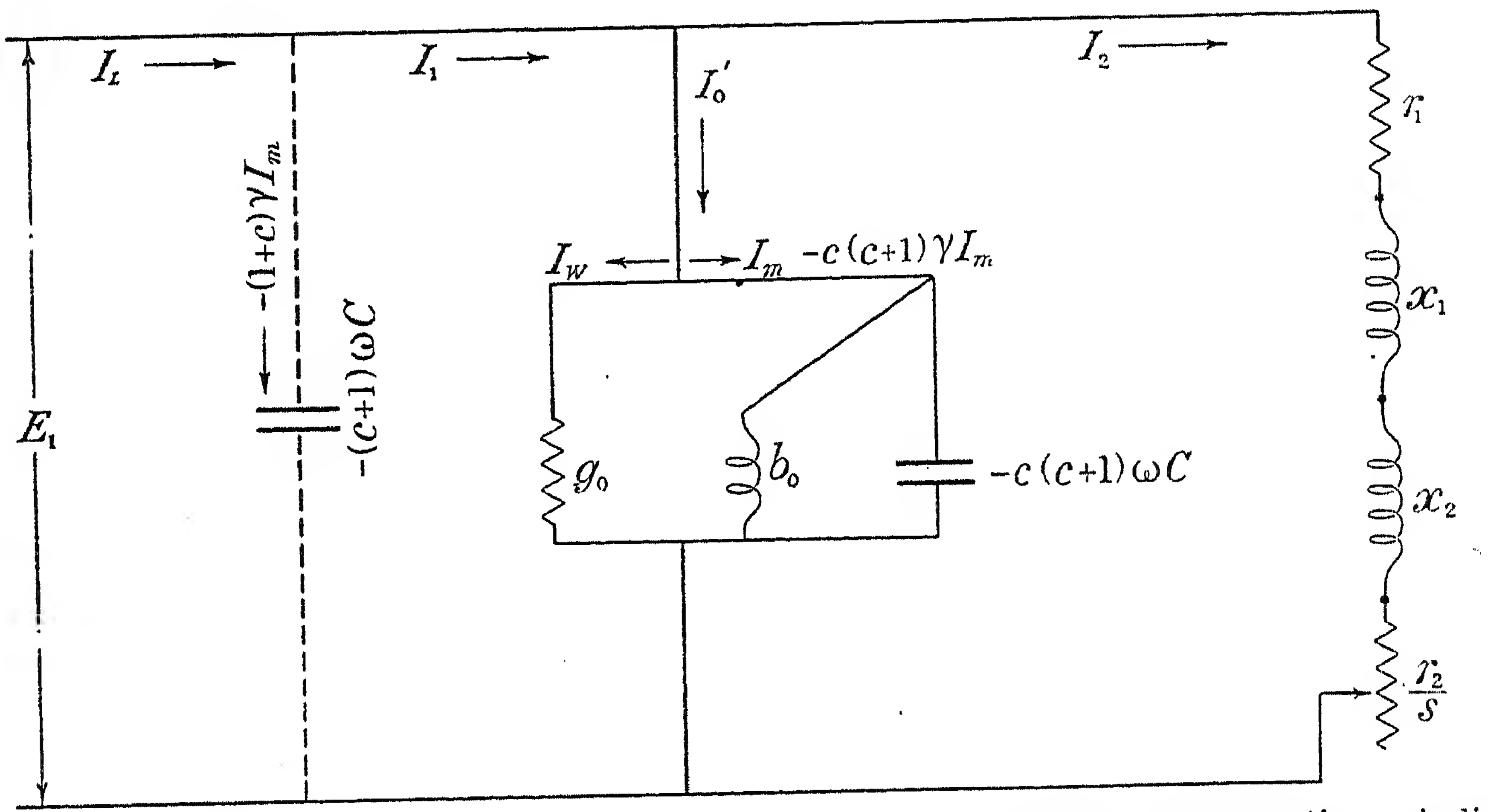

Fig. 7.—Equivalent circuit of the polyphase capacitor motor with compensating winding connected in the normal way.

line current is reduced from OQ_1 to OQ_4 and the primary current from OQ_1 to OQ_3.

(5) Behaviour of the Motor with Connections of Compensating Winding to Primary Reversed (Case 3).

In this case the voltage impressed on the condenser is $-cE_1 + E_1$ and the current in the compensating winding is $(c-1)\gamma I_m$, lagging 90° behind E_1. But its ampere-turns produce flux in phase with the air-gap flux and reduce the magnetizing current of the primary to $I_m - c(c-1)\gamma I_m$. Since the current drawn by the compensating winding is lagging, the wattless component of the line current on open-circuit is $I_m - c(c-1)\gamma I_m + (c-1)\gamma I_m = I_m - (c-1)^2\gamma I_m$. The line current is therefore greater than the primary current and, although compensation is poor compared with Cases 1 and 2, nothing disastrous happens. Proceeding as in Case 2, it can be seen that the equivalent circuit in this case is similar to that in Fig. 7, the susceptance $-(c+1)\omega C$ being replaced by a susceptance $(c-1)\omega C$, and the susceptance $-c(c+1)\omega C$ being replaced by a susceptance $-c(c-1)\omega C$.

Summarizing the results given above, it may be said that the use of compensating winding and condenser with an induction motor reduces the wattless current of the primary by $c^2\gamma I_m$, $c(c+1)\gamma I_m$, or $c(c-1)\gamma I_m$, and the motor the voltage impressed on the condenser was taken as $cE_1 + E_1$, which is not correct. The voltage induced in the compensating winding is cE and not cE_1, and, as is well known, E decreases with load but is practically equal to E_1 on no-load. Moreover, there is a drop of voltage in the compensating winding equal to $r_cI_c + jx_c(I_c + I_1/c)$. Hence the voltage impressed on the condenser is $cE + E_1 - r_cI_c - jx_c(I_c + I_1/c)$ and the condenser current decreases with load. In the 5-h.p. motor referred to below, the condenser voltage was found to vary from 274 volts on no-load to 260 volts on the maximum load that was put on the motor. This, however, only resulted in variation of the condenser current from 2·56 to 2·42 amperes and, considering that the full-load current of the motor is about 27 amperes, the effect of this variation is negligible. Another approximation that has been made is due to the assumption that $I_0 = (g_0 - jb_0)E_1$ instead of $(g_0 - jb_0)E$, but the error due to this is of the same order as in an ordinary induction motor and is negligible.

(7) Test-results.

Fig. 8 shows the characteristics of a 100-h.p., 400-volt, 8-pole, 50-cycle polyphase capacitor motor and of the ordinary induction motor. The undotted curves relate to the former and the dotted ones to the latter. The efficiency curve of the induction motor is practically the

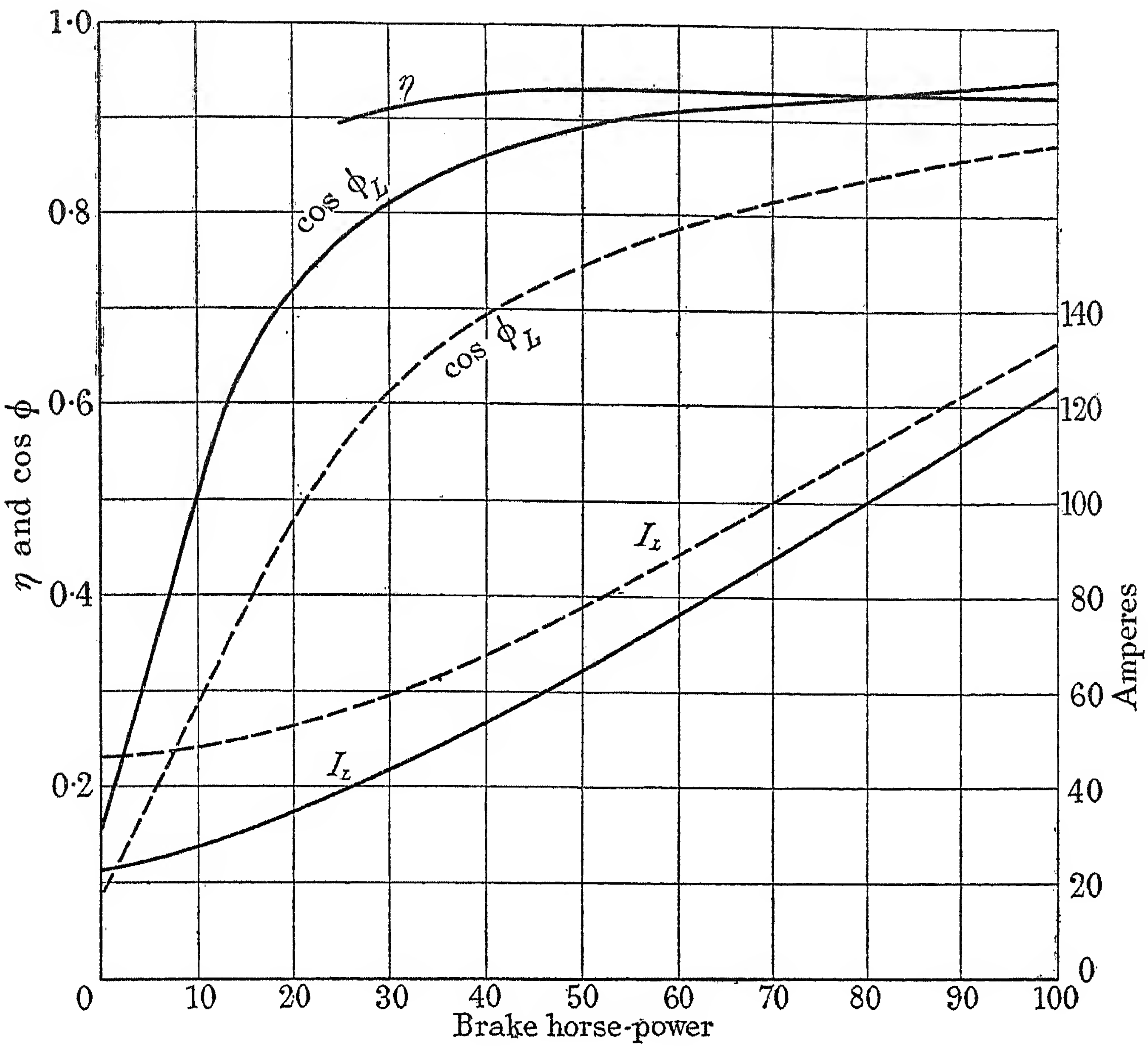

FIG. 8.—Performance curves of a 100-h.p. polyphase capacitor motor.

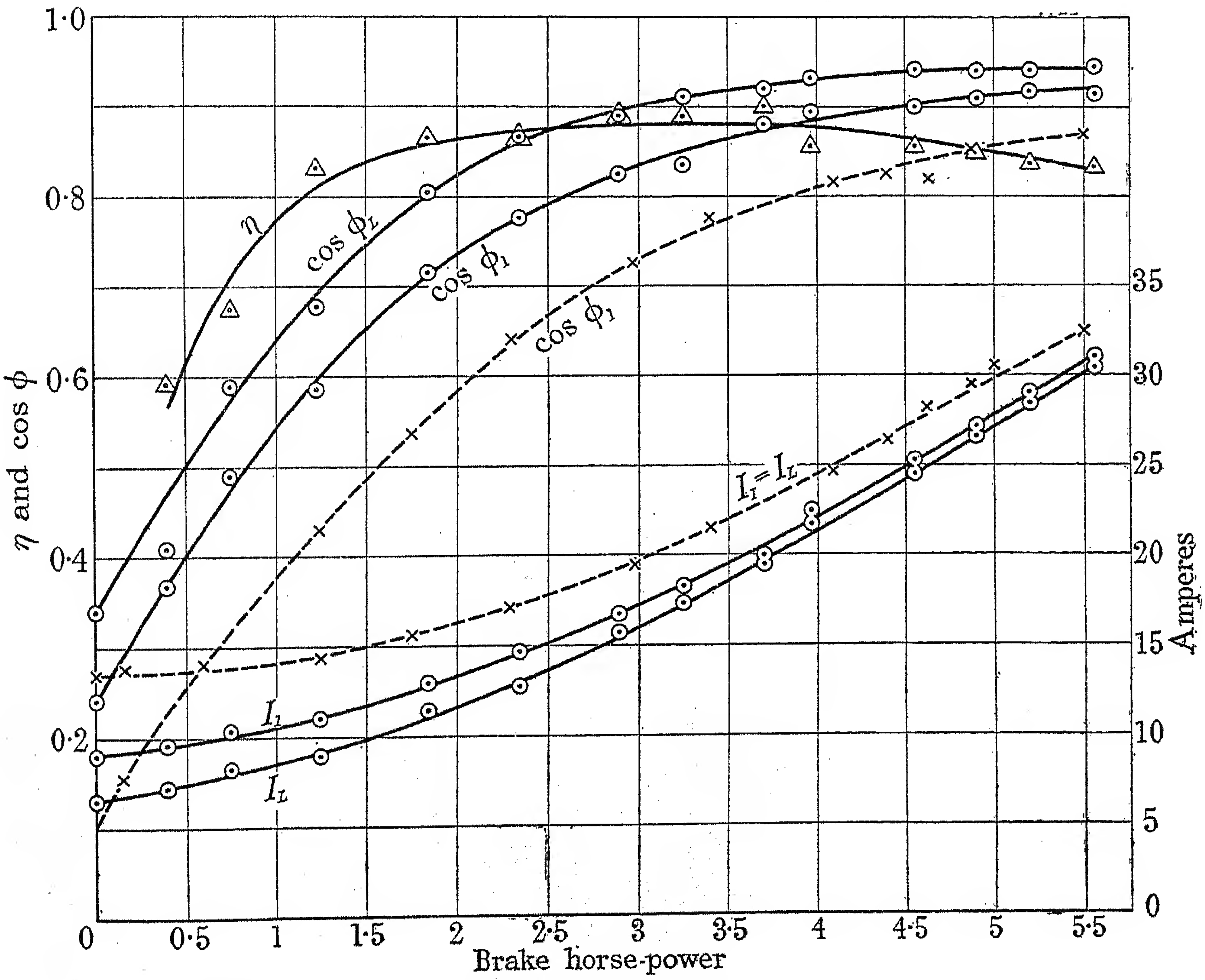

FIG. 9.—Performance curves of a 5-h.p. polyphase capacitor motor.

same as that of the polyphase capacitor motor shown in the figure. The no-load and full-load line currents are 48 per cent and 92 per cent respectively of those of the ordinary induction motor. The curves are taken from a catalogue of a manufacturer of this motor and are based on test-results.

Fig. 9 shows the experimental performance curves of a 5-h.p., 100-volt, 4-pole, 25-cycle polyphase capacitor motor and of an ordinary induction motor investigated by the authors. The dotted curves relate to the latter and those with full lines to the former. I_1 and $\cos\phi_1$ stand respectively for primary current and primary power factor, and I_L and $\cos\phi_L$ for those of the line. The line and primary currents of the polyphase capacitor motor are respectively 48 per cent and 67 per cent on no-load, and 91 per cent and 92 per cent on full load, of those of the ordinary induction motor. It is interesting to note that the percentage reduction in primary current on full load is practically the same as that of the line, or, in other words, the primary gets practically the same benefit from power-factor improvement as the line. The efficiency curve shown in the figure is of the polyphase capacitor motor. The efficiency curve of the induction motor is slightly higher and has been omitted to avoid overcrowding. The line voltage impressed on the motor during the tests was 100 volts by the voltmeter used, and its corrected value is 99 volts.

It will be observed that in both these motors the full-load power factor is about 0·95 lagging. This is obviously to keep the cost of the motors down. A reference to Fig. 5 will show that by the use of a suitable compensating winding and condenser, unity or leading power factor on full load can be obtained.

(8) Predetermining the Performance of the Motor.

Current Loci.—The constants of a 5-h.p., 4-pole, 25-cycle polyphase capacitor motor are: $E_1 = 57·25$ volts; $I_0 = 13·5$ amps.; $I_m = 13·45$ amps.; $I_w = 1·24$ amps.; $b_0 = 0·236$ mho; $g_0' = 0·0216$ mho; $x_1 + x_2 = 0·256$ ohm; $r_1 + r_2 = 0·26$ ohm; $c = 1·76$; $C = 103\ \mu$F; $r_c = 1·92$ ohms; $x_c = 0·66$ ohm; $\gamma = 0·0683$; $I_K = 0·92$ amp. The motor is slightly unbalanced and the constants refer to the mean phase. With the help of these constants the no-load points, centres, and radii, of the circular current loci for various cases, can be determined as explained above and the circles drawn in the usual way. Fig. 10 shows the theoretical circular current loci of the motor for various connections. Points a, a′, b, and c, are centres of circles A, A′, B, and C respectively. A and A′ are the loci of line and primary currents of the polyphase capacitor motor with normal connections. B is the current locus with the compensating winding starred as in Case 1, and C is the current locus of the ordinary induction motor. Points shown in the figure are experimental. The loci for the line and primary currents when the compensating winding is reversed are very close to the induction-motor locus and are not shown, in order to avoid crowding.

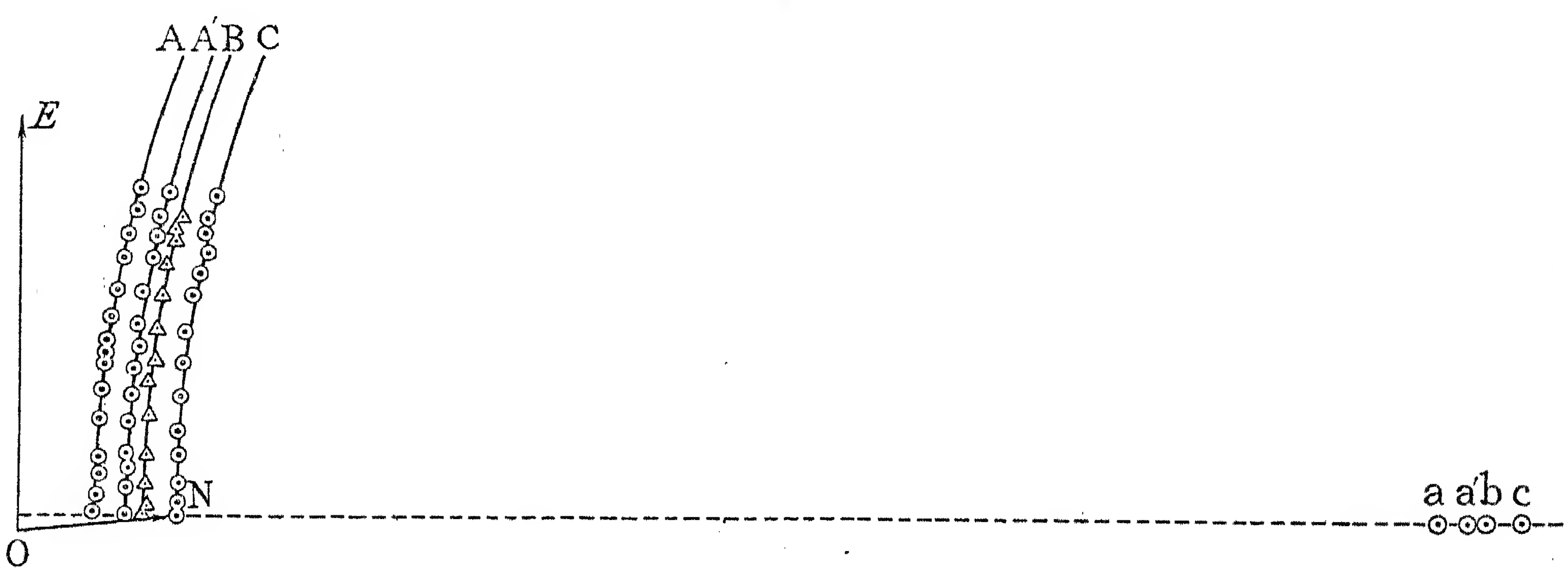

Fig. 10.—Current loci of a 5-h.p. polyphase capacitor motor. Scale: 1 cm = 10 amps.

The entire performance of the motor for any case may be obtained graphically from the circle diagram as for the ordinary induction motor. The graphical method, however, has certain inherent defects, and for convenience and accuracy such as are, for example, demanded in a design office, it is preferable to have a method involving the use of slide-rule only. Such a method and its application are given below with reference to the 5-h.p. polyphase capacitor motor referred to above. In Fig. 11, let

ϕ_0 = phase difference between E_1 and I_0'.
ϕ_{0L} = phase difference between E_1 and I_{0L}.
ϕ_1 = phase difference between E_1 and I_1.
ϕ_L = phase difference between E_1 and I_L.
ϕ_2 = phase difference between E_1 and I_2.

No-load Primary Current.—$I_m' = I_m - c(c+1)\gamma I_m = 13·45 - 4·45$ or 9 amps., and $I_w' = I_w = 1·24$ amps. Therefore $I_0' = \sqrt{(I_w'^2 + I_m'^2)} = 9·1$ amps., $\cos\phi_0 = I_w'/I_0' = 0·136$, and $\phi_0 = 82°$.

No-load Line Current.—$I_{mL} = I_m' - (c+1)\gamma I_m = 9·0 - 2·54$ or $6·46$ amps., and $I_{wL} = I_w = 1·24$ amps. Therefore $I_{0L} = \sqrt{(I_{wL}^2 + I_{mL}^2)} = 6·5$ amps., $\cos\phi_{0L} = I_{wL}/I_{0L} = 0·19$, and $\phi_{0L} = 79°$.

Performance on Load.—Let ON_3 and ON_4 (Fig. 11) represent respectively the no-load primary and line currents so obtained. Then the loci of the primary and line currents are circles through N_3 and N_4 respectively of radius equal to 112 amperes. Draw N_3E' parallel to OE and let N_3Q_3 be the secondary current vector at any load. Then $Q_3N_3E' = \phi_2$ and

$$I_2 = 2\rho \sin\phi_2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

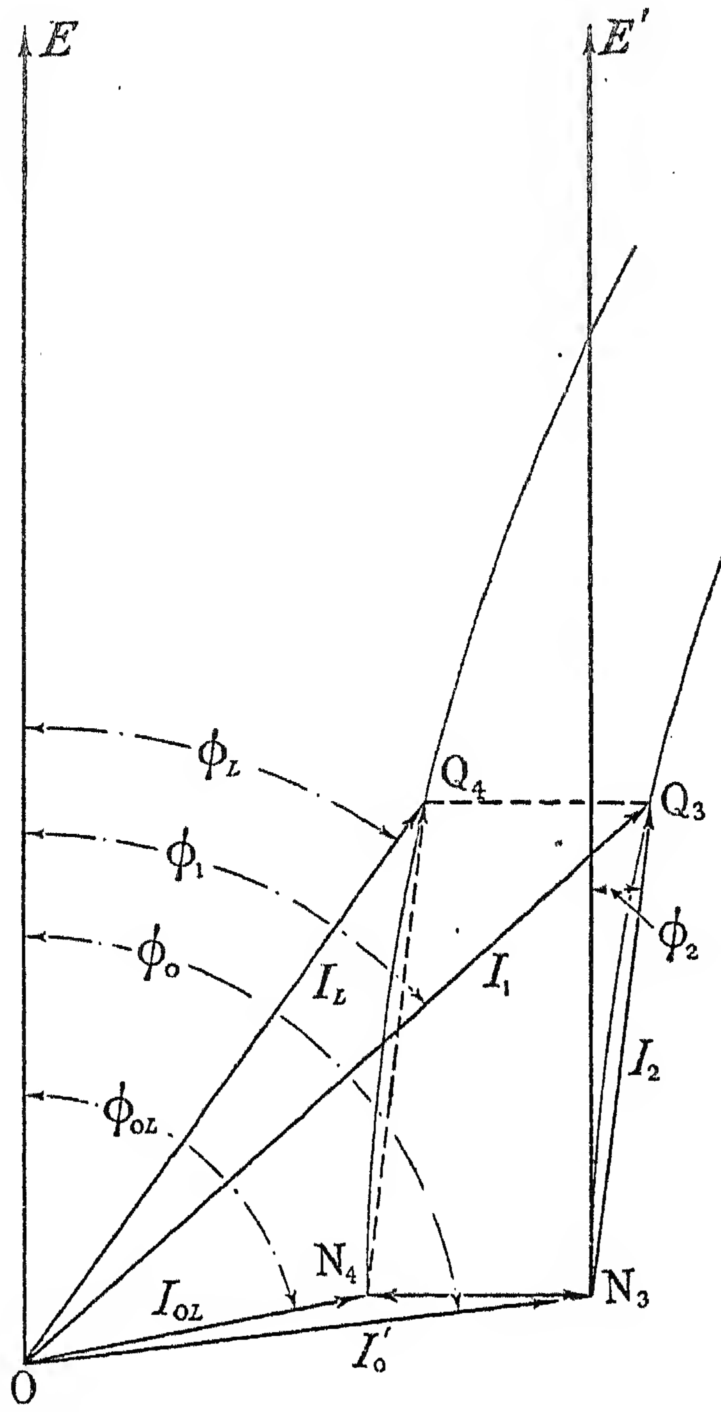

FIG. 11.—Vector diagram of a polyphase capacitor motor.

Therefore $\cos\phi_2 = \sqrt{\left(1 - \frac{I_2^2}{4\rho^2}\right)}$ . . . . (2)

From equation (2) values of $\cos\phi_2$ for various values of I_2 can be found and used as shown below. For example, suppose that $I_2 = 30$ amps. Then, $\cos\phi_2 = 0\cdot99$ and $\phi_2 = 7\cdot7°$.

Output per phase

$$P = E_1 I_2 \cos\phi_2 - (r_1 + r_2)I_2^2 = 1\,466 \text{ watts.}$$

Output for the three phases $= 4\,398$ watts
$= 5\cdot9$ h.p.

Input per phase

$$Q = E_1 I_2 \cos\phi_2 + I_w E_1 = 1\,770 \text{ watts}$$

Efficiency per phase

$$\eta = \frac{P}{Q} = 0\cdot83$$

Primary current

$$I_1 = \sqrt{\left\{(I_0')^2 + I_2^2 + 2I_0' I_2 \cos(\phi_0 - \phi_2)\right\}} = 33\cdot6 \text{ amperes}$$

Since $I_1 \cos\phi_1 = I_0' \cos\phi_0 + I_2\cos\phi_2$
Primary power factor is

$$\cos\phi_1 = \frac{I_0'\cos\phi_0 + I_2\cos\phi_2}{I_1} = 0\cdot923$$

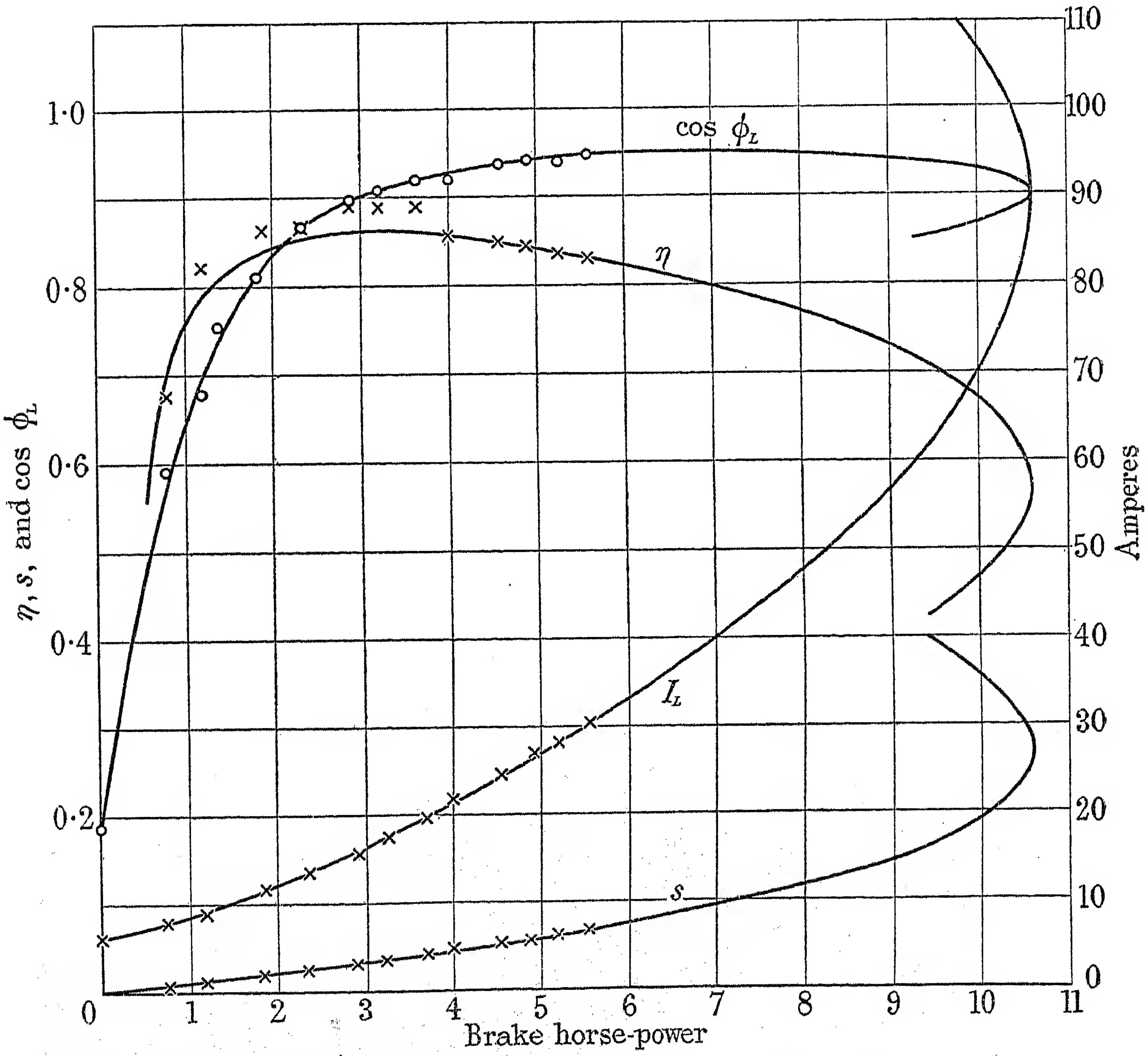

FIG. 12.—Calculated performance curves of a 5-h.p. polyphase capacitor motor.

From the power circuit in Fig. 7,

$$\text{Total resistance drop} = \left(r_1 + \frac{r_2}{s}\right)I_2 = E_1 \cos \phi_2$$

Hence, slip of the motor

$$s = \frac{r_2 I_2}{E_1 \cos \phi_2 - r_1 I_2} = 0 \cdot 74$$

Torque in synchronous watts per phase

$$\mathbf{T} = (E_1 I_2 \cos \phi_2 + I_w E_1) - (I_w E_1 + r_1 I_2^2) = E_1 I_2 \cos \phi_2 - r_1 I_2^2 = 1\ 583$$

Line current

$$I_L = \mathrm{ON}_4 + \mathrm{N}_4\mathrm{Q}_4 = \sqrt{\left\{I_{0L}^2 + I_2^2 + 2I_{0L}I_2 \cos(\phi_{0L} - \phi_2)\right\}} = 32 \cdot 8 \text{ amperes}$$

Line power factor

$$\cos \phi_L = \frac{I_2 \cos \phi_2 + I_{0L} \cos \phi_{0L}}{I_L} = 0 \cdot 945$$

Proceeding in this manner for various values of I_2, the entire performance of the motor from no-load to standstill may be obtained. The curves in Fig. 12 show the calculated performance of the motor at various outputs. Points on or near the curves are experimental.

(9) Comparison of Various Compensated Induction Motors.

The polyphase capacitor motor may be regarded as a combination of induction motor, auto-transformer, and condenser. The iron and the primary of the motor, together with the compensating winding, constitute an auto-transformer. Again, not only is the entire iron and copper of the primary necessary for an external auto-transformer saved, but also its iron and primary copper losses are done away with. Moreover, compensation results in a decrease of the primary current, which is not true in the case of an external auto-transformer and condenser used with ordinary induction motor, and the saving in the copper and space required by the primary of the motor can be utilized for the compensating winding.

Comparing the polyphase capacitor motor with an induction motor and built-in or separate phase-advancer, it will be seen that for the same line current the primary copper losses are less in the latter; but this is offset by the increase in the secondary copper losses due to increase in the secondary current, and the losses of the phase advancer. The increase in the secondary copper losses may be taken as roughly equal to the decrease in the primary copper losses. It might be expected that, owing to the extremely high efficiency of the condenser, the losses in it and in the compensating winding would be less than those of the phase advancer. From these considerations it would appear that the polyphase capacitor motor is more efficient than an induction motor and phase-advancer. However, this difference will not be appreciable because, in general, the efficiency of an induction-motor phase-advancer combination is only about 1 or 2 per cent less and that of the polyphase capacitor motor slightly less than that of the uncompensated motor. A more important difference between the polyphase capacitor motor and the induction-motor phase-advancer combination lies in the simplicity and the probable lower cost of the compensating winding and condenser compared with the phase-advancer. The costly item in the compensating device of the polyphase capacitor motor is the condenser, and the position of the motor will be improved with the progress in the manufacture and demand of condensers and the consequent fall in their price.

The authors wish to record their thanks to Prof. F. N. Mowdawalla for the facilities given to enable this work to be carried out, to the Electric Construction Co. for permission to publish the curves in Fig. 8, and to the Director, Indian Institute of Science, for permission to publish this paper.

THERMIONIC PEAK VOLTMETERS FOR USE AT VERY HIGH FREQUENCIES.

By Prof. C. L. FORTESCUE, O.B.E., M.A., Member.

(*Paper received* 29*th April*, 1935.)

SUMMARY.

This paper refers to the two sources of error experienced when using peak voltmeters at frequencies of 20 megacycles per sec. or higher, namely, the error due to resonance and that due to the time of flight of the electrons from anode to cathode in the rectifying diode. In the case of the parallel-plane diode the latter appears to impose the severer limitation, and it is shown that very short times of flight are necessary. An approximate method of estimating this error with diodes of other geometric form is suggested, and a convenient arrangement of peak voltmeter for use at high frequencies is described.

(1) INTRODUCTION.

The use of a rectifying valve, reservoir condenser, and voltmeter, for peak-voltage measurement is now common practice for any frequency up to 1 million cycles per sec., or even higher. The design of the components to give the best results is well known and the possibility of certain errors—for example, that due to the initial velocity of the electrons—being unavoidable is generally appreciated. At frequencies of 50 million, however, more serious difficulties are experienced on account of resonance effects and of the time of flight of the electrons from cathode to anode within the rectifying valve. For all those cases where it is desired to measure voltages to earth, the first of these difficulties may be overcome, up to any frequency for which the second error is not serious, by the arrangement of apparatus described later in this paper.

(2) THE NATURE OF THE ERRORS AT HIGH FREQUENCIES.

(*a*) The circuit consisting of the inter-electrode capacitance of the valve and the inductance of the leads from earth to the valve and from the anode to the point at which the potential is to be measured, constitutes a series resonant circuit, and at any frequency approaching the natural frequency of this circuit the potential between the electrodes will be different from that which it is desired to measure, owing to what is commonly called the "resonance step-up."

(*b*) The time of flight of the electrons from cathode to anode within the valve may be comparable with the period of the alternating potential being measured, and consequently the time during which the maximum potential is applied may be insufficient for any electrons actually to reach the anode, with the result that the reservoir condenser will not be fully charged and the voltmeter will give indications which are too low.

(*c*) In some cases a third source of error may arise from the fact that the conditions in the circuit on which the measurements are being made may be disturbed by the introduction of the capacitance of the rectifying valve, and by bringing up to the circuit earthed conductors not previously there.

(3) THE RESONANCE ERROR.

If C_v is the capacitance of the anode to the filament and the screen to which it is connected, and L_v is the combined inductance of the lead to the anode and the leads to the surrounding earth, then resonance occurs at

$$f_R = \frac{1}{2\pi\sqrt{(L_v C_v)}}$$

The ratio of the maximum values of the voltage to be measured and the voltage across the anode of the valve, at any other frequency f, depends upon the resistance of the circuit. So long as it is small compared with the reactances,

$$\frac{v_{max.}}{v_{a_{max.}}} = 1 - \frac{f^2}{f_R^2}$$

If V is the corresponding steady voltage across the reservoir condenser, as observed by the electrostatic voltmeter, then $V = v_{a_{max.}}$, provided that the reservoir condenser is satisfactory. Consequently

$$K = \frac{v_{max.}}{V} = \frac{v_{max.}}{v_{a_{max.}}} = 1 - \frac{f^2}{f_R^2}$$

This result is, of course, based on the assumptions that the electrode capacitance is a definite constant and that the distributed capacitance of the leads can be neglected. Since the anode of the valve is negative to the cathode for almost the whole cycle in a peak voltmeter of this kind, there will be no space charge throughout almost the whole cycle and the assumption of a constant C_v is thus justifiable. The second assumption can be made only when the inductance of the earth connections is low and the capacitance to earth of the leads from the anode is reduced to a minimum. Both of these requirements are met in the arrangement of the instrument described in Section (6).

In order that this resonance error should not exceed 1 per cent, it follows that f/f_R must not exceed $0 \cdot 1$. Thus if a maximum frequency of 75 megacycles is required, the natural frequency of this circuit must not

* The Papers Committee invite written communications, for consideration with a view to publication, on papers published in the *Journal* without being read at a meeting. Communications (except those from abroad) should reach the Secretary of the Institution not later than one month after publication of the paper to which they relate.

be less than 750 megacycles. But if $C_v = 0{\cdot}5 \times 10^{-12}$ and $L_v = 0{\cdot}1 \times 10^{-6}$, f_R is still only just over 700 megacycles. The necessity of reducing both C_v and L_v is thus obvious. This limitation alone probably renders it impossible to make an accurate portable peak voltmeter for any frequency as high as 75 megacycles. For higher frequencies still, voltages between given conductors and adjacent earth points may be measured by permanently connected and very compactly arranged valves and condensers. The indicating voltmeter and the filament batteries may, of course, be at any convenient distance away, provided that the terminal of the reservoir condenser is connected directly to earth. In such cases, L_v may possibly be reduced to $0{\cdot}01 \times 10^{-6}$, and, if C_v has the value $0{\cdot}5 \times 10^{-12}$, f_R becomes about 2 250 megacycles and reasonable accuracy from 200 to 250 megacycles may then be possible, so far as this source of error is concerned.

It might appear at first sight that the introduction of resistance to damp out the resonance effects is permissible, since the internal resistance of the valve is high. This is not so, however, since the fall of potential due to the alternating current in the resistance would cause the voltage between the valve electrodes to be much below that which it is required to measure.

(4) The Error due to the Time of Flight of the Electrons.

This error is much more difficult to calculate, and even with valves of simple geometric form only an indication of the magnitude of the error can be found.

(a) *Diodes with Plane Electrodes.*

No electrons leave the cathode until the instant at which the anode potential reaches the steady potential between the plates of the reservoir condenser. For a short interval subsequent to this, electrons are attracted across by a small excess of anode potential above that of the condenser. If any of these electrons actually reach the anode, the condenser voltage rises, and during the next cycle the excess of anode potential is reduced. Steady conditions are reached when the first electrons attracted away from the cathode just fail to reach the anode. If it be assumed that these first electrons are not subject to any space-charge effects, their flight is easily calculated and the conditions for the first ones just to fail to reach the cathode can be found.

Let V be the steady potential across the reservoir condenser and $v = v_{max.} \sin(\omega t + \theta)$ be the alternating potential applied to the electrodes of the valve, the units being electrostatic.

Measuring time from the instant P (Fig. 1), we have

$$v_a = v_{max.} \sin(\omega t + \theta) - V$$

If d is the spacing of the electrodes and x is the distance measured from the cathode towards the anode, the equation of motion of an electron subject only to the field produced by the potential v_a is

$$\frac{d^2x}{dt^2} = \frac{e}{md}\left\{v_{max.} \sin(\omega t + \theta) - V\right\}$$

Integrating twice and inserting the boundary conditions that when t is zero both x and dx/dt are zero (or negligibly small), gives:—

$$\frac{dx}{dt} = \frac{1}{\omega} \cdot \frac{e}{md}\left\{v_{max.} \cos\theta - v_{max.} \cos(\omega t + \theta) - V\omega t\right\} \quad (1)$$

and

$$x = \frac{1}{\omega^2} \cdot \frac{e}{md}\left\{v_{max.} \sin\theta - v_{max.} \sin(\omega t + \theta) + v_{max.}\, \omega t \cos\theta - \tfrac{1}{2} V\omega^2 t^2\right\} \quad . \quad (2)$$

In the case of electrons leaving at $t = 0$ and just coming to rest outside the anode after a time t_a without falling into it, equations (1) and (2) give:—

$$\frac{V}{v_{max.}}\omega t_a = \cos\theta - \cos(\omega t_a + \theta) \quad . \quad . \quad (3)$$

$$\frac{\omega^2 d^2}{(e/m) v_{max.}} = \left\{\sin\theta - \sin(\omega t_a + \theta) + \omega t_a \cos\theta - \tfrac{1}{2}\frac{V}{v_{max.}}\omega^2 t_a^2\right\} \quad . \quad . \quad (4)$$

Also

$$\sin\theta = \frac{V}{v_{max.}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Equations (3), (4), and (5), enable ωt_a and θ to be found,

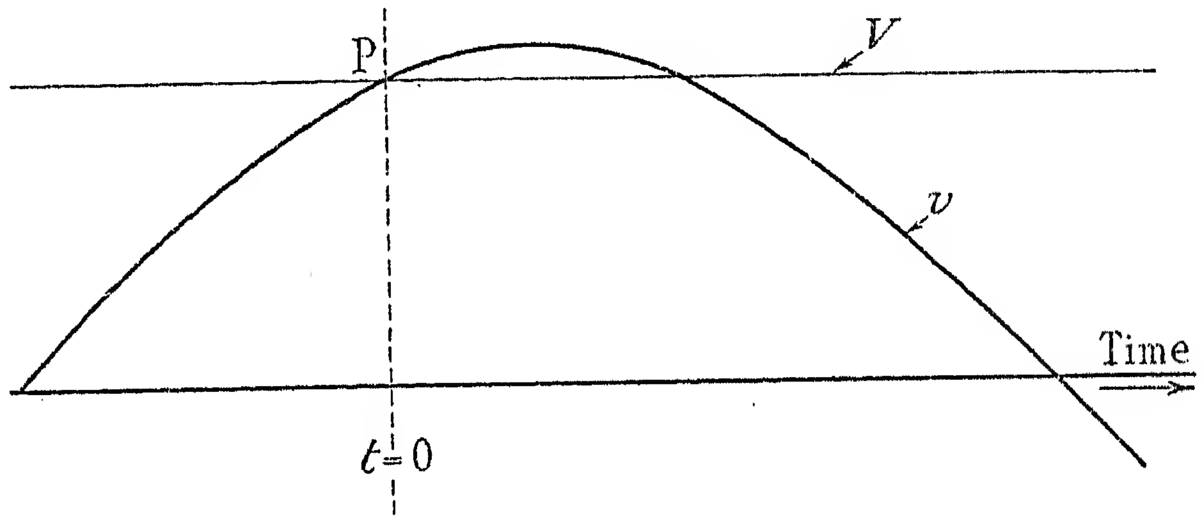

FIG. 1.

though not very conveniently. Algebraical rearrangement leads to

$$\sin\theta - \sin(\omega t_a + \theta) = 2\frac{\omega d}{v_{max.}}\left(\frac{2e}{m}\right)^{-\frac{1}{2}} V^{\frac{1}{2}} \quad . \quad (6)$$

and then to

$$\omega\frac{d}{\left(\frac{2e}{m}V\right)^{\frac{1}{2}}} = 1 - \frac{\psi}{\tan\psi} \quad . \quad . \quad . \quad . \quad (7)$$

where $\psi = \frac{1}{2}\omega t_a$

The time of flight of an electron under a steady anode potential V and without space charge is

$$T_a = \frac{2d}{\left(\frac{2e}{m}V\right)^{\frac{1}{2}}}$$

Equation (7) may therefore be written

$$\tfrac{1}{2}\omega T_a = 1 - \frac{\psi}{\tan\psi} \quad . \quad . \quad . \quad . \quad (8)$$

Equation (8) enables ψ to be found, and, rearranging (6) in the form $\tan\theta = \sin^2\psi/(\psi - \sin\psi\cos\psi)$, θ, and therefore the ratio $K = v_{max.}/V$, can be found.

Since $\sin\theta = 1/K$ it is therefore possible to calculate, and either to plot or to tabulate, K in terms of $\left(1 - \frac{\psi}{\tan\psi}\right)$ and therefore in terms of ωT_a. The following Table is calculated in this way.

TABLE.

ωT_a	K
0·00	1·000
0·04	1·014
0·10	1·034
0·20	1·068

The departure of K from unity is almost directly proportional to ωT_a.

This result shows that with a plane diode an error of at least 1 per cent would be expected when $\omega T_a = 0{\cdot}04$. Thus for a frequency of 75 megacycles, $\omega = 472 \times 10^6$ and the valve must be so designed that T_a does not exceed about $0{\cdot}85 \times 10^{-10}$ sec. Now $(2e/m)^{\frac{1}{2}} = 1{\cdot}03 \times 10^9$, and if $V = 2{\cdot}00$ (i.e. 600 volts) d must not be greater than 0·06 cm.

(*b*) *Diodes of other Geometric Forms.*

No treatment of this problem with valves of other forms seems to be possible, though with a cylindrical diode a graphical integration could, no doubt, be applied to determine, approximately, the ratio of $v_{max.}/V$ in any particular case. This would, however, be very laborious.

Some general conclusions may, however, be drawn from the result just given for the plane diode. In the first place it is probable that a given time of flight under a given voltage will cause errors of the same order, whatever may be the geometric form of the electrodes. Secondly, whatever the form of the electrodes, an approximation to the time of flight can be found by comparison with either a plane diode having the same current-voltage characteristic per unit area of anode, or by comparison with a cylindrical diode having the same diameter of cathode and a diameter of anode giving the same current-voltage characteristic per unit length of cathode. Both of these times of flight are calculable and give an indication of the limiting frequency up to which any required accuracy may be expected.

(5) INFLUENCE OF THESE ERRORS ON THE CHOICE OF DIODES FOR USE AT HIGH FREQUENCIES.

In order to reduce both errors to a minimum, it is necessary to design the valves for minimum electrode capacitance with a minimum time of flight. This involves the smallest possible electrodes with the smallest possible clearances. The limitations arise from difficulty of manufacture and the necessity of providing sufficient mechanical strength to withstand the electrostatic forces of attraction due to the doubled peak voltage between the electrodes during the negative half-cycle. Double-ended diodes having hairpin filaments slightly flattened at the end, and small disc-shaped anodes, were specially made by the General Electric Co. at their Research Laboratories at the author's request some years ago and have proved satisfactory for moderately high frequencies. Mounted in a screened portable instrument, the electrode capacitance was found to be between 0·8 and 0·9 $\mu\mu$F. It is probable that this design can now be greatly improved upon, and the figure of 0·5 $\mu\mu$F mentioned in a previous paragraph should be easily obtainable. The clearance between the anodes and filaments of these diodes is about 0·2 cm and is probably unnecessarily large, with the result that the time of flight is longer than that which could possibly have been obtained. From the characteristics this appears to be in the neighbourhood of $1{\cdot}6 \times 10^{-9}$ sec. with $V_a = 600$ volts, and for an accuracy of 1 per cent the frequency limit thus appears to be only 4·0 megacycles per sec.

The limitation due to the time of flight of the electrons is thus much more serious than that due to the resonance effects. The future design of small diodes for very high-frequency measurements must therefore concentrate on the reduction of the time of flight, even if some increase of inter-electrode capacitance is unavoidable.

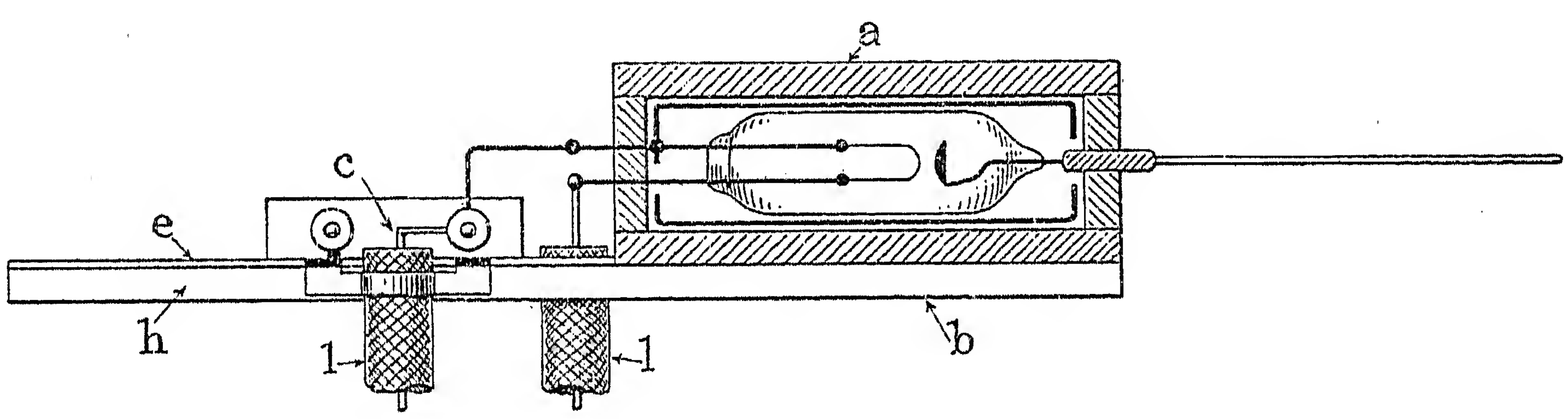

FIG. 2.

(6) ARRANGEMENT OF A PORTABLE INSTRUMENT.

The construction of this instrument is shown diagrammatically in Fig. 2. The little diode is mounted in an ebonite box (a), on the inside of which is a copper screen connected to the negative end of the filament. This screen surrounds the valve except at the seal of the anode lead. This lead is soldered to a small piece of brass rod mounted in the end of the box. This rod also supports the projecting copper wire, about 6 cm long, by means of which contact can be made at the various points at which the potential is to be measured. The valve in its box is mounted on a base (b) with any convenient form of handle (h). The two screened flexible leads (1, 1) connect the filament to a 2-volt

battery on an insulating stand on an earth plate making direct contact with surrounding earthed conductors. The reservoir condenser (c) has one terminal connected to the negative end of the filament and the other to an earthed strip (e) which is in soldered connection with the earthed braiding of the two battery leads. The capacitance to earth of the battery and battery leads is thus in parallel with the reservoir condenser, and the insulation of both must be equal to that of the condenser. The electrostatic voltmeter is connected across the reservoir condenser and may be mounted either on the battery stand or alongside the reservoir condenser.

By this arrangement, the inductance of the earth connections is reduced owing to the large diameter of the outer conductors of the two battery leads, and the inductance of the anode lead is only that of a length of 6 cm of No. 20 S.W.G. copper wire.

The effective inductance is thus as low as it possibly can be with a portable instrument.

The author wishes to express his indebtedness to the Director of the General Electric Research Laboratories for undertaking the construction of the special diodes; to several past-students of the City and Guilds College who have assisted with this work, particularly Messrs. J. R. F. Jarvis, E. C. S. Megaw, and D. G. Reid; and to the Clothworkers' Company, whose grants for research rendered the experimental work possible.

PROCEEDINGS OF THE INSTITUTION.

876TH ORDINARY MEETING, 20TH DECEMBER, 1934.

Prof. W. M. Thornton, O.B.E., D.Sc., D.Eng., President, took the chair at 6 p.m.

The minutes of the Ordinary Meeting held on the 6th December, 1934, were taken as read and were confirmed and signed.

A paper by Mr. F. T. M. Kissel, B.Sc., Member, entitled "The Organization of Electricity Supply in New Zealand" (see page 63), was read, in the absence of the author, by Mr. W. P. Gauvain and discussed.

The meeting terminated at 7.35 p.m. with a vote of thanks, moved by the President, to the author for his paper and to Mr. Gauvain for reading it on his behalf.

877TH ORDINARY MEETING, 10TH JANUARY, 1935.

Prof. W. M. Thornton, O.B.E., D.Sc., D.Eng., President, took the chair at 6 p.m.

The minutes of the Ordinary Meeting held on the 20th December, 1934, were taken as read and were confirmed and signed.

The following list of donors to the Library was taken as read, and the thanks of the meeting were accorded to them:—Air Ministry; R. Amberton; American Institute of Electrical Engineers; American Society of Mechanical Engineers; Association of Engineers in Burma; Association Suisse des Électriciens; K. Aston, B.Sc.Tech.; The Astronomer Royal; Librairie J. B. Baillière et fils; Messrs. Ernest Benn, Ltd.; Messrs Ed. Bennis and Co., Ltd.; G. S. Berkeley; A. W. Beuttell; Messrs. P. Blakiston's Son and Co., Inc.; Board of Education; British Broadcasting Corporation; British East African Meterological Service; British Electrical and Allied Industries Research Association; British Standards Institution; Canadian Department of Trade and Commerce; Central Electricity Board; Messrs. Chapman and Hall, Ltd.; H. J. B. Chapple, B.Sc.(Eng.); B. C. Chatterjee; The Chief Inspector of Factories and Workshops; A. E. Clayton, D.Sc.; Commercial Motor Users Association (Inc.); Messrs. Constable and Co., Ltd.; H. Cotton, M.B.E., D.Sc.; Department of Scientific and Industrial Research; Derby Society of Engineers; H. M. Dowsett; Electric Supply Authority Engineers' Association of New Zealand; Electrical Association for Women; Electrical Association of Japan; Electrical Contractors' Association (Inc.); "The Electrical Review"; The Electricity Board for Northern Ireland; The Electricity Commissioners; Electricity Supply Commission, S. Africa; "Engineering"; Federation of British Industries; W. Fennell; Prof. Sir Ambrose Fleming, M.A., D.Sc., F.R.S.; Messrs. Gauthier-Villars et Cie; Messrs. V. Gollancz, Ltd.; Sir R. A. Hadfield, Bart., F.R.S.; H. R. Harper; H. H. Harrison, M.Eng.; J. Henderson, M.C., B.Sc.; Messrs. Hermann et Cie; Hydro-Electric Power Commission of Ontario; W. S. Ibbetson, B.Sc.; Incorporated Municipal Electrical Association; Indian Posts and Telegraphs Department; Institute for Research in Agricultural Engineering, Oxford; Institution of Engineers, Australia; International Electrotechnical Commission; International Tin Research and Development Council; Prof. A. E. Kennelly, D.Sc.; The Ketton Portland Cement Co., Ltd.; Messrs. W. King, Ltd.; London and Home Counties Joint Electricity Authority; Messrs. Longmans, Green and Co.; Messrs. Macmillan and Co., Ltd.; Massachusetts Institute of Technology; J. W. Meares, C.I.E.; Meteorological Office; Messrs. Methuen and Co., Ltd.; F. L. Milne; Mines Department; E. Molloy; S. G. Monk, M.Sc.(Eng.), B.Sc.; National Electrical Manufacturers' Association, N.Y.; Messrs. Neumeyer Aktiengesellschaft; New Zealand Hydro-Electric Development; New Zealand Post and Telegraph Department; Messrs. George Newnes, Ltd.; Norway Watercourse and Electricity Service; Messrs. Odhams Press, Ltd.; Oxford

University Press; Prof. L. S. Palmer, M.Sc., Ph.D.; The Hon. G. L. Parsons; Sir Leonard Pearce, C.B.E., D.Sc.; Messrs. Sir Isaac Pitman and Sons, Ltd.; Public Works, Roads and Transport Congress and Exhibition; The Quasi-Arc Co., Ltd.; Rand Water Board, S.A.; J. L. Rowbotham; Royal Alfred Observatory, Mauritius; Royal Society of Arts; A. R. Rubin; Science Museum; J. Scott-Taggart, M.C.; Messrs. Siemens-Schuckert (Great Britain), Ltd.; Prof. S. P. Smith, D.Sc.; Messrs. E. and F. N. Spon, Ltd.; Svenska Teknologforeningen; Tasmanian Hydro-Electric Commission; S. P. Tchernoguboski; Union des Exploitations Électriques en Belgique; Union des Syndicats de l'Électricite; Prof. M. Walker, M.A., D.Sc., F.R.S.; A. R. Watson; Messrs. Williams and Norgate, Ltd.; T. P. Wilmshurst, M.B.E.; World Petroleum Congress; and A. P. Young, O.B.E.

A list of candidates for election and transfer, approved by the Council for ballot, was taken as read and was ordered to be suspended in the Hall.

The President announced that, during the month of December, 67 donations and subscriptions to the Benevolent Fund had been received, amounting to £129. A vote of thanks was accorded to the donors.

A paper by Mr. A. Monkhouse, Member, entitled "Electrical Developments in the U.S.S.R." (see vol. 76, page 601), was read and discussed.

The meeting terminated at 8 p.m. with a vote of thanks to the author, which was moved by the President and carried with acclamation.

878TH ORDINARY MEETING, 17TH JANUARY, 1935.

Prof. W. M. Thornton, O.B.E., D.Sc., D.Eng., President, took the chair at 6 p.m.

The minutes of the Ordinary Meeting held on the 10th January, 1935, were taken as read and were confirmed and signed.

A paper by Messrs. J. L. Pearson, B.A., Ph.D., G. Nonhebel, B.A., B.Sc., and P. H. N. Ulander, entitled "The Removal of Smoke and Acid Constituents from Flue Gases by a Non-Effluent Water Process" (see page 1), was read and discussed.

The meeting terminated at 8 p.m. with a vote of thanks to the authors, which was moved by the President and carried with acclamation.

879TH ORDINARY MEETING, 24TH JANUARY, 1935.

Prof. W. M. Thornton, O.B.E., D.Sc., D.Eng., President, took the chair at 6 p.m.

The minutes of the Ordinary Meeting held on the 17th January, 1935, were taken as read and were confirmed and signed.

The President announced that the Council had elected the Rt. Hon. Lord Hirst of Witton an Honorary Member of the Institution, and that the thirteenth award of the Faraday Medal had been made to Dr. F. B. Jewett (New York).

Messrs. S. B. Jackson and J. M. Wallace were appointed scrutineers of the ballot for the election and transfer of members and, at the end of the meeting, the President reported that the members whose names appeared on the list (see vol. 76, page 347), had been duly elected and transferred.

A list of candidates for election and transfer, approved by the Council for ballot, was taken as read and was ordered to be suspended in the Hall.

A paper by Mr. T. P. Preist entitled "Electrical Control of Road Traffic by Vehicle Actuation" (see page 149), was read and discussed.

The meeting terminated at 7.55 p.m. with a vote of thanks to the author, which was moved by the President and carried with acclamation.

880TH ORDINARY MEETING, 14TH FEBRUARY, 1935.

Prof. W. M. Thornton, O.B.E., D.Sc., D.Eng., President, took the chair at 6 p.m.

The minutes of the Ordinary Meeting held on the 24th January, 1935, were taken as read and were confirmed and signed.

Messrs. H. Duckworth and P. P. Wheelwright were appointed scrutineers of the ballot for the election and transfer of members and, at the end of the meeting, the President reported that the members whose names appeared on the list (see vol. 76, page 459) had been duly elected and transferred.

The President announced that, during the month of January, 2 977 donations and subscriptions to the Benevolent Fund had been received, amounting to £1 401. A vote of thanks was accorded to the donors.

A paper by Mr. Ralph Poole, Associate Member, entitled "The Application of Propeller Fans to the Cooling of Electrical Machines" (see page 293), was read and discussed.

The meeting terminated at 7.35 p.m. with a vote of thanks to the author, which was moved by the President and carried with acclamation.

881ST ORDINARY MEETING, 28TH FEBRUARY, 1935.

Mr. H. T. Young, Vice-President, in the absence of the President through indisposition, took the chair at 6 p.m.

The minutes of the Ordinary Meeting held on the 14th February, 1935, were taken as read and were confirmed and signed.

A list of candidates for election and transfer, approved by the Council for ballot, was taken as read and was ordered to be suspended in the Hall.

A paper by Mr. J. A. Sumner, Associate Member, entitled "Private Plants and Public Supply Tariffs" (see page 310), was read and discussed.

The meeting terminated at 7.35 p.m. with a vote of thanks to the author, which was moved by the chairman and carried with acclamation.

INSTITUTION NOTES.

I.E.E. Wiring Regulations.

A supplement (dated 20th June, 1935), containing alterations and additions to various clauses, has been issued in connection with the Tenth Edition of the I.E.E. Regulations for the Electrical Equipment of Buildings. Copies of the supplement can be obtained free of charge, on application to the Secretary, for insertion in existing copies of the Regulations.

Transmission Section.

The scrutineers appointed at the meeting of the Transmission Section held on the 15th May, 1935, have reported to the Chairman of the Section that the result of the ballot to fill the vacancies which will occur in the Committee on the 30th September next is as follows:—

Chairman: Mr. W. Fennell.
Vice-Chairman: Dr. P. Dunsheath, O.B.E., M.A.
Ordinary Members of Committee: Mr. W. C. Bexon, Mr. F. W. Main, Mr. F. W. Purse, and Mr. S. R. Siviour.

Conversazione of Overseas Members.

A Conversazione of members from overseas and their ladies was held in the Institution building on Thursday, 11th July, the total attendance being over 150. The proceedings commenced with a reception by the President (Prof. W. M. Thornton, O.B.E., D.Sc., D.Eng.) and the Council. This was followed by short addresses delivered in the lecture theatre by Mr. H. R. Harper (Chairman and Hon. Secretary, Victoria Local Committee) on "Brown Coal Development in Victoria," and by Mr. R. A. Watson Watt, B.Sc.(Eng.), on "The Cathode-Ray Oscillograph." A reunion then took place in the library, during which there were the following demonstrations:—

A repetition of the experiments illustrating Mr. Watson Watt's address on the cathode-ray oscillograph. (These illustrated the principal properties of the tube, with its facilities for two-dimensional indication and for the electrical modulation of the light intensity.)

A demonstration illustrating some investigations into the economic production of effective street lighting, arranged by Mr. Clifford C. Paterson, O.B.E.

A demonstration of effects capable of being produced by certain electric discharge lamps, arranged by Mr. Clifford C. Paterson, O.B.E.

A demonstration of a reflection meter, arranged by Mr. W. P. Digby, serving to measure the tarnishing and corrosion of metals, variations in the content of sulphur and certain other impurities in the atmosphere, and the fading of paints due to ultra-violet light, immersion in fluids, etc.

A demonstration of a synchronous time system with automatic control for marine use, arranged by Mr. Patrick Hamilton.

The following members from overseas were present: Mr. I. M. E. Aitken (*British Guiana*), Mr. E. B. Banks (*India*), Mr. H. G. Barker (*Straits Settlements*), Mr. H. C. Bowker, B.Sc.(Eng.), Ph.D. (*Nigeria*), Mr. F. la T. Budgett, B.Sc.(Eng.) (*United States*), Mr. H. Burkinshaw (*India*), Mr. M. P. Croxford, B.Sc. (*Federated Malay States*), Mr. E. S. Evans (*Australia*), Mr. G. Floyd (*India*), Mr. A. G. Forgan, B.Sc.(Eng.) (*India*), Mr. A. D. Foster, M.Sc. (*South Persia*), Mr. R. H. Gray (*Belgium*), Mr. A. Hakim (*India*), Mr. H. R. Harper (*Australia*), Lieut.-Col. N. Harrison, C.M.G., D.S.O. (*South Africa*), Mr. G. B. Hayward, B.A. (*Spain*), Mr. A. W. C. Hirst, B.Sc.(Eng.) (*Switzerland*), Mr. F. T. Homan (*India*), Mr. L. A. Hoyle (*India*), Mr. R. E. McInnes (*New Zealand*), Mr. P. G. Moore (*India*), Mr. C. W. W. Prescott (*Australia*), Mr. H. W. Puttick (*India*), Mr. F. H. Robinson (*Straits Settlements*), Mr. W. B. Roe (*India*), Mr. H. B. Sale (*Argentina*), Mr. H. G. Sale (*India*), Mr. W. A. Taylor (*Straits Settlements*), Mr. L. G. H. Turner (*India*), Mr. A. R. Tyrer (*Southern Rhodesia*), Mr. L. T. Wakeford (*Straits Settlements*), Prof. W. A. Wales, B.Sc.(Eng.) (*India*), Mr. A. W. M. Watkins (*New Zealand*), Mr. C. R. Webb (*China*), Mr. J. Weissenbach (*Switzerland*), and Mr. V. H. Winson, B.Sc.(Eng.) (*Federated Malay States*).

A similar Conversazione will be held each year towards the middle of the summer, and members from overseas who expect to be in England about that time and would like to be present should notify the Secretary in order that invitations may be sent to them.

Overseas Members and the Institution.

During the period 1st June to 31st August, 1935, the following members from overseas called at the Institution and signed the "Attendance Register of Overseas Members":—

Baly, W. F. (*Buenos Aires*).
Batty, H. (*Port of Spain, Trinidad*).
Bayley, G. G. (*Rawalpindi, India*).
Beaglehole, K., B.Sc. (*Wellington, N.Z.*).
Berenbaum, A. (*Haifa, Palestine*).
Borissow, B. G., B.Sc. (Eng.) (*Buenos Aires*).
Boscolo, G. R. (*Calcutta*).
Bowker, H. C., B.Sc.(Eng.), Ph.D. (*Lagos*).
Buckley, C. F. (*Sliema, Malta*).
Cherry, D. M. (*Auckland, N.Z.*).
Croxford, M. P., B.Sc. (*Batu Gajah, F.M.S.*).
Dickens, T. A. J. (*Kurow, N.Z.*).
Dowson, H. (*Swanbourne, W. Australia*).
Flemons, S. (*Shanghai*).
Forsyth, J. L. (*Krugersdorp, Transvaal*).
Garry, F. K., B.E. (*Auckland, N.Z.*).
Geare, H. W. (*Cape Town*).
Hambleton, E. P. (*Greymouth, N.Z.*).
Harris, L. M., B.Sc. (*Melbourne*).
Harris, V. A., B.Sc.Tech. (*São Paulo, Brazil*).
Henderson, G. P. (*Buenos Aires*).
Homan, F. T. (*Calcutta*).
Inwood, G. S. (*Christchurch, N.Z.*).
Kennelly, Prof. A. E., D.Sc. (*Cambridge, Mass.*).

King, W. L. (*Bloemfontein*).
Kirsten, K. H., B.Sc. (*Johannesburg*).
Leak, B. W. (*Cairo*).
Logan, T. B. (*Christchurch, N.Z.*).
Macdonald, H. B. (*Christchurch, N.Z.*).
Moore, P. G. (*Cawnpore*).
Muir, E. C. (*Shanghai*).
Munday, G. W. (*Buenos Aires*).
Page, T. H. D. (*Johannesburg*).
Pheasant, J. W. A. (*Calcutta*).
Phillips, W. E., B.Sc. (*Durban*).
Puttick, H. W. (*Dhanbad, India*).
Sale, H. G. (*Bombay*).
Sharpley, Prof. F. W., F.R.S.E. (*Dhanbad, India*).
Steel, C. S. (*Calcutta*).
Tyrer, A. R. (*Salisbury, S. Rhodesia*).
Vaux, A. (*Cape Town*).
Websdale, Major G. J., M.C. (*Almeria, Spain*).

Scholarships.

The following Scholarships have been awarded by the Council for 1935:—

Ferranti Scholarship (*Annual Value* £250; *tenable for* 2 *years*).

C. H. W. Clark, M.Sc.(Eng.) (Queen Mary College, London).

Duddell Scholarship (*Annual Value* £150; *tenable for* 3 *years*).

D. J. G. Richards (Port Talbot County Schools).

Silvanus Thompson Scholarship (*Annual Value* £100, *plus tuition fees*; *tenable for* 2 *years*).

G. King (Messrs. Callender's Cable and Construction Co., Ltd.).

Swan Memorial Scholarship (*Value* £120; *tenable for* 1 *year*).

A. C. W. V. Clarke, B.Sc. (King's College, London).

David Hughes Scholarship (*Value* £100; *tenable for* 1 *year*).

T. F. Monahan (Leeds University).

Salomons Scholarships (*Value* £50 *each*; *tenable for* 1 *year*).

C. G. Longford (Birmingham University).
J. E. Parton (Birmingham University).

Paul Scholarship (*Annual Value* £50; *tenable for* 2 *years*).

F. E. Burton (Regent Street Polytechnic, London).

Thorrowgood Scholarship (*Annual Value* £25; *tenable for* 2 *years*).

H. W. Hadaway (London Passenger Transport Board).

War Thanksgiving Education and Research Fund (No. 1).

Grants of £50 each have been made for 1935–1936 to the following for research purposes:—

H. Barker (Leeds University).
W. E. Harper (Birmingham University).

Graduateship Examination Results: May, 1935.

*Passed.**

Allerston, P. (South Farnborough, Hants).
Angus, J. H. (Mitcham).
Anwar-Ali, C. (Newcastle-on-Tyne).
Ashworth, J. E. N. (Lytham).
Bahree, I. C. (Wolverton).
Beckett, D. R. (Exeter).
Bennett, F. I. (Cardiff).
Blakeley, A. R. (London).
Blomfield, F. W. R. (Norwich).
Blue, J. L. (Glasgow).
Chapman, J. H. (Sileby).
Clark, P. F. (Shere).
Collins, J. (Birmingham).
Craig, W. M. (Stafford).
Crook, W. E. (Hamble, Hants).
Dabbs, S. W. (London).
Edgecombe, P. J. E. (Kingston-upon-Thames).
George, J. R. (Rhondda).
Halliday, D. H. (Stafford).
Harris, J. B. (London).
Hibell, A. R. (Hastings).
Hubbard, N. S. (Norwich).
Jones, N. W. (Dursley).
Joyner, W. S. (Nottingham).
Keen, J. H. (Cheadle Hulme).
Kennedy, B. (Nottingham).
Kennedy, J. D. (Peterborough).
Lacey, R. E. (London).
Latham, A. (Blackburn).
Leek, T. W. (Liverpool).
Lumsdaine, W. N. (Edinburgh).
MacHutchon, I. F. (Hebburn-on-Tyne).
McKie, N. A. M. (Annan).
Matthews, K. R. (Eastbourne).
Meredith, D. L. (Cheltenham).
Mills, W. H. (Aberdare).
Minter, G. (Birmingham).
Morgan, F. (Wolverhampton).
Nicholls, W. J. (Bristol).
Nurse, R. A. (Nottingham).
Oram, G. C. (Llanelly).
Parker, G. P. (Wallington).
Peirson, G. F. (Coventry).
Pulvermacher, F. H. (Cheshunt).
Roberts, W. G. (Sutton, Surrey).
Rowland, F. G. (Seighford, Stafford).
Roy, D. W. (Edinburgh).
Sharpe, E. (Gloucester).
Simmonds, J. C. (Lincoln).
Slatter, J. R. (Swindon).
Smith, A. (Port Talbot).
Smith, A. N. (Welling, Kent).
Tapson, E. L. (St. Leonards-on-Sea).
Taylor, C. H. (London).
Thompson, F. A. (Hartlepool).
Tickell, F. H. (Darlington).
Turner, T. F. (Newcastle-on-Tyne).
Wakefield, J. A. P. (Newcastle-on-Tyne).
Walker, E. H. (Long Eaton).
Whyte, A. McG. (Barrow-in-Furness).
Wilkinson, F. R. (Sutton Coldfield).
Wilson, L. J. (Ashford, Kent).
Wilson, W. (Chester).
Winstanley, E. (Stafford).
Woods, D. (London).

Passed Part I only.

Adlington, L. (Chesterfield).
Boud, F. H. (Bedford).
Brownjohn, R. (Manchester).
Burgess, R. A. (Bristol).
Chao, S. Y. (Lincoln).
Coutts, J. A. (Gourock).
Coysh, J. W. (Hull).
Davies, R. J. (Newport, Mon.).
Eldred, E. M. (Dundee).
Fletcher, H. D. (Loughborough).
Gray, J. A. (Boston).
Hayward, B. T. (Bournemouth).
Holden, H. S. (Sheffield).

* This list also includes candidates who are exempt from, or who have previously passed, a part of the Examination and have now passed in the remaining subjects.

Passed Part I only—continued.

Hughes, C. J. (Liverpool).
Loraine, T. E. (Darlington).
Macdonald, N. A. (Belfast).
McKibbin, E. L. (Douglas, Isle of Man).
Moore, G. J. (Wattsville, Mon.).
Pace, H. O. (Ellesmere Port).
Renton, J. E. (Blyth).
Ruffhead, H. E. (London).
Shashoua, S. (Blackpool).
Watson, S. N. (Whitley Bay).

Passed Part II only.

Ball, R. (Coalville).
Baume, H. (Dewsbury).
Brown, F. O. (Birmingham).
Burke, A. J. (Leadgate).
Byrnes, J. W. (Hull).
Caister, P. R. (Ashford, Kent).
Chambers, A. G. (Loughborough).
Collins, I. L. (Cardiff).
Davis, J. H. (Sherwood, Nottingham).
Dawson, M. W. (Derby).
Driscoll, L. (Darley Abbey, Derby).
Foster, M. G. (Croydon).
Hazell, S. W. (London).
Hope, R. (Hyde).
Parsons, A. J. (Dursley).
Purvey, A. (Loughborough).
Shaw, T. (Wallsend-on-Tyne).
Shuttleworth, H. T. (Woolston).
Simpkins, R. A. (Hull).
Sood, B. S. (Newcastle-on-Tyne).

Informal Meetings.

176th Informal Meeting (14th January, 1935).

Chairman: Mr. M. Whitgift.

Subject of Discussion: "Economics of Electric Cooking" (introduced by Mr. W. N. C. Clinch).

Speakers: Mesdames V. Brice, V. O. Ripley, and H. E. Bell, and Messrs. A. Morgan, W. A. Gillott, A. N. D. Kerr, W. A. Erlebach, B.Sc.(Eng.), H. M. Sayers, A. C. Cramb, N. F. Saunders, H. Hobbins, H. H. Long, L. A. Harris, F. Jackson, D. G. W. Acworth, J. I. Bernard, B.Sc.Tech., J. Aylmer, M.C., and H. Bourne.

177th Informal Meeting (28th January, 1935).

Chairman: Mr. G. F. Bedford, B.Sc.

Subject of Discussion: "The Electrical Warming and Air-conditioning of, and the Supply of Hot Water to, Large Buildings" (introduced by Mr. R. Grierson and Mr. D. Betts).

Speakers: Messrs. J. R. Bedford, T. G. Partridge, F. Jackson, J. Lesser, A. N. D. Kerr, D. B. Williamson, H. E. Barnett, J. M. Kennedy, P. L. Newman, and W. J. Minton.

178th Informal Meeting (11th February, 1935).

Chairman: Lieut.-Col. A. G. Lee, O.B.E., M.C.

Subject of Discussion: "The I.E.E. Wiring Regulations, 10th Edition" (introduced by Mr. H. J. Cash).

Speakers: Messrs. J. R. Bedford, N. S. Richardson, A. N. D. Kerr, H. White, H. H. Long, F. Jackson, —. Smith, H. C. Cooper, A. Davidson, H. Bright, F. B. Walbery, E. G. Goble, —. Starling, J. I. Bernard, B.Sc.Tech., E. Jacoby, H. Southey, E. H. Freeman, P. Higgs, W. Rawlings, A. F. W. Richards, A. Morgan, F. H. D. Sewell, P. Good, and S. M. H. Evans.

The following replied to the points raised in the discussion: Messrs. R. W. L. Phillips, S. W. Melsom, E. Ridley, and H. J. Cash.

179th Informal Meeting (4th March, 1935).

Chairman: Mr. F. Jervis Smith.

Subject of Discussion: "Small Rectifiers" (introduced by Mr. G. F. Bedford, B.Sc., Mr. S. A. Stevens, and Mr. H. Rissik, B.Sc.(Eng.).

Speakers: Messrs. A. Morgan, P. G. A. H. Voigt, B.Sc.(Eng.), R. S. G. Terry, B.Sc.(Eng.), G. H. Fowler, A. J. Bousfield, N. Edwards, E. S. Ritter, J. H. Greenwood, —. Donaldson, J. R. Bedford, P. P. Wheelwright, and L. S. Crutch, B.Sc.(Eng.).

180th Informal Meeting (18th March, 1935).

Chairman: Mr. A. F. W. Richards.

Subject of Discussion: "Safety in Urban and Rural Consumers' Installations" (introduced by Mr. H. Leyburn, B.Sc.).

Speakers: Messrs. A. Morgan, F. Jackson, W. Cross, J. F. Shipley, F. W. Shilstone, F. E. Rowland, K. M. Mackenzie, H. Bright, C. F. Mounsdon, C. W. Prescott, D. A. Butcher, H. S. Moody, B.Sc.(Eng.), A. V. Cross, F. B. P. Pearce, and A. F. W. Richards.

181st Informal Meeting (1st April, 1935).

Chairman: Mr. E. L. Hefferman.

Subject of Discussion: "Change-over" (introduced by Mr. H. Brierley and Mr. A. F. W. Richards).

Speakers: Messrs. R. H. Rawll, A. F. Harmer, R. H. Upton, A. N. D. Kerr, P. H. Cobbold, A. Morgan, A. G. Hilling, L. M. Jockel, D. A. Butcher, and F. Jervis Smith.

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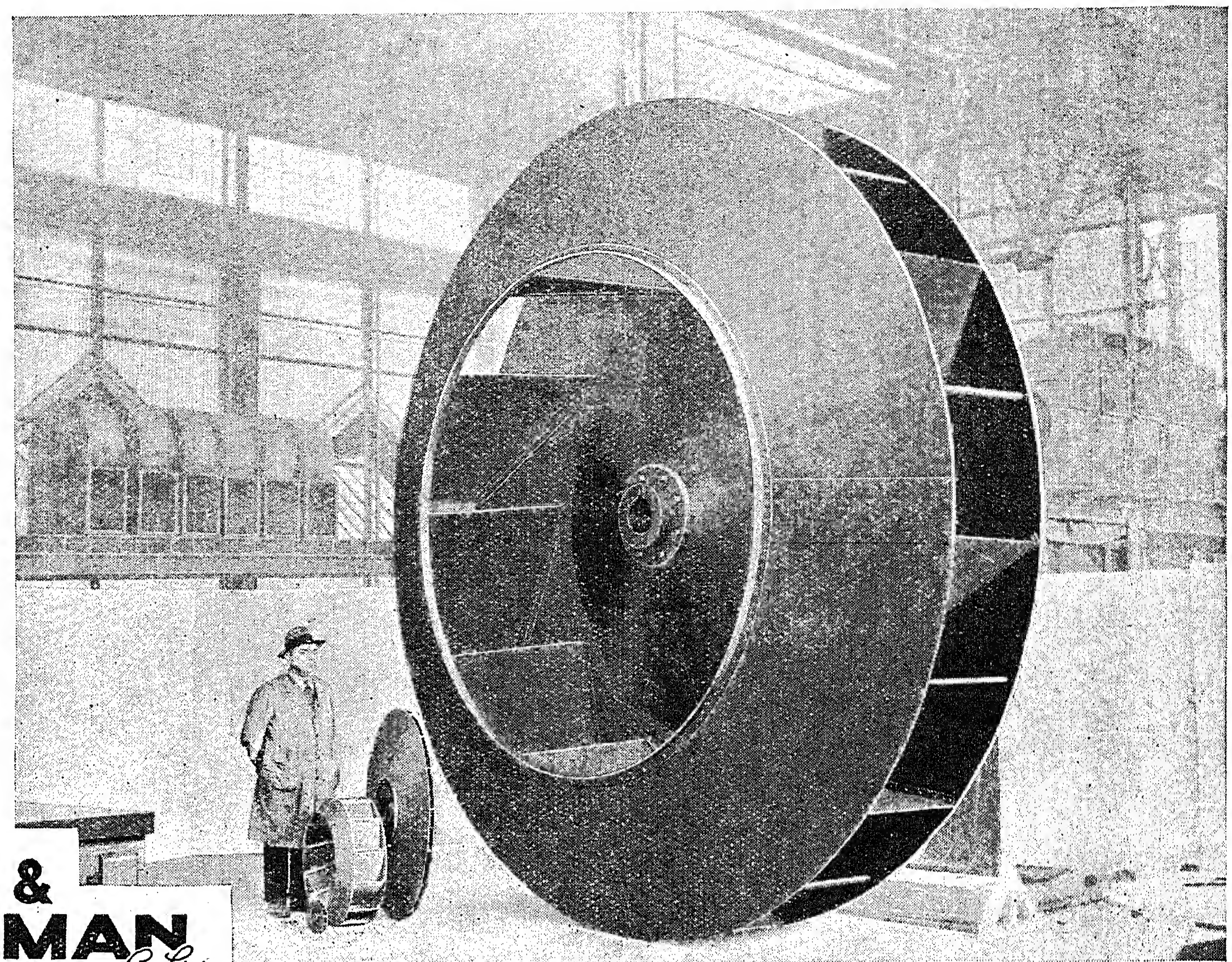

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0-1.2 "	0- 120 "	0- 10,000 "	0- 1.2 "	0- 480 "
0- 600 m.a.	0- 60 "	0- 1,000 "	0- 0.6 "	0- 240 "
0- 120 "	0- 12 "		0-120 milliamps.	0- 120 "
0- 60 "	0- 6 "		0- 60 "	0- 60 "
0- 12 "	0- 1.2 "			0- 12 "
0- 6 "	0- 600 millivolts.			0- 6 "
	0- 120 "			
	0- 60 "			

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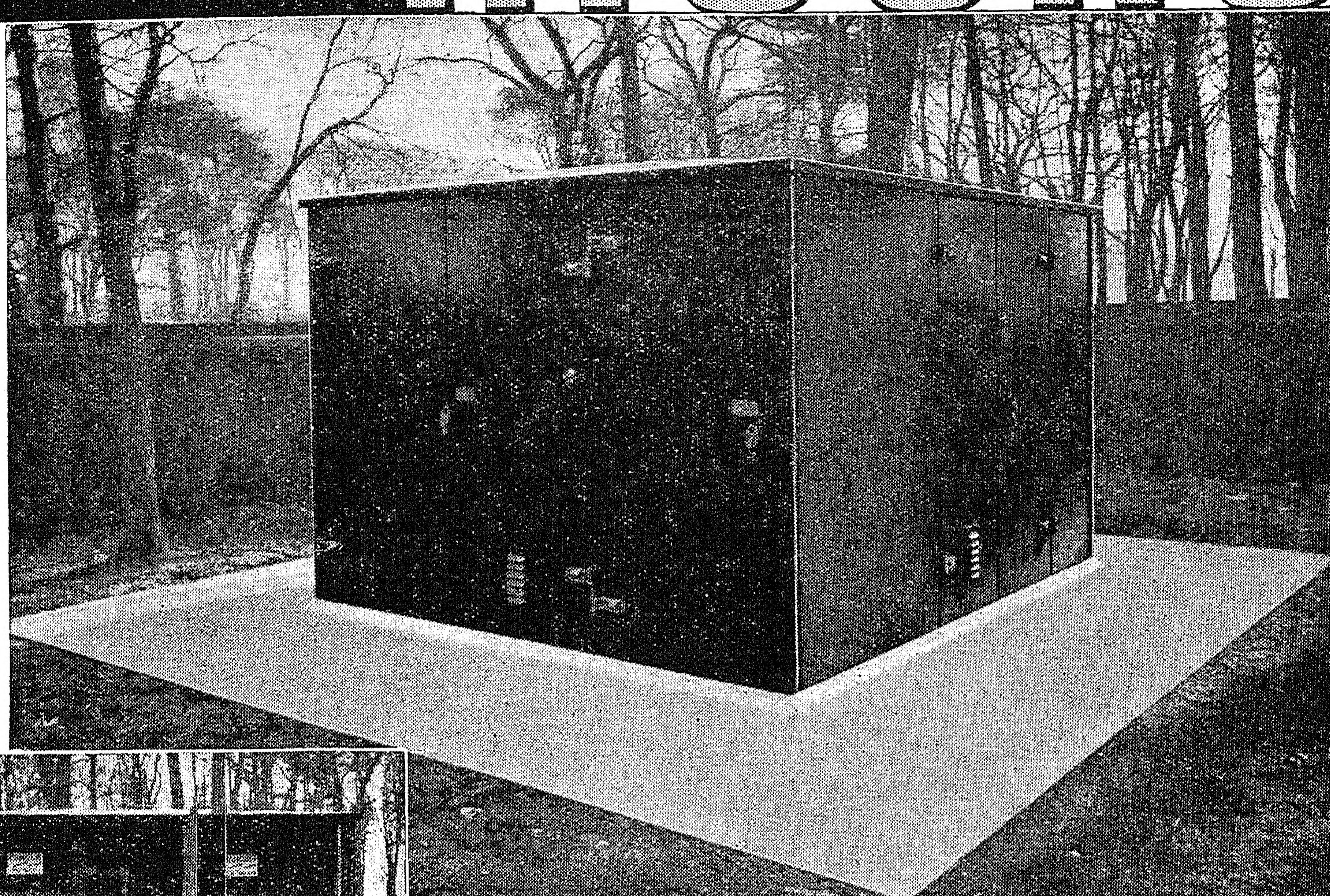

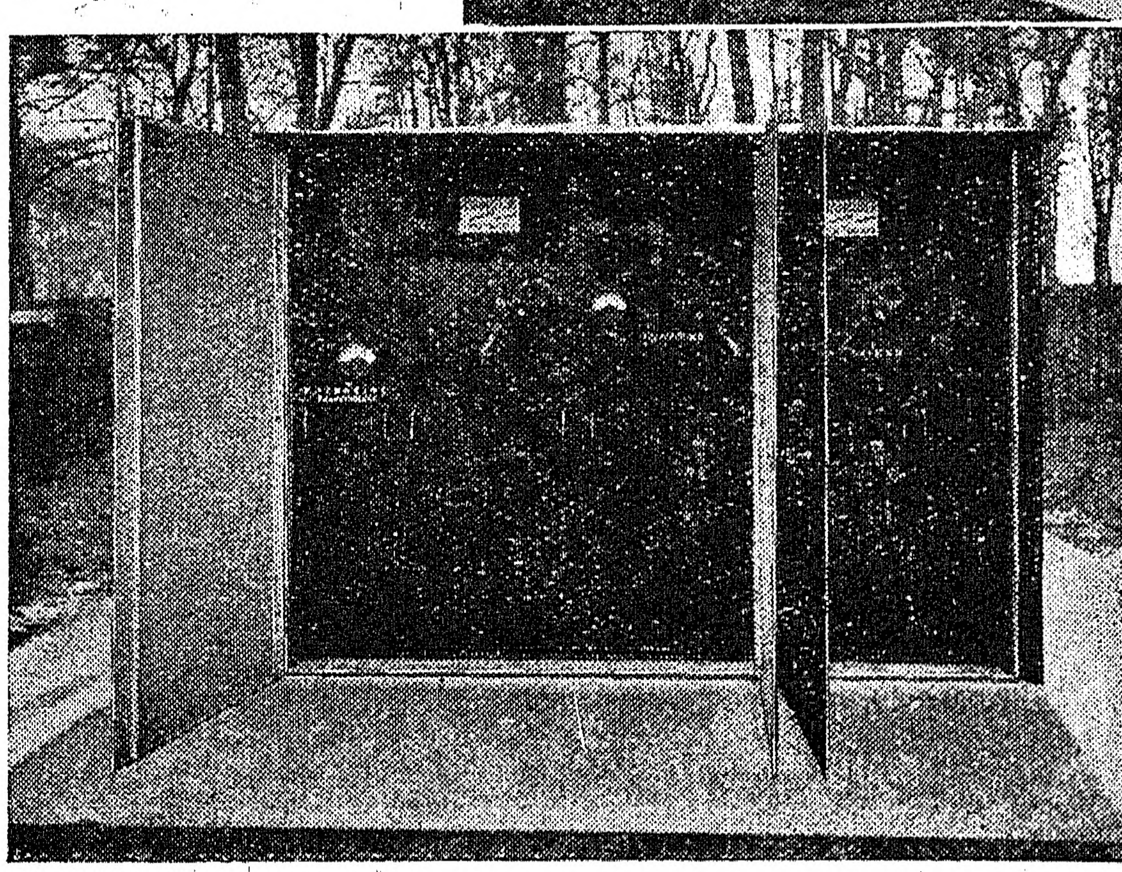

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